CompSci 201, L13: Recursion (and more Linked List)

Person in CS: Ellen Ochoa

- BS physics (‘75), PhD Electrical Eng. (‘85).
- Starting working on software for optical recognition systems (computer vision)
- Applied to be an astronaut in…
  - ‘85…rejected
  - ‘87…rejected
  - ‘90…accepted!!!
- Worked on flight software, computer hardware, and robotics
- First Hispanic woman in space ’93
- Director of NASA Johnson Space Flight Center (Houston) ‘13

Announcements, Coming up

- Today, Monday 2/27
  - Nothing due
- Wednesday 3/1
  - APT 5 (linked list problems) due
- Next Monday 3/6
  - Project P3: DNA (linked list project) due
- Midterm exam 2? Pushed back to Wed. 3/22 after spring break.

Today's outline

1. Wrapping up DIYLinkedList:
   - Add to front of linked list
   - Writing an Iterator

2. Introducing Recursion
   - Counting ListNodes
   - Reversing a LinkedList

3. Sorting algorithms: Recursive Mergesort

More DIYLinkedList

Live Coding 

- Add to front?
- Efficient iterator?

Toward Recursion by counting nodes: Iterative vs. Recursive

- Standard linked list iteration
  - Advance local pointer, do something at each node

```
public int countIter(ListNode list) {
    int total = 0;
    while (list != null) {
        total += 1;
        list = list.next;
    }
    return total;
}
```

- Recursion?
  - Base Case?
  - General case?
  - Define `size` using `size`?

```
public int size(ListNode list) {
    if (list == null) return 0;
    return 1 + size(list.next);
}
```

Key ideas in recursion

1. Base case: Easy answer when input small
2. Recursive call(s): Get answer on subset of input
3. Do something with the result of the recursive call(s) and then return

- Note: Method does not call itself
  - Calls identical clone, with its own state
  - Methods/calls stacked, like all methods

