Lab 5: Count Min-Sketch: The Heavy Hitters Problem

Monday, October 15

CompSci 531, Fall 2018

Outline

- Review Big Data Streaming Model
  - Bloom Filters
- Application: The Heavy Hitters Problem
  - (Detecting Viral Google Searches)
- Streaming Data Structure: Count Min-Sketch

Big Data

- **Problem.** Too much data to fit in memory (e.g., who can store the internet graph?

Big Data

- **Problem.** Alternatively, maybe we *could* store our data, but it would take too long to process it, and we want a real time (or near real time) application.

Streaming Model

- **Solution.** In the **streaming model** of computation, we process the data one piece at a time, with limited memory.

- Equivalently: we develop algorithms that run in a *single* left to right pass over an array, with a small amount of auxiliary storage.

Football	Duke	Politics	News	...	Weather
0	1	2	3	…	T

Auxiliary Storage of size n
(n << T)

Bloom Filter

- We have already seen how to construct a *bloom filter,* a form of *lossy compression* (as opposed to lossless compression, e.g., Huffman).

- Answers *membership queries*; i.e., "Have I seen element x before in the stream?"

- Applications include:
  - Web browser checking for known malicious urls
  - Checking for "one hit wonders" in web caching (remember consistent hashing?)

Bloom Filter

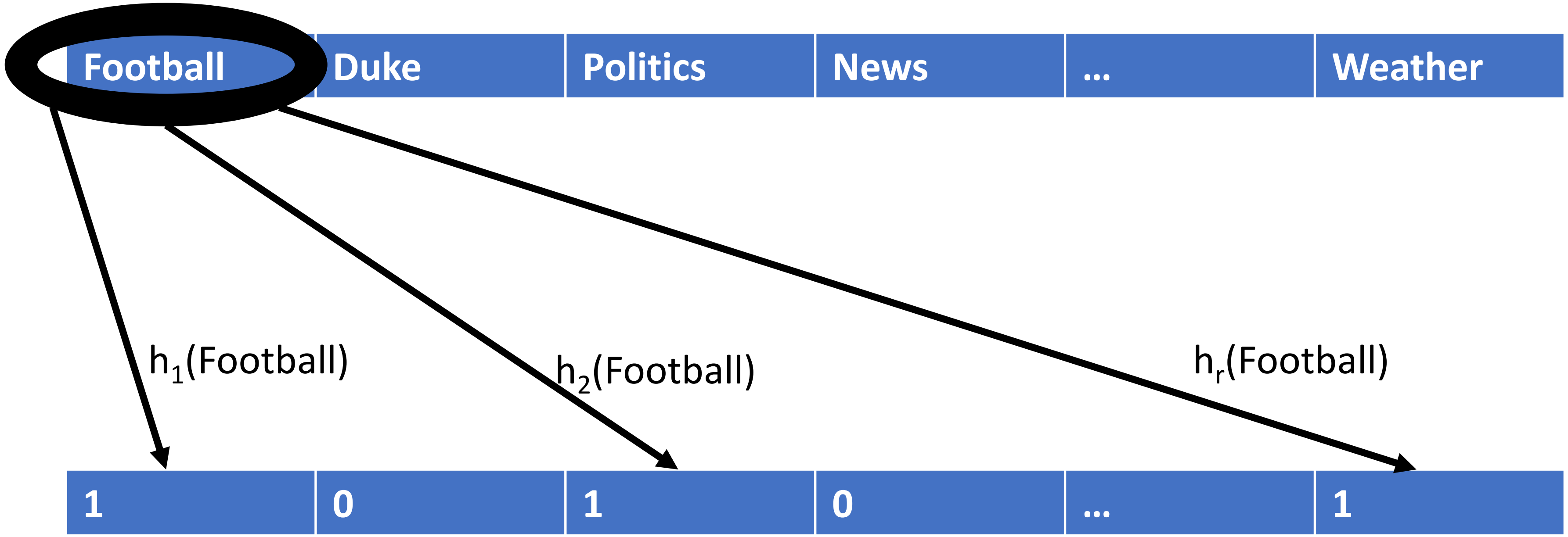
- Our auxiliary storage is just a hash table of size n. Initialize all values to 0.

- We also use r independent hash functions $h_1, \ldots, h_r$.

- Whenever we see an element x in the stream, set $h_1(x) = \ldots = h_r(x) = 1$.

- To check whether we have seen an element y:
  - If $h_1(y) = \ldots = h_r(y) = 1$, return True.
  - Else, return False.

