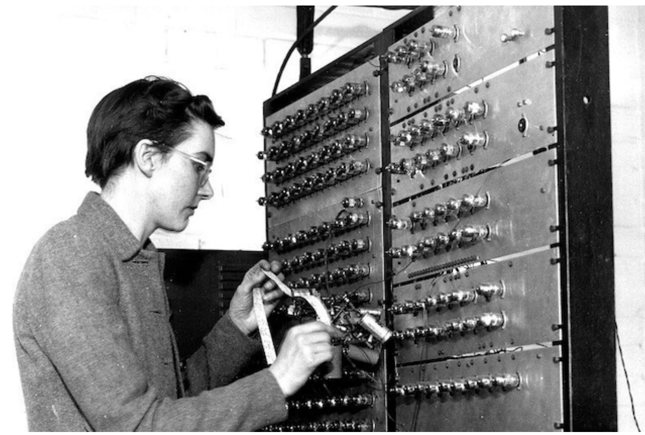
CompSci 201, L18: Tree Recursion

Person in CS: Kathleen Booth

- 1922 – 2022
- British Mathematician, PhD in 1950
- Worked to design the first *assembly language* for early computer designs in the 1950s
- May have been the first woman to write a book on programming
- Early interest in *neural networks*



Logistics, Coming up

- P4 Autocomplete due today 10/31
- APT8 (Tree problems) due this Wed., 11/2
- P5: Huffman releasing this week, due Monday 11/14

Three ways to recursively traverse a any binary tree (search or not)

- Difference is in where the non-recursive part is

inOrder

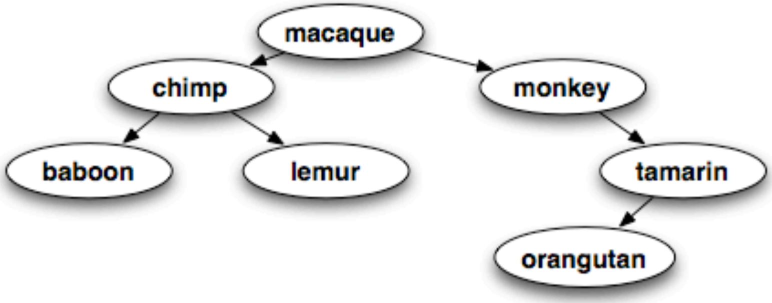
```
void inOrder(TreeNode t) {
  if (t != null) {
    inOrder(t.left);
    System.out.println(t.info);
    inOrder(t.right);
  }
}
```

preOrder

```
void preOrder(TreeNode t) {
  if (t != null) {
    System.out.println(t.info);
    preOrder(t.left);
    preOrder(t.right);
  }
}
```

postOrder

```
void postOrder(TreeNode t) {
  if (t != null) {
    postOrder(t.left);
    postOrder(t.right);
    System.out.println(t.info);
  }
}
```



Wrapper and recursive helper method to return List

```java
public ArrayList<String> visit(TreeNode root) {
    ArrayList<String> list = new ArrayList<>();
    doInOrder(root,list);
    return list;
}

private void doInOrder(TreeNode root, ArrayList<String> list) {
    if (root!= null) {
        doInOrder(root.left,list);
        list.add(root.info);
        doInOrder(root.right,list);
    }
}
```

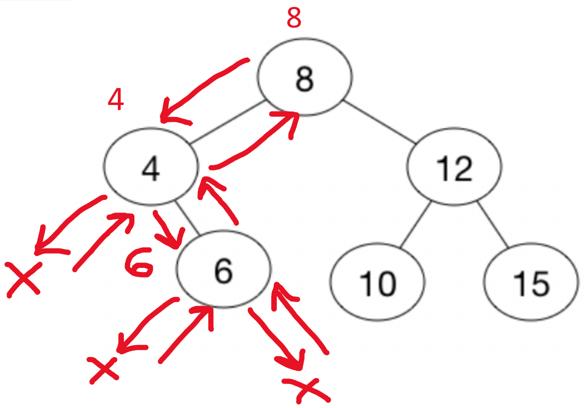
- In order traversal → list?
- Create list, call helper, return list
- values in returned list in order

