CompSci 201, L20: Binary Heaps

Logistics, Coming up

- APT9 (more tree problems) due this Wed., 11/9
- P5 Huffman due next Monday 11/14
- Wrapping up binary heaps and balanced binary search trees this week, then on to graphs!

People in CS: Clarence “Skip” Ellis

- Born 1943 in Chicago. PhD in CS from U. Illinois UC in 1969
  - First African American anywhere in US to complete a PhD in CS
- Founding member of the CS department at U. Colorado, also worked in industry.
  - Developing original graphical user interfaces, object-oriented programming, collaboration tools.

“People put together an image of what I was supposed to be,” he recalled. “So I always tell my students to push.”

Read more here

Huffman Compression

Representing data with bits: Preferably fewer bits

- Zip

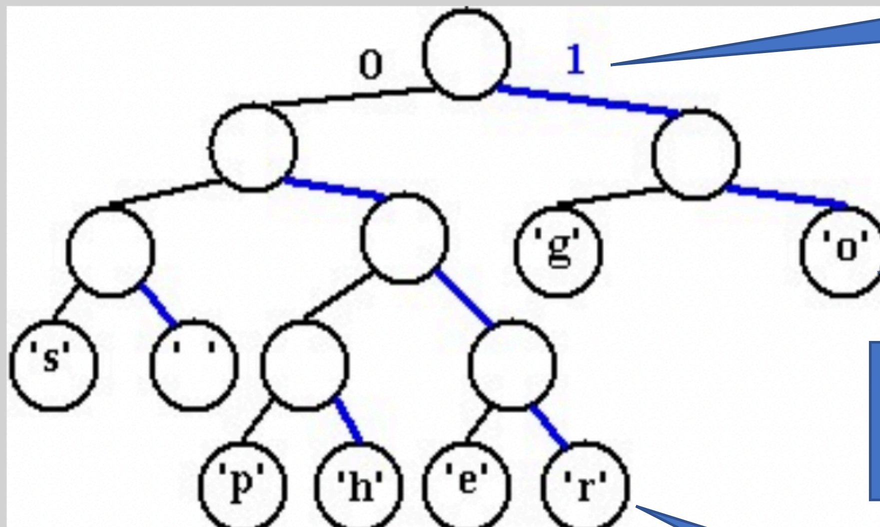
- Unicode
- JPEG

- MP3

Huffman compression used in all of these and more!

Prefix property encoding as a tree

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



Convention: 0 for left and 1 for right

Values you want to encode are leaves: Ensures prefix property.

Encoding is the sequence of 0's and 1's on root to leaf path

Values deeper in tree encoded with more bits than those earlier in the tree.