Thinking recursively

1. When is the input small enough that the answer is trivial? Base case.

   Base case

2. Otherwise, suppose an *oracle* could solve the *exact some problem* on a smaller subset of the input.

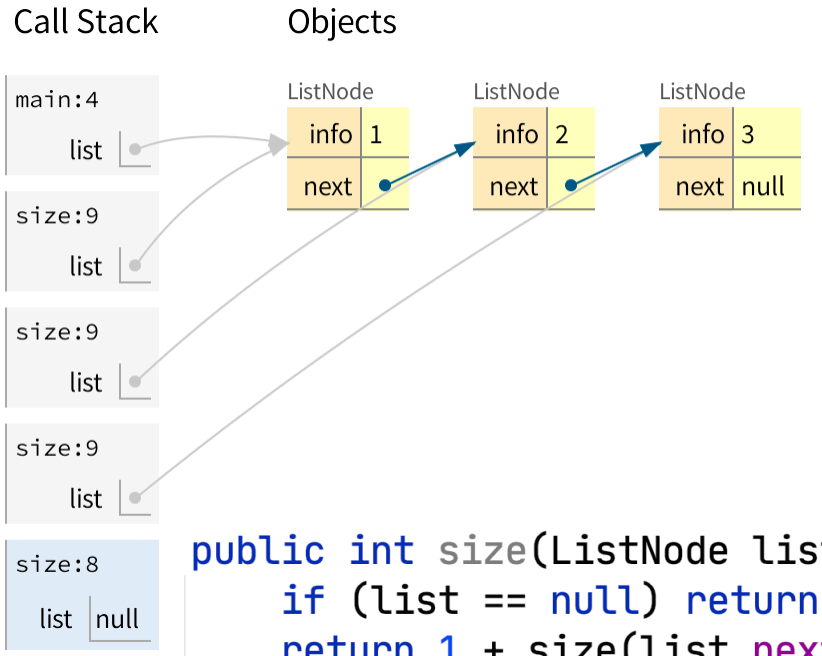
   Recurse

3. Could you solve the larger problem given what the oracle told you?

   Use result

The call stack: How recursion works on a machine

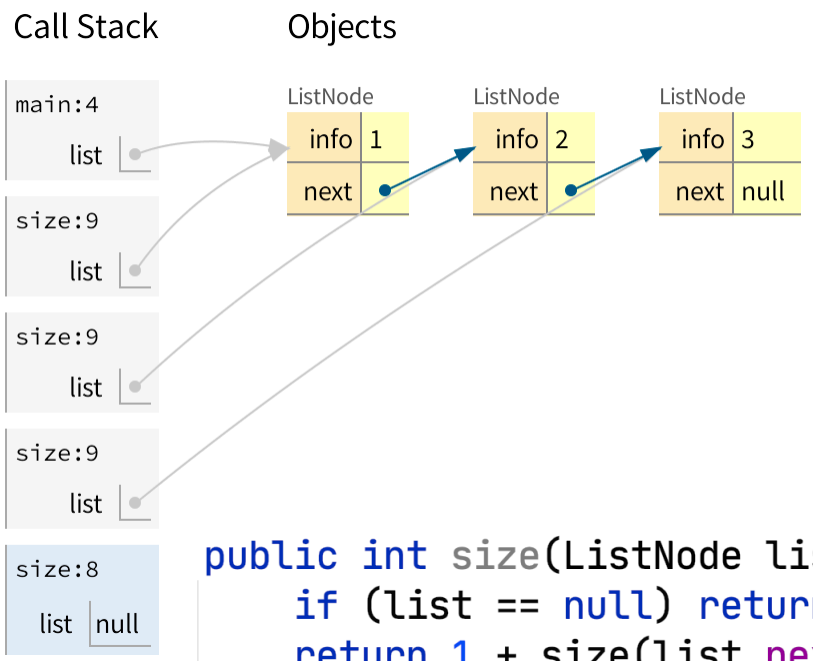
- Each method call gets its own ***call frame*** (local variables, etc.)
- ***Eager evaluation***: Invoking method does not resume until invoked method returns.




```
public int size(ListNode list) {
    if (list == null) return 0;
    return 1 + size(list.next);
}
```