TreeCount APT

Problem Statement

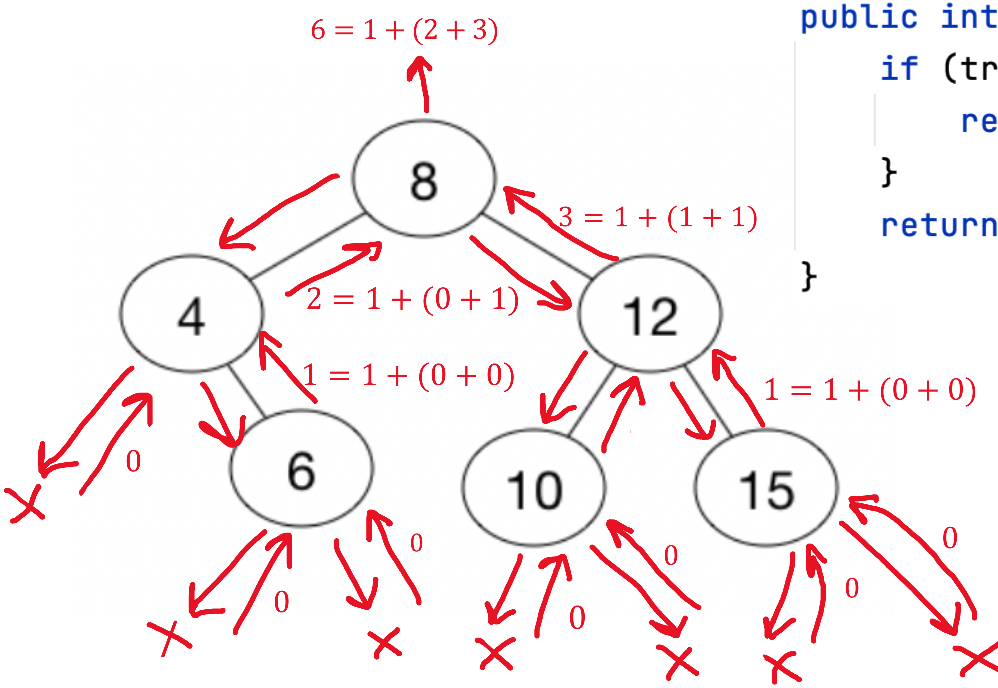
Write a method that returns the number of nodes of a binary tree. The TreeNode class will be accessible when your method is tested.

```
public class TreeCount {
    public int count(TreeNode tree) {
        // replace with working code
        return 0;
    }
}
```



is characterized by the pre-order string **8, 4, x, 6, x, x, 12, 10, x, x, 15, x, x**

Solving TreeCount in Picture & Code




```java
public int count(TreeNode tree) {
    if (tree == null) {
        return 0;
    }
    return 1 + count(tree.left) + count(tree.right);
}
```

WOTO

Go to duke.is/8xcjw

Not graded for correctness, just participation.

Try to answer *without* looking back at slides and notes.

But do talk to your neighbors!

Complexity of tree traversal

- Intuition: visit every node once and print it
  - If there are N nodes, should be O(N)
  - But what about recursive calls?