Bloom Filter



Bloom Filter

- Guarantees:
  - If we *have* seen x, we always correctly output True.
  - If we *have not* seen x, we correctly output False with high probability.
- What if we want to remember more than just whether we have seen x?
- How about “How many times have we seen x?”

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Heavy Hitters Problem

- In particular, suppose we want to construct an algorithm for detecting viral google searches.
- There are a few billion google searches every day, and we'll say that a search is viral if it constitutes a constant fraction of those searches (e.g., 1%).
- Can we detect these viral google searches with a *single* pass over the stream of searches?

Heavy Hitters Problem

- We can formalize this as the **heavy hitters problem**.

- We are given a stream of length T and a parameter k.
  - Think of T >> k.

- In a single pass over the stream, we want to find any elements that appear at least T/k times.

Heavy Hitters Problem

- Bloom filters gets us part of the way there.

- In particular, if we had k=T, the heavy hitters problem is the membership problem.

- Thus, the heavy hitters problem is *at least* as hard (computationally, more on reductions later in the course) as the membership problem.

- Since we only had a correct algorithm with high probability for membership, we shouldn’t expect an exact answer here.

Heavy Hitters Problem

- Thus, we consider the ***ϵ*-approximate heavy hitters problem.** Still given a stream of length T and a parameter k (T >> k), but we are also given an "error tolerance" parameter ϵ.

- In a single pass over the stream using just O(1/ϵ) auxiliary storage, we want to output a list L of elements such that:
  - If x occurs at least T/k times in the stream, then x is in L.
  - If x is in L, then with high probability, x occurs at least T/k - ϵT times in the stream.
  - (e.g., if ϵ = 1/(2k), then we get O(k) storage and should satisfy: if x is in L, with high probability, x occurs at least T/2k times in the stream).

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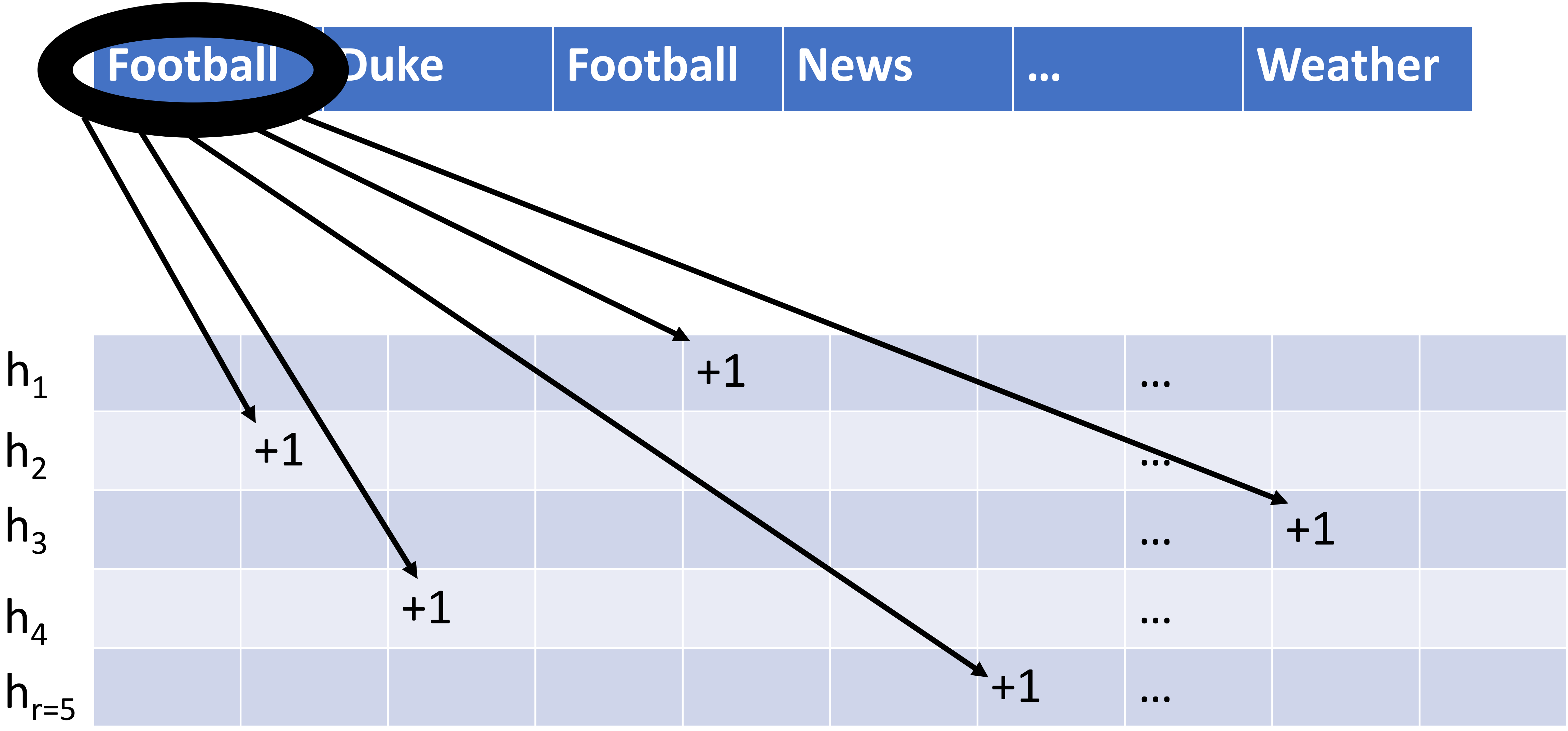
Count Min-Sketch