Eager evaluation and substitution

- Return value will be substituted into the expression calling the method.




```
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    if (list == null) return 0;
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}
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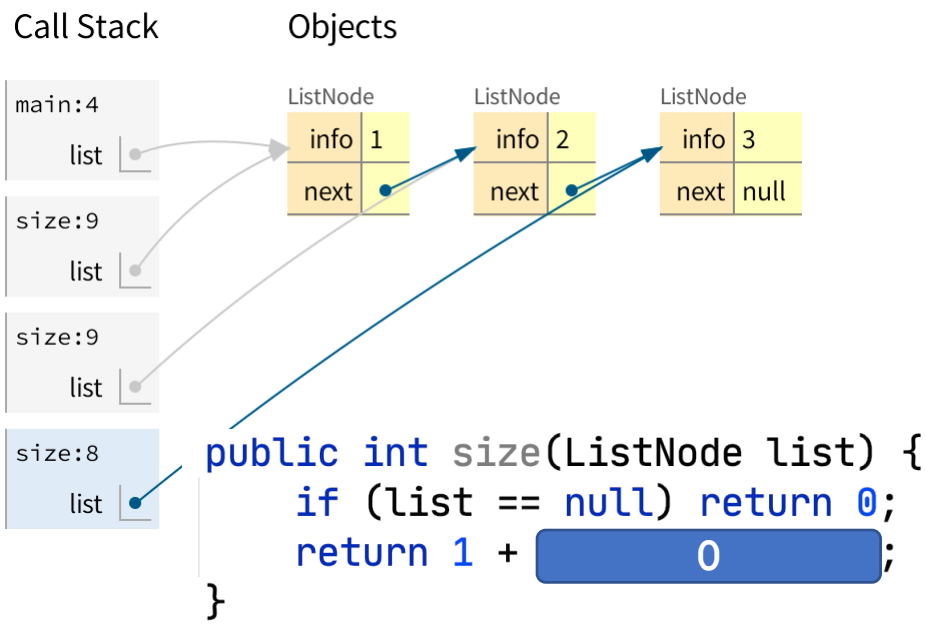
Eager evaluation and substitution

- Return value will be substituted into the expression calling the method.