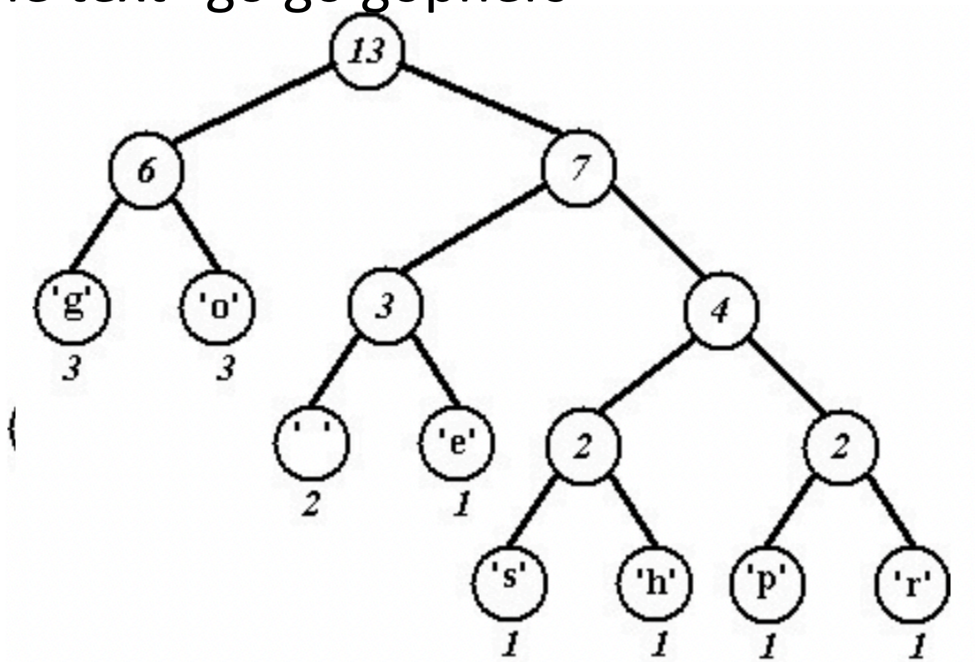
Huffman Coding

- Greedy algorithm for building an optimal variable length encoding tree.

- High level idea:
    - Start with the leaves/values you want to encode with weights = frequency.
    - Iteratively choose the ***lowest weight nodes*** to “connect up” to a new node with weight = sum of children.

Visualizing the algorithm

Encoding the text “go go gophers”



char	binary
'g'	00
'o'	01
'p'	1110
'h'	1101
'e'	101
'r'	1111
's'	1100
' '	100

WOTO

Go to duke.is/pge7f

Not graded for correctness, just participation.

Try to answer *without* looking back at slides and notes.

But do talk to your neighbors!

P5 Outline

1. Write Decompress first
    - Takes a compressed file (we give you some)
    - Reads Huffman tree from bits
    - Uses tree to decode bits to text

2. Write Compress second
    - Count frequencies of values/characters
    - Greedy algorithm to build Huffman tree
    - Save tree and file encoded as bits

Heaps Revisited

java.util.PriorityQueue Class

- Kept in sorted order, smallest out first
  - Objects must be Comparable OR provide Comparator to priority queue

```
PriorityQueue<String> pq = new PriorityQueue<>();
pq.add("is");
pq.add("Compsci 201");
pq.add("wonderful");
while (! pq.isEmpty()) {
    System.out.println(pq.remove());
}
```

```
Compsci 201
is
wonderful
```

```
PriorityQueue<String> pq = new PriorityQueue<>(
        Comparator.comparing(String::length));
pq.add("is");
pq.add("Compsci 201");
pq.add("wonderful");
while (! pq.isEmpty()) {
    System.out.println(pq.remove());
}
```

```
is
wonderful
Compsci 201
```

Tradeoffs, Heaps, and Trees

- Fast add and remove?
- Binary Heap: Implements a priority queue with:
  - Peek: O(1)
  - Remove: O(log(N))
  - Add: O(log(N))
- `java.util PriorityQueue` is implemented as a binary heap

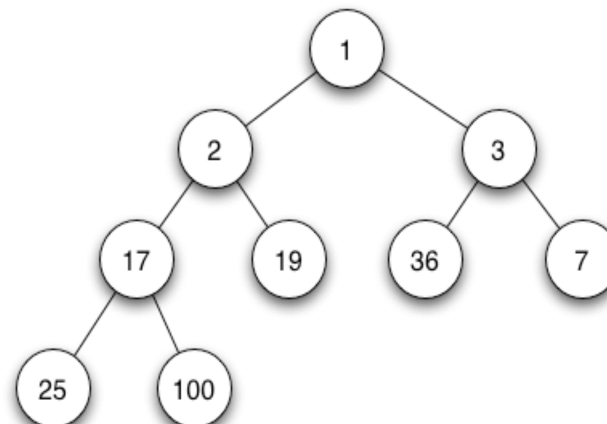
N	$\log_2(N)$
100	6.6
200	7.6
400	8.6
800	9.6
1600	10.6
3200	11.6
6400	12.6
12800	13.6
25600	14.6
51200	15.6

Binary Heap at a high level

- Maintain the **heap property** *that every node is less than or equal to its successors*, and
- The **shape property** *that the tree is complete* (full except perhaps last level, fill from left to right*)*

Operations:

- Peek: Return value of root node
- Remove: Remove root node and fix tree to reestablish properties.
- Add: Insert at first open position, fix to reestablish properties.