- **Big Idea.** Just build a bloom filter that can count.

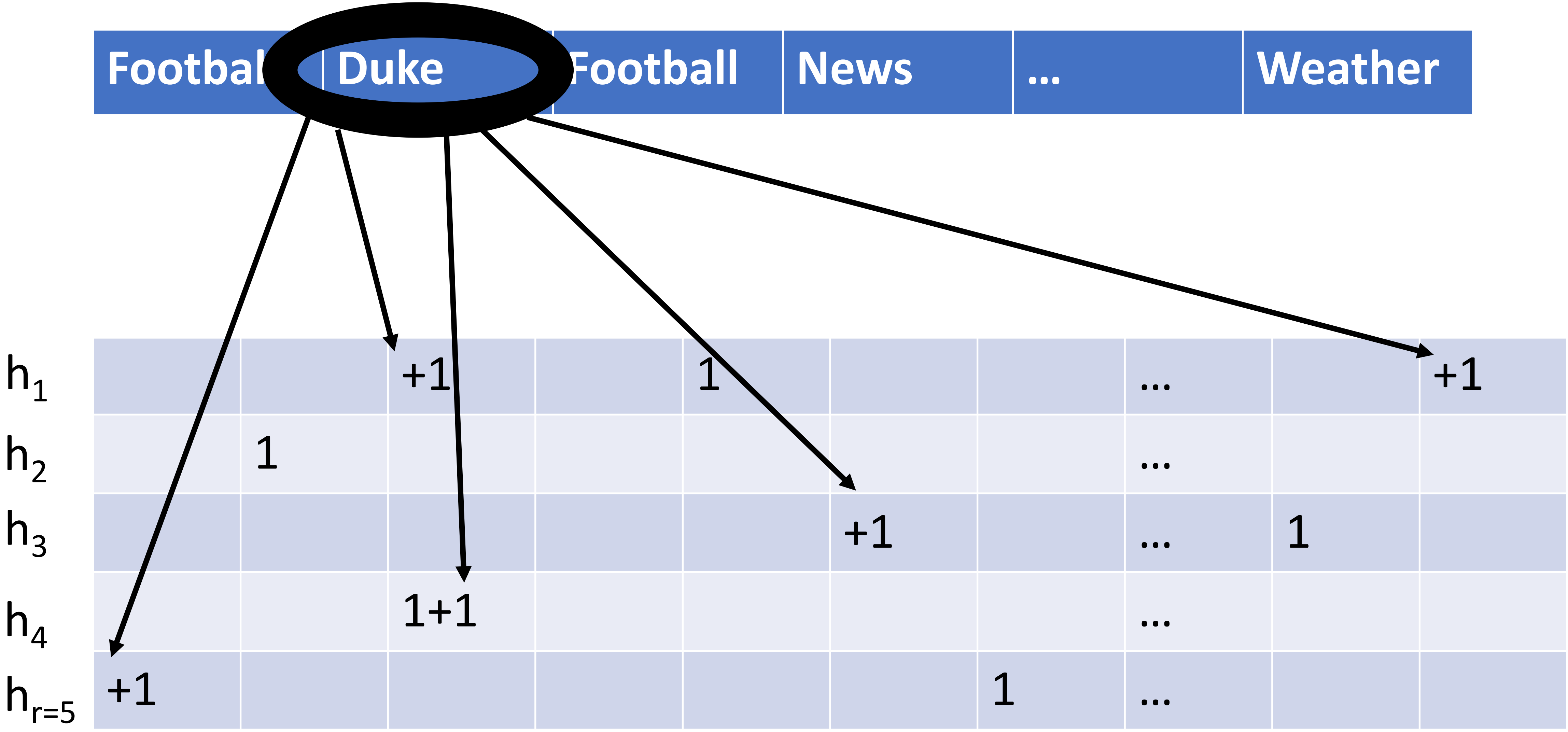
- Our auxiliary storage consists of *r* hash tables, each of size *n* and initialized to 0's, with corresponding *r* independent hash functions h_1, …, h_r.