Eager evaluation and substitution

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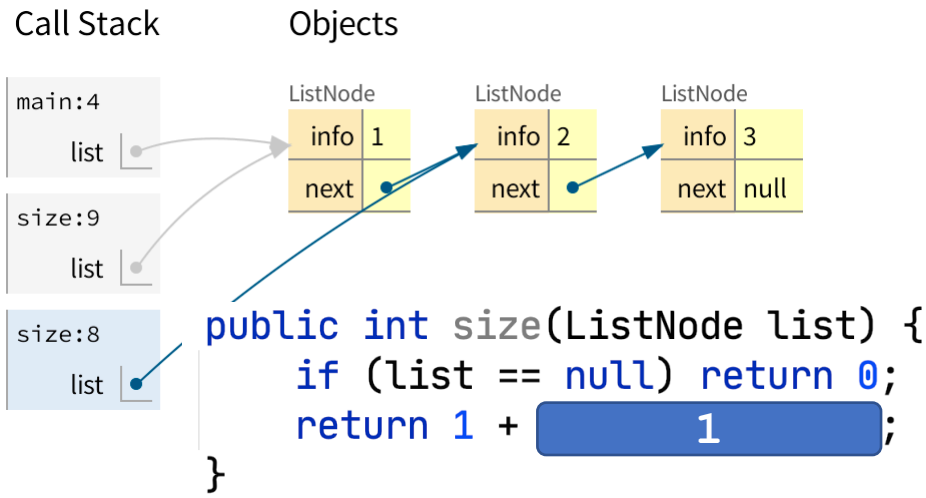

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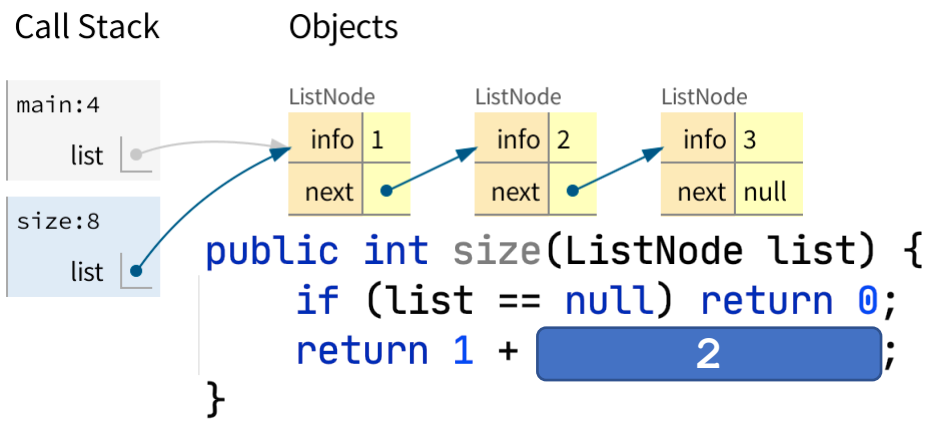
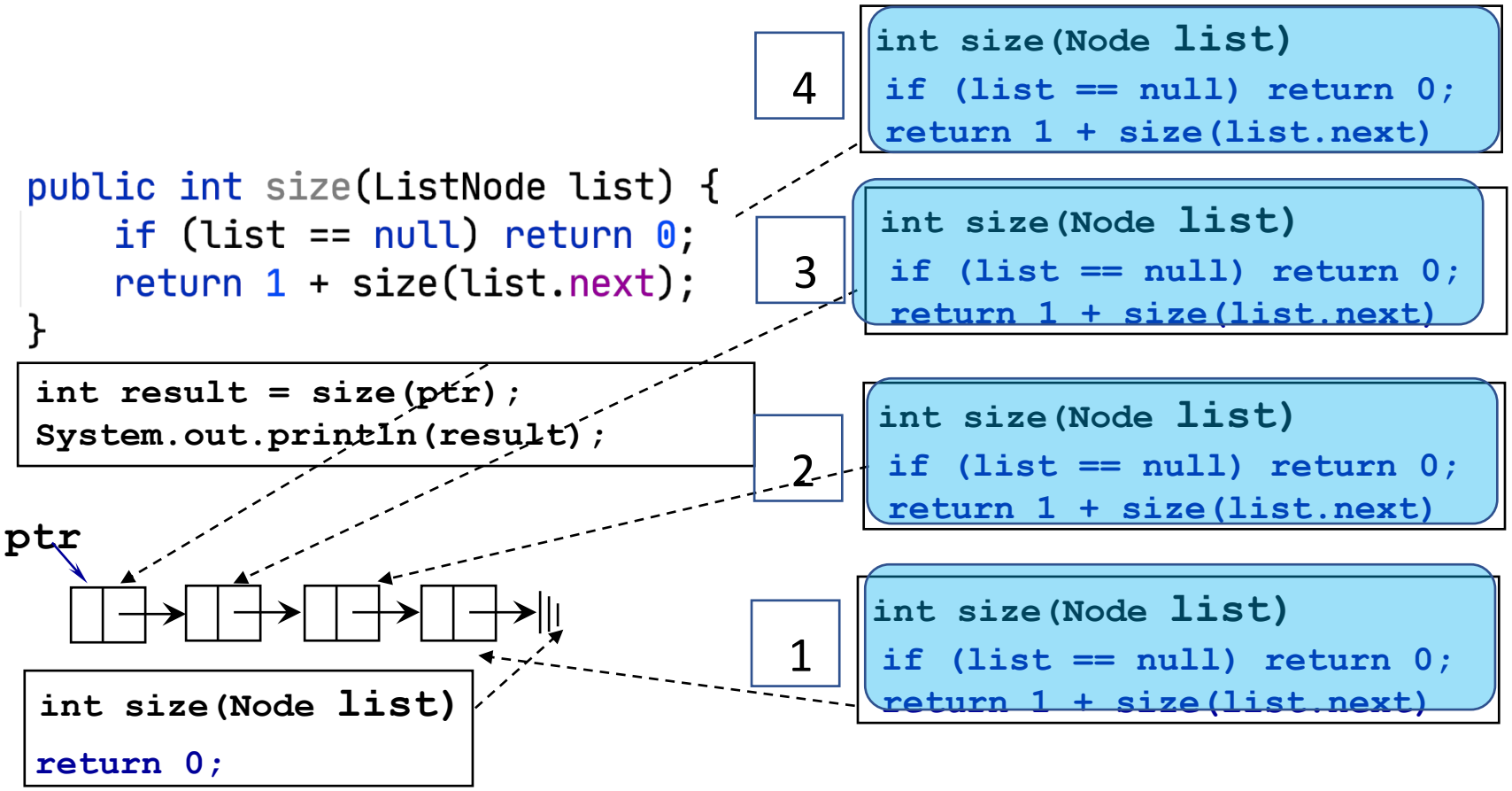


Counting Nodes



Recursive runtime

- Concept is the same: Count the number of constant time operations…across all recursive calls!

- **E*nsure each recursive call gets closer to the base case,*** else code may run forever.