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How are PriorityQueues implemented?

- Normally think about binary tree used for implementing priority queues, supports:
  - insert/add
  - findMin/peek
  - deleteMin/remove

- Actually implement with an array
  - minimizes storage (no explicit points/nodes)
  - simpler to code, no explicit tree traversal
  - faster too (constant factor, not asymptotically)---children are located by index/position in array

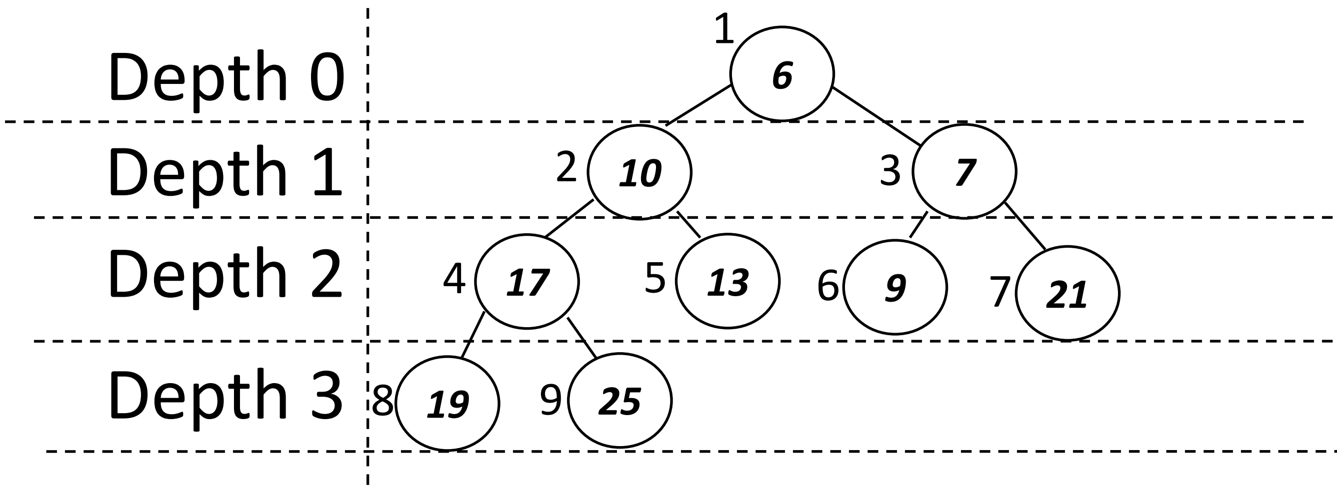
Aside: How much less memory?

- Storing an int takes 4 bytes = 32 bits on most machines.
- Storing one *reference to an object* (a memory location) takes 8 bytes = 64 bits on most machines.

- For a heap storing N integers…
  - Array of N integers takes ~ 4N bytes.
  - Binary tree where each node has an int, left, and right reference takes ~20N bytes.
  - So maybe a 5x savings in memory (just an estimate). Not an asymptotic improvement.

Using an array for a Heap

- Hard to keep track of the last node in the tree.
- Index positions in the tree level by level, left to right:



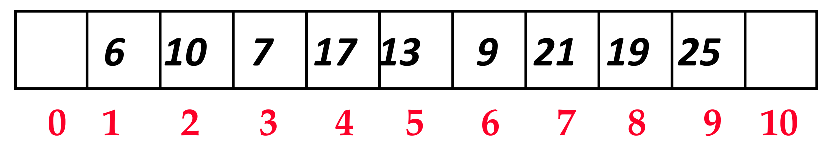
- Last node in the heap is always just the largest index
- Can use indices to represent as an array!