- Whenever we see an element x in the stream:
  - For all i=1 to i=r: {$h_i(x) = h_i(x) + 1$}
  - if $\min_i h_i(x) \geq T/k$, add x to L.

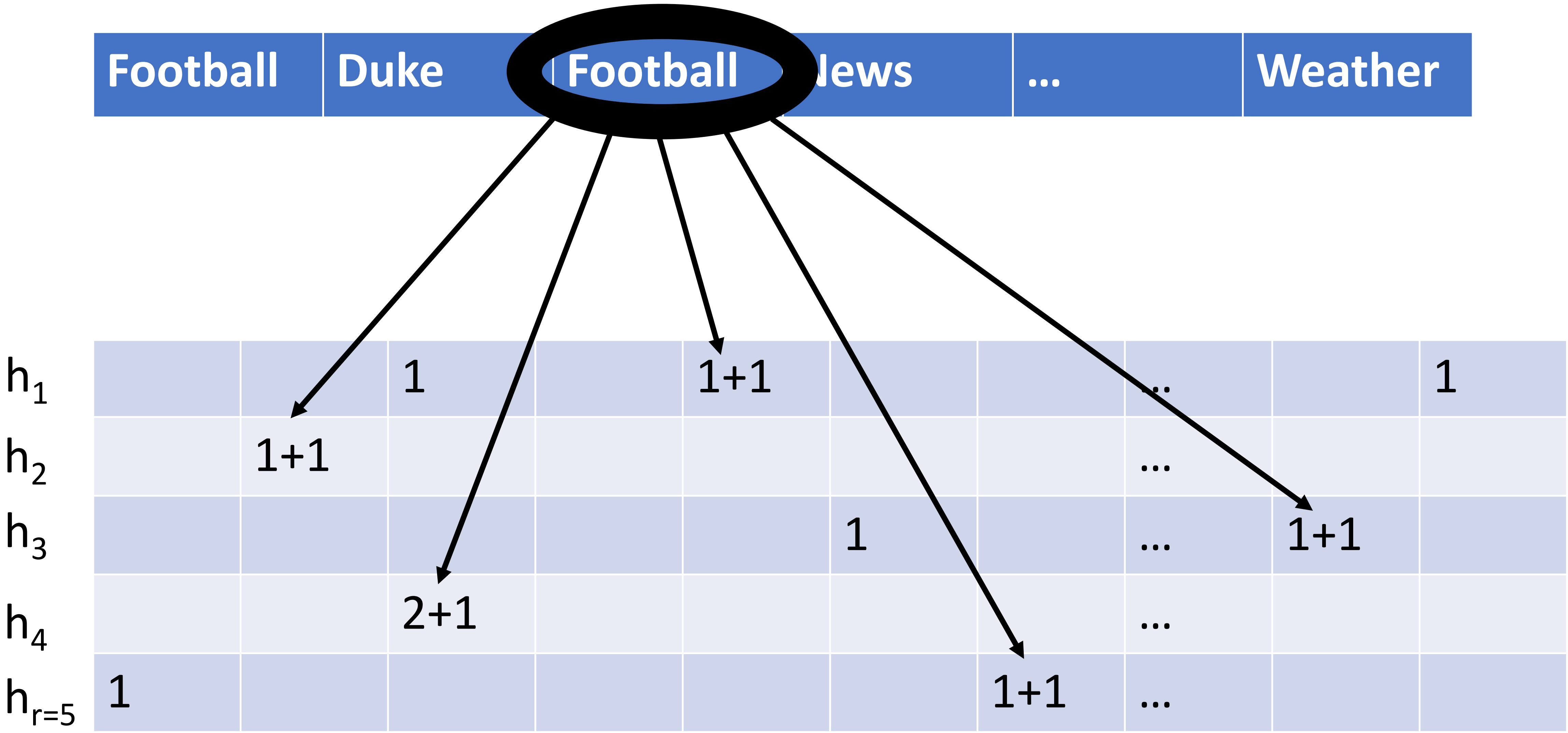
Count Min-Sketch



Count Min-Sketch



Count Min-Sketch



Count Min-Sketch

- Note that we occasionally *overestimate* frequencies, but we never *underestimate* frequencies.