```
public int size(ListNode list) {
    if (list == null) return 0;
    return 1 + size(list.next);
}
```

- Moves one node toward the base case at each step.
- List of N nodes, makes O(N) recursive calls, each is O(1).
- Overall O(N) runtime complexity.

Recall the reverse problem

- How do we reverse nodes in a linked list
  - Go from A->B->C to C->B->A
  - Typical interview style question
  - https://leetcode.com/problems/reverse-linked-list/
  - https://www.hackerrank.com/challenges/reverse-a-linked-list

Base case, words and code

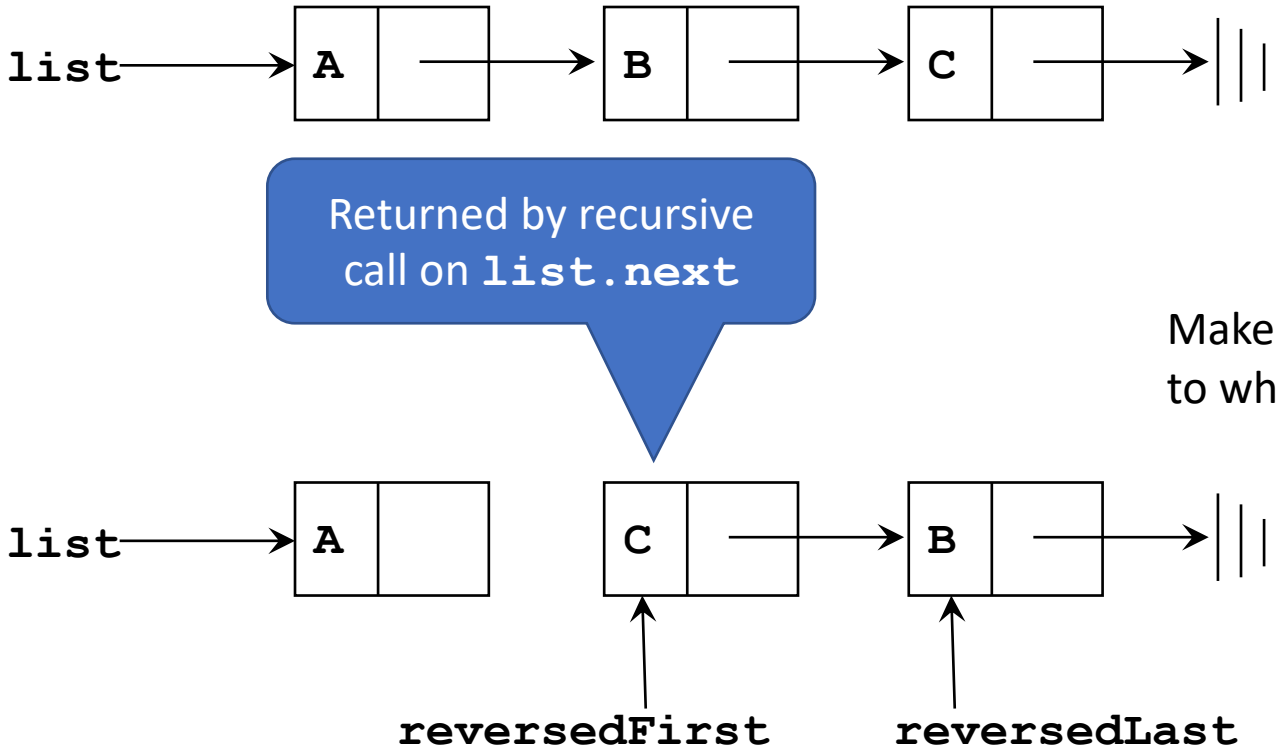
- Base case: When is there nothing to do?
  - A list with 0 or 1 nodes is its own reverse