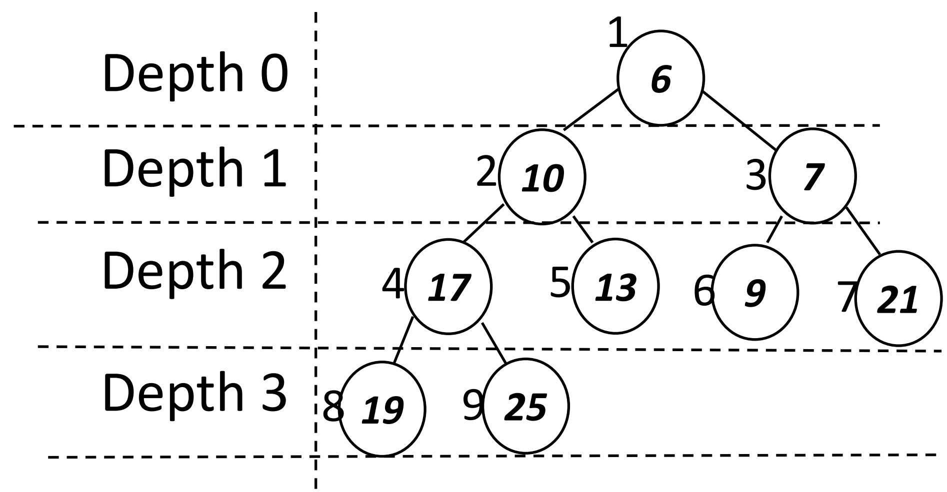
	6	*10*	*7*	*17*	*13*	*9*	*21*	*19*	*25*	
0	1	2	3	4	5	6	7	8	9	10

(`ArrayList` if you want it to be growable)

Properties of the Heap Array

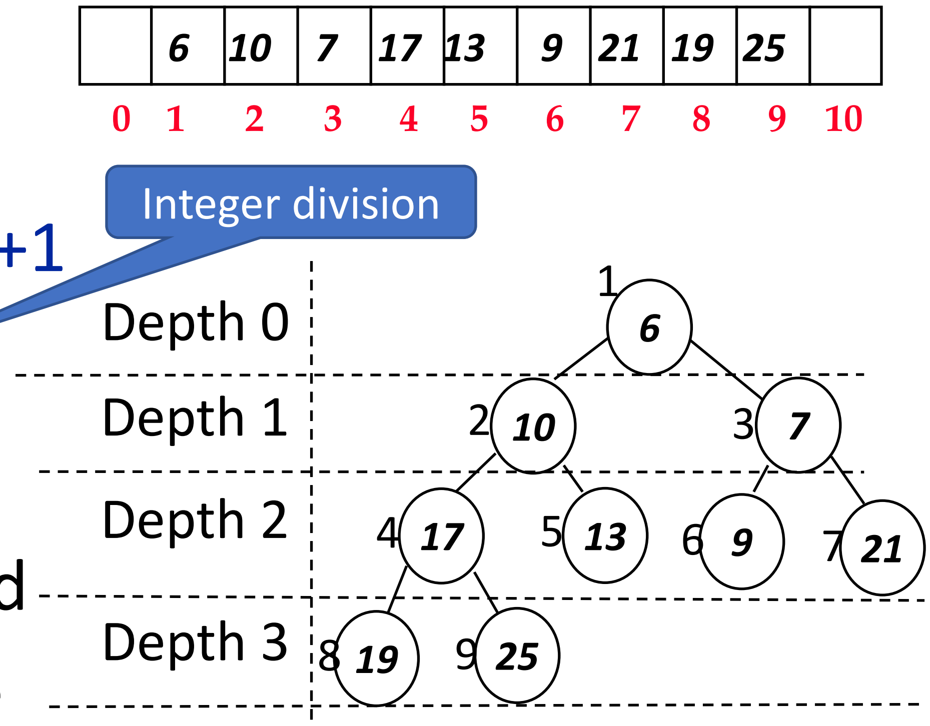
- Store “node values” in array beginning at index 1
  - Could 0-index, Zybook does this
- Last node is always at the max index
- Minimum node is always at index 1
- peek is easy, return first value.
  - How about add?
  - Remove?