- More generally/formally:
  - We create a recurrence relation (an equation)
  - Solving the equation yields runtime

Analyzing Recursive Runtime

Develop a recurrence relation of the form

$$T(N) = a \cdot T\big(g(N)\big) + f(N)$$

Total runtime

Recursive call(s)

Non-recursive runtime

Where:

- $T(N)$ - runtime of method with input size *N*
- a is the number of recursive calls
- $g(N)$ - how much input size decreases on each recursive call
- $f(N)$ - runtime of non-recursive code on input size N

LinkedList Example: Runtime of Recursive Reverse

```java
public Node reverse(Node list) {
    if (list == null || list.next == null) {
        return list;
    }
    // in A->B->C->D, what does A point at?
    // afterMe -> D->C->B->null
    Node afterMe = reverse(list.next);
    list.next.next = list;
    list.next = null;
    return afterMe;
}
```

$g(N) = N - 1$

$f(N) = O(1)$

$$T(N) = T(N-1) + O(1)$$

Solving Recurrence Relation

$$
\begin{aligned}
T(N) &= T(N-1) + O(1) \\
&= \big(T(N-2) + O(1)\big) + O(1) \\
&= (T(N-3) + 3 \cdot O(1) \\
&\vdots \\
&= T(1) + N \cdot O(1) \\
&= O(N)
\end{aligned}
$$

Apply recurrence again to T(N-1)

Total runtime

T(1) is base case, just O(1)

Table of Recurrences

Recurrence	Algorithm	Solution
`T(n) = T(n/2) + O(1)`	binary search	`O(log n)`
`T(n) = T(n-1) + O(1)`	sequential search	`O(n)`
`T(n) = 2T(n/2) + O(1)`	tree traversal	`O(n)`
`T(n) = T(n/2) + O(n)`	qsort partition ,find k^{th}	`O(n)`
`T(n) = 2T(n/2) + O(n)`	mergesort, quicksort	`O(n log n)`
`T(n) = T(n-1) + O(n)`	selection or bubble sort	$O(n^2)$

We expect you to be able to derive a recurrence relation from an algorithm, but not necessarily to solve. We will provide a table of solutions like this for exams.

Recurrence relation and runtime for traversing a balanced tree

- ***T(n)*** time **`inOrder(root)`** with ***n*** nodes
  - T(n) = T(n/2) + O(1) + T(n/2) = O(n)

```java
public void inOrder(TreeNode root) {
    if (root != null) {
        inOrder(root.left);
        System.out.println(root.info);
        inOrder(root.right);
    }
}
```

- Why T(n/2)?

Assumes the tree is *balanced:* Same number of nodes in the left subtree as the right.

Recurrence relation and runtime for traversing an unbalanced tree

- If every node has a right child but no left…
  - $T(n) = T(0) + O(1) + T(n-1) = O(n)$

```java
public void inOrder(TreeNode root) {
    if (root != null) {
        inOrder(root.left);
        System.out.println(root.info);
        inOrder(root.right);
    }
}
```

- So Tree *traversal* is O(n) regardless of balance.
- What about search/contains and insert for a binary search tree?

Balance and runtime for search and insert in a binary search tree

```
public boolean contains(TreeNode tree, String target) {
    if (tree == null) return false;
    int result = target.compareTo(tree.info);
    if (result == 0) return true;
    if (result < 0) return contains(tree.left,target);
    return contains(tree.right, target);
}
```

If balanced?

- $T(n) = T(n/2) + O(1) = O(\log(n))$

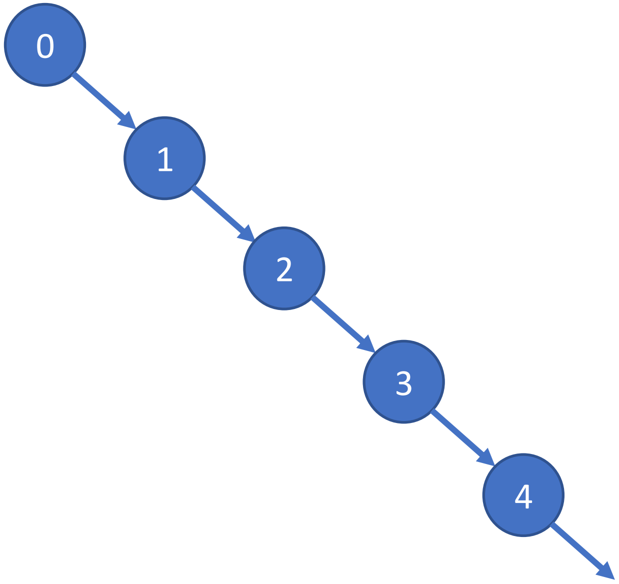
If unbalanced?

- $T(n) = T(n-1) + O(1) = O(n)$

Why would a tree not be balanced?

Worse case:

- What if we insert sorted data?




```
For(int i=0; i<n; i++) {
    myTree.insert(i);
}
```

- Average case height O(log(n)) for random-ish order
- AVL trees, red-black trees (later) can dynamically ensure good balance.

How much balance is enough?

Approximate balance: Say that a binary tree is (a, b) −approximately balanced if…

- For every *node* rooting a subtree of size $n \geq a$,
- The left and right subtrees of the *node* both contain at most b $\left(\frac{n}{2}\right)$ nodes.

Then the recurrence relation for contains is…

- $T(n) \leq T\left(b\,\frac{n}{2}\right) + O(1)$ for $n \geq a$, and
- $T(n) \leq T(n-1) + O(1)$ for $n < a$.

How much balance is enough?

So for example, if a binary tree is (5, 1.5)-approximately balanced…

Then the height of the tree is at most
$5 + (\log_{2/1.5} n + 1)$
$= 6 + \log_{4/3} n$
$O(\log(n))$

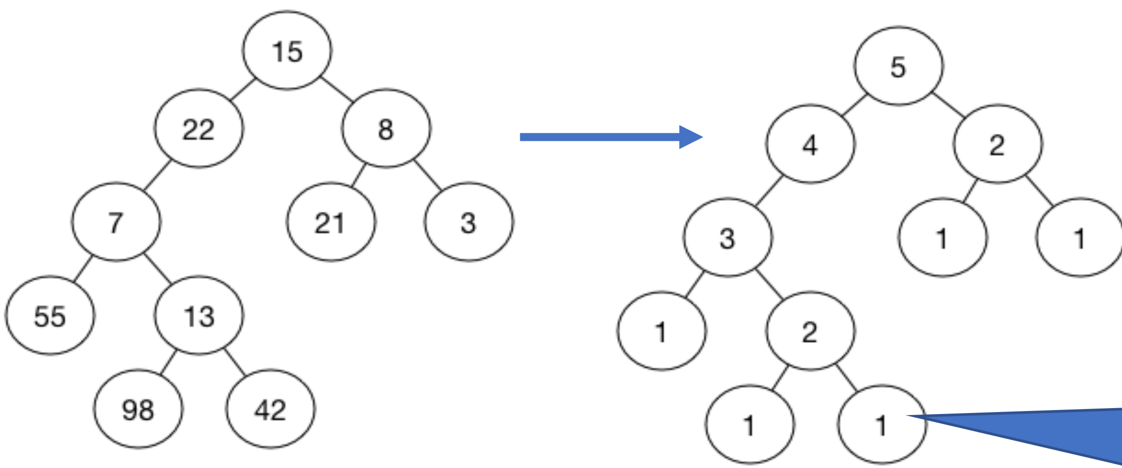
Don't need to worry about the derivation.

Takeaway: Approximate balance is good enough for good asymptotic performance.

HeightLabel APT

https://www2.cs.duke.edu/csed/newapt/heightlabel.html

- Create a new tree from a tree parameter
  - Same shape, nodes labeled with height
  - Use **`new TreeNode`**. With what values …



Note that this APT 1-indexes height/depth. We introduced it 0-indexed and will otherwise stick to that convention.

FAQ: Can I make a tree?