```
3    public static ListNode reverse(ListNode list) {
4        if (list == null || list.next == null) {
5            return list;
6        }
```

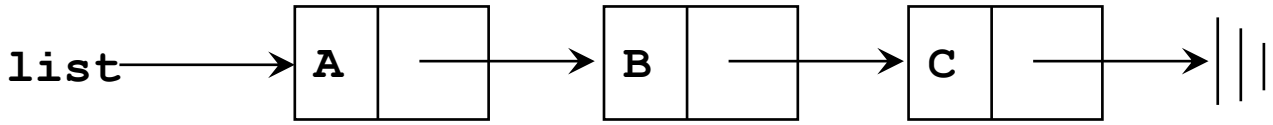
Recursive step in words

- Suppose the recursion oracle (a recursive call) reverses the list *after the first node*.
- How to use? Just put the first node at the end!
- Restated: The reverse of a list is *the reverse of all but the first element*, with the first element added to the end.

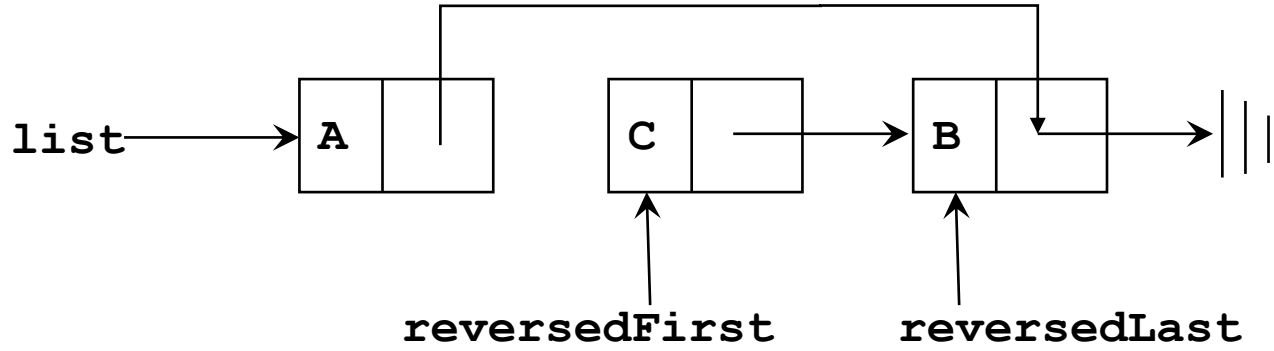
Recursive step in pictures



Recursive step in code

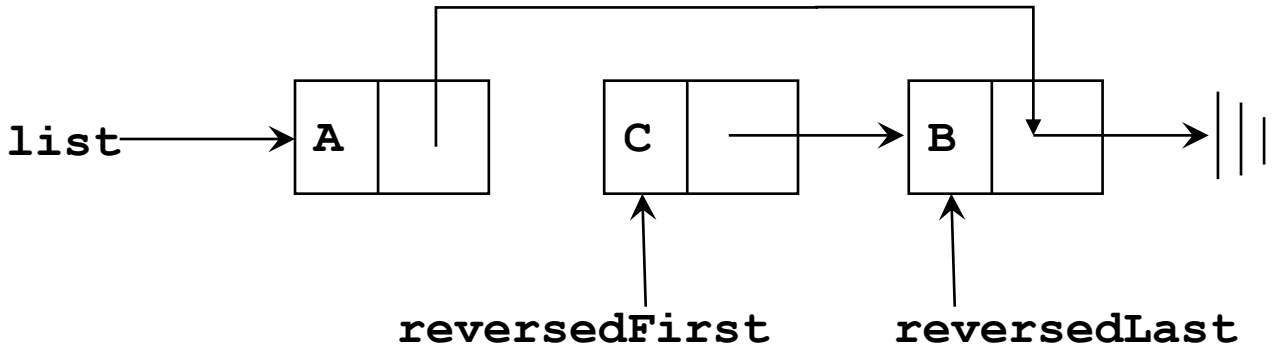



```
7       ListNode reversedLast = list.next;
8       ListNode reversedFirst = reverse(list.next);
```



Note that **`list.next`** still refers to **`reversedLast`**

Recursive step in code (continued)