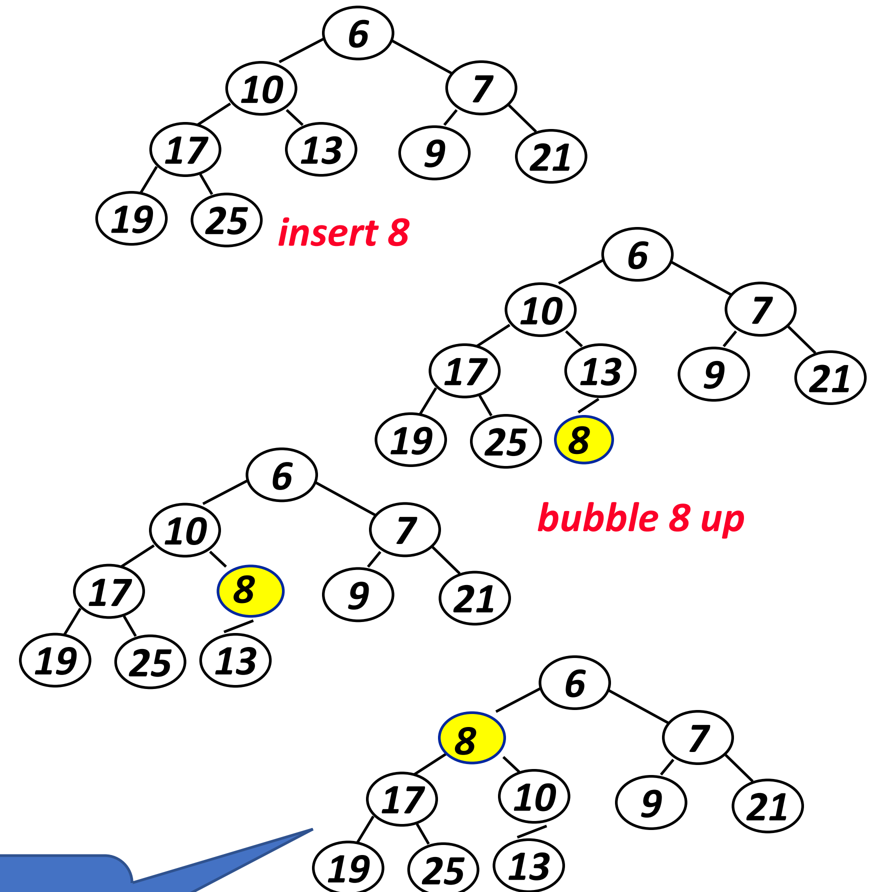
Relating Nodes in Heap Array

- When 1-indexing: For node with index k
  - left child: index 2*k
  - right child: index 2*k+1
  - parent: index k/2
- Why? Follows from:
  - Heap is *complete*, and
  - Complete binary tree has 2^d nodes at depth d (except last)



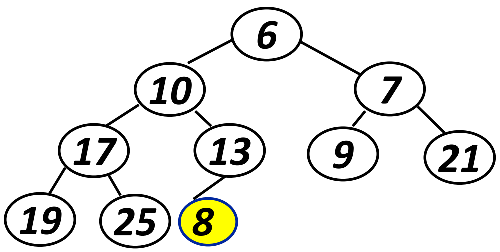
Adding values to heap in pictures

- Add to first open position in last level of the tree
  - (really, add to end of array)
- Swap with parent if heap property violated
  - stop when parent is smaller
  - Or you reach the root