- So it is easy to satisfy the first part of the heavy hitter's problem: "If x occurs at least T/k times in the stream, then x is in L."

- **Problem.** We need to argue that it is unlikely we overestimate so badly that we violate the other part: "If x is in L, then with high probability, x occurs at least T/k - ϵT times in the stream."

Count Min-Sketch

- Let f_x be the frequency (# of times appearing in stream) of element *x*.
- Let $\widehat{f_x}[1], \ldots, \widehat{f_x}[r]$ be our estimated frequencies, that is, $\widehat{f_x}[i] = h_i(x)$ at the end of our pass through the stream.
- Let $I_{x,y}[i]$ be an indicator random variable equal to 1 if $h_i(x) = h_i(y)$, and 0 otherwise.
- What is $\mathbb{E}\left[\widehat{f_x}[i]\right]$?

Count Min-Sketch

- We make the assumption of *universal hashing*: For all $x \neq y, \Pr\big(h(x) = h(y)\big) \leq \frac{1}{n}$.

$$\mathbb{E}\left[\widehat{f_x}[i]\right] = f_x + \mathbb{E}\left[\sum_{y \neq x} f_y \times I_{x,y}[i]\right]$$

$$= f_x + \sum_{y \neq x} f_y \, \mathbb{E}[\, I_{x,y}[i]]$$

$$= f_x + \sum_{y \neq \mathrm{x}} \frac{f_y}{n} \leq \; f_x + \frac{T}{n}$$

Count Min-Sketch

- Recall we want to use O($1/\epsilon$) storage: set n (size of each hash table) to $3/\epsilon$. Let ϵ = 1/(2k). Then
$$\mathbb{E}\left[\widehat{f_x}[i]\right] \leq f_x + \epsilon \frac{T}{3} = f_x + \frac{T}{6k}.$$

- To bound the probability that we get a large overestimate, we can use Markov's inequality: For any constant c > 1 and random variable X, $\Pr(X > c\,\mathbb{E}[X]) \leq \frac{1}{c}$. For c = 3/2,
$$\Pr\left(\widehat{f_x}[i] > \frac{3}{2}\,\mathbb{E}\left[\widehat{f_x}[i]\right] = \frac{3}{2} f_x + \frac{T}{4k}\right) \leq \frac{2}{3}.$$

Count Min-Sketch

- Recall however, that we output the *minimum* estimate. Exploiting the fact that the *r* hash functions are chosen *independently*:

$$\Pr\left(\min_i \widehat{f_x}[i] > \frac{3}{2}\,\mathbb{E}\left[\widehat{f_x}[i]\right]\right) = \Pr\left(\forall i,\ \widehat{f_x}[i] > \frac{3}{2}\,\mathbb{E}\left[\widehat{f_x}[i]\right]\right)$$

$$= \prod_i \Pr\left(\widehat{f_x}[i] > \frac{3}{2}\,\mathbb{E}\left[\widehat{f_x}[i]\right]\right)$$

$$\leq \left(\frac{2}{3}\right)^r$$

Count Min-Sketch

- Recall that the problem for ϵ = 1/2k is: we get O(k) storage and should satisfy: if x is in L, with high probability, x occurs at least T/(2k) times in the stream).
- Consider some x with $f_x < \frac{\mathrm{T}}{2\mathrm{k}}$. We have shown that

$$\Pr\left(\min_i \widehat{f_x}[i] > \frac{3T}{4k} + \frac{T}{4k} = \frac{T}{k}\right) \leq \left(\frac{2}{3}\right)^r .$$

- So if x is in L, then it occurs at least T/(2k) times in the stream with probability at least 1-(2/3)^r.
- So if we want an error with probability at most 2% (say), we just need to use $r = \lceil \log_{3/2}(50) \rceil = 10$ independent hash functions.

Count Min-Sketch

- In summary, we can use $20/\epsilon = O(1/\epsilon)$ space to:
  - find *all* elements that appear at least T/k times in the stream, and
  - output elements that appear less than T/2k times in stream with probability at most 2%.
  - And in practice, even fewer hash functions often suffice for good performance.
- Note that we can do all of this with just a *single* linear scan over the stream (and only constant time operations per element), and just $O(1/\epsilon)$ storage.
  - The amount of auxiliary storage we use is *completely* independent of T!

Count Min-Sketch

- **Food for Thought.** What if you didn’t know T beforehand?
  - Maybe this is just a real time application, and you want to maintain a list of any elements that are heavy hitters among what you have seen so far.