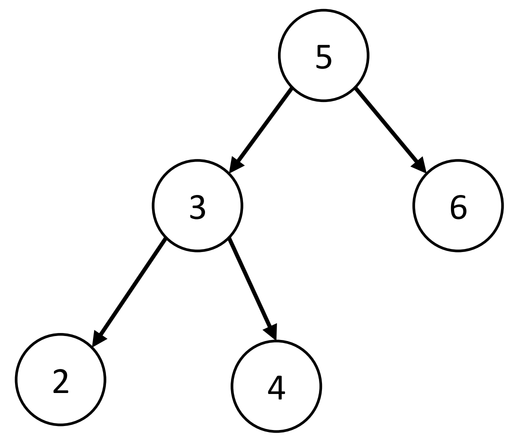
```
public class TreeNode {
    int info;
    TreeNode left;
    TreeNode right;
    TreeNode(int x){
        info = x;
    }
    TreeNode(int x, TreeNode lNode, TreeNode rNode){
        info = x;
        left = lNode;
        right = rNode;
    }
```

Just call the TreeNode constructor for each new node and connect them.

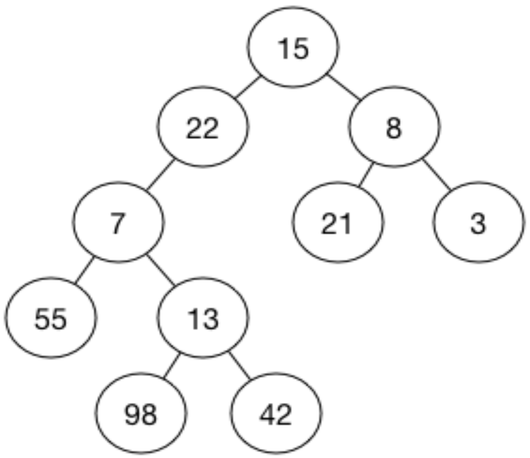
```
TreeNode root = new TreeNode( x: 5);
root.left = new TreeNode( x: 3);
root.right = new TreeNode( x: 6);
root.left.left = new TreeNode( x: 2);
root.left.right = new TreeNode( x: 4);
```

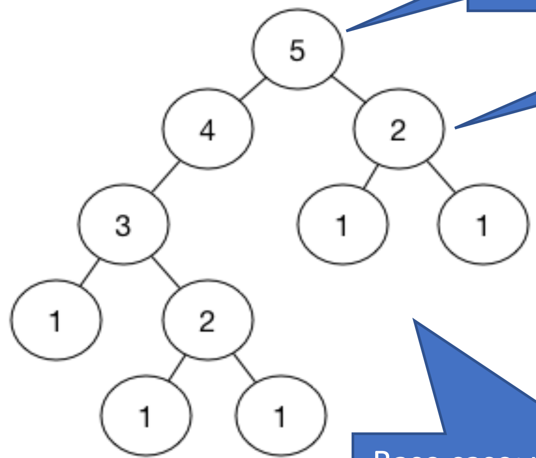
More terse version

```
TreeNode myTree = new TreeNode( x: 5,
          new TreeNode( x: 3,
                   new TreeNode( x: 2),
                   new TreeNode( x: 4)),
          new TreeNode( x: 6));
```



Solving HeightLabel in Pictures





5 = 1 + max(4, 2)

2 = 1 + max(1, 1)

1 = 1 + max(0, 0)

When null? 0?

Base case: when null, 0

Recursive case: height of node is
1 + max(height of node.left
height of node.right)

Solving HeightLabel in Code

```
private int height(TreeNode t) {
     if  (t == null) return 0;
     return 1 + Math.max(height(t.left),
                         height(t.right));
}
```

Base case: when null, 0

Recursive case: height of node is 1 + height of node.left + heigh of node.right

```
public class HeightLabel {
    public TreeNode rewire(TreeNode t) {
        // replace with working code
        return null;
    }
}
```

Method doesn't just calculate height, is supposed to create and return new tree with new nodes…

```
public TreeNode rewire(TreeNode t) {
    if (t == null) return null;
     return new TreeNode(height(t),
                         rewire(t.left),
                         rewire(t.right));
}
```

Using height helper method, get height, create new node, return.

Tree Recursion tips / common mistakes

1. Draw it out! Trace your code on small examples.
2. Return type of the method. Do you need a helper method?
3. Base case first, otherwise infinite recursion / null pointer exception.
4. If you make a recursive call, make sure to use what it returns.

WOTO

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Rewire runtime?

- recurrence of this all-green code? T(n) =
- **2T(n/2)+O(n)**
  - Balanced tree
- **T(n-1)+O(n)**
  - Unbalanced

```
public TreeNode rewire(TreeNode t) {
     if (t == null) return null;
      return new TreeNode(height(t),
                          rewire(t.left),
                          rewire(t.right));
}

private int height(TreeNode t) {
     if  (t == null) return 0;
     return 1 + Math.max(height(t.left),
                         height(t.right));
}
```

T(n)

O(n)

T(n/2) if balanced

T(n/2) if balanced

HeightLabel Complexity

- Balanced? **`O(N log N)`**,
  - **`2T(n/2) + O(n)`**
- Unbalanced, **`O(N`**^{2}**`)`**,
  - **`T(N) = T(N-1) + O(N)`**

- Do in **`O(N)`** time? Yes, if we don't call height
  - Balanced: **`T(N) = 2T(N/2) + O(1)`**
  - Unbalanced: **`T(N) = T(N-1) + O(1)`**

HeightLabel in O(N) time

- If recursion works, subtrees store heights!
- Balanced? **O(N)** ,
  - **2T(n/2)+O(1)**
- Unbalanced, **O(N)**,
  - **T(N-1)+O(1)**

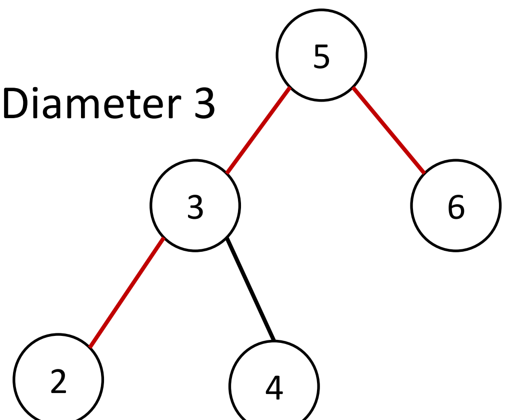
```java
public TreeNode rewire(TreeNode t) {
    if (t == null) { return null; }
    TreeNode leftOfMe = rewire(t.left);
    TreeNode rightOfMe = rewire(t.right);
    int lHeight = 0;
    int rHeight = 0;
    if (leftOfMe != null) { lHeight = leftOfMe.info; }
    if (rightOfMe != null) { rHeight = rightOfMe.info; }
    return new TreeNode(
            x: 1+Math.max(lHeight, rHeight),
            leftOfMe,
            rightOfMe);
}
```

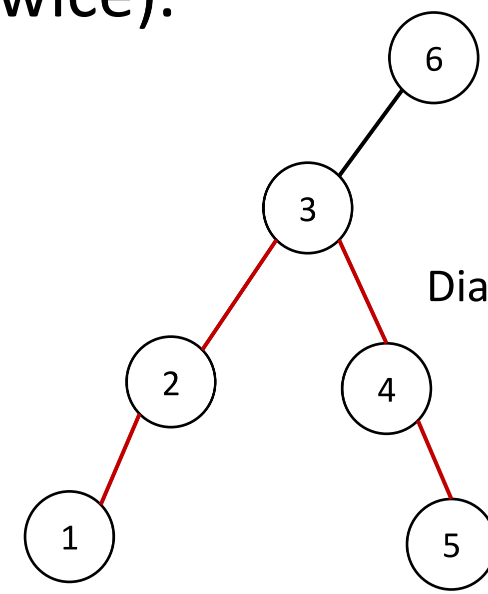
Diameter Problem

leetcode.com/problems/diameter-of-binary-tree

Calculate the *diameter* of a binary tree, the length of the longest path (maybe through root, maybe not, can't visit any node twice).

Diameter 3





Diameter 4

Live Coding