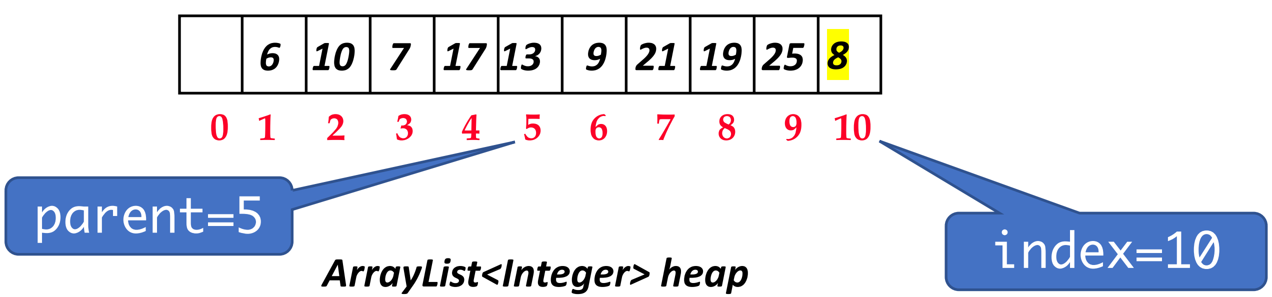


Heap property re-established

Heap add implementation

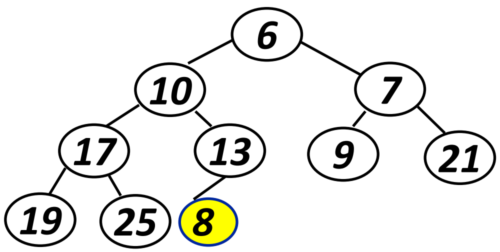


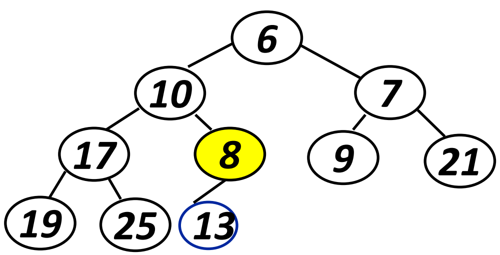

```java
public void add(Integer value) {
    heap.add(value); // add to last position
    size++;

    int index = size; // note we are 1-indexing
    int parent = index / 2;

    while(parent >= 1 && heap.get(parent) > heap.get(index)) {
        swap(index, parent);
        index = parent;
        parent /= 2;
    }
}
```



ArrayList<Integer> heap

Heap add implementation



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    }
}
```

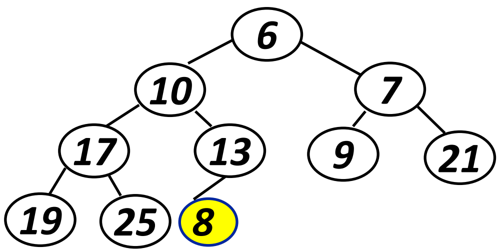
	6	10	7	17	8	9	21	19	25	13
0	1	2	3	4	5	6	7	8	9	10

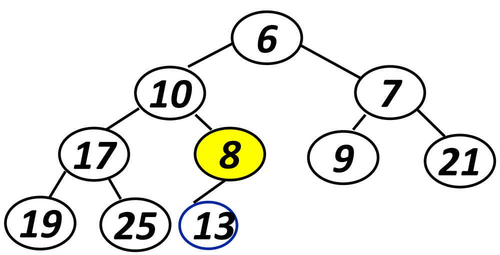
parent=2

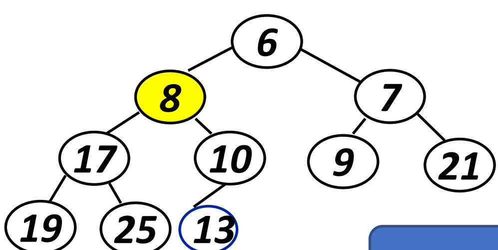
index=5

ArrayList<Integer> heap

Heap add implementation





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        parent /= 2;
    }
}
```

	6	8	7	17	10	9	21	19	25	13
0	1	2	3	4	5	6	7	8	9	10

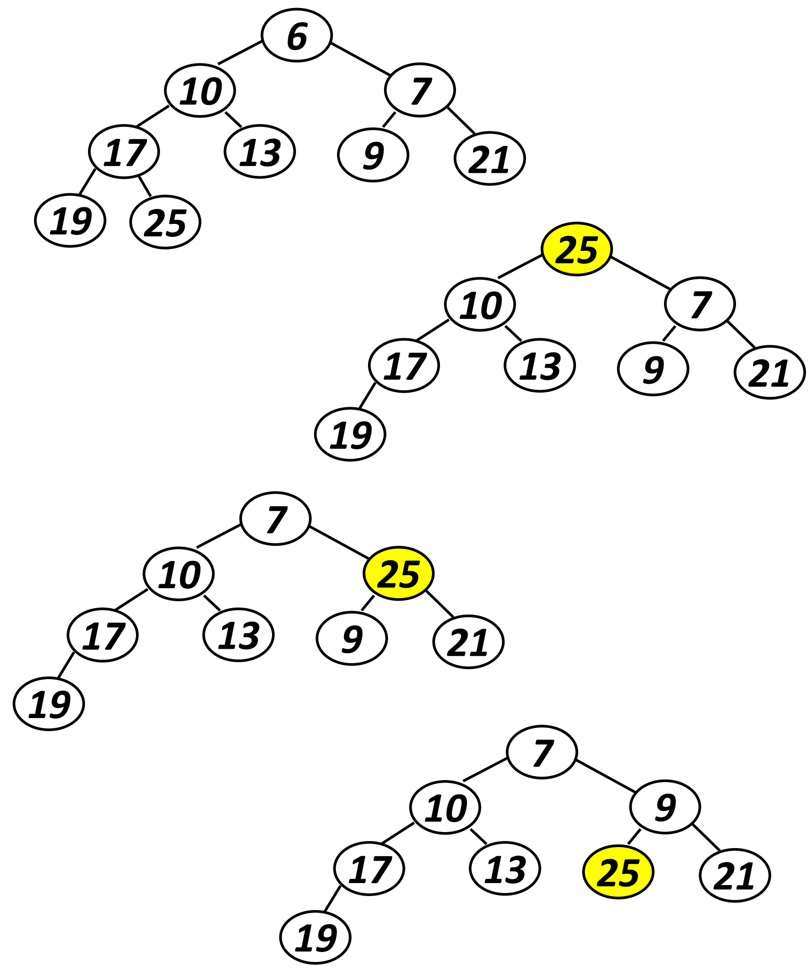
index=2

parent=1

ArrayList<Integer> heap

Heap remove in pictures

- Always return root value
- Replace root with last node in the heap
- While heap property violated, swap with *smaller* child.



Heap Complexity

- Claimed that:
  - Peek: O(1)
  - Add: O(log(N))
  - Remove: O(log(N))
- On a heap with N values. Why?
  - Peek: Easy, return first value in an Array
  - Complete binary tree always has height O(log(N)).
  - add and remove “traverse” ***one*** root-leaf path, at most O(log(N)).

WOTO

Go to duke.is/guzv2

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But do talk to your neighbors!

Choose your own adventure

1. Look at Generic DIYBinaryHeap: coursework.cs.duke.edu/cs-201-fall-22/diybinaryheap , or
2. Solve Greedy Coding Problem with PriorityQueue: leetcode.com/problems/non-overlapping-intervals/

Live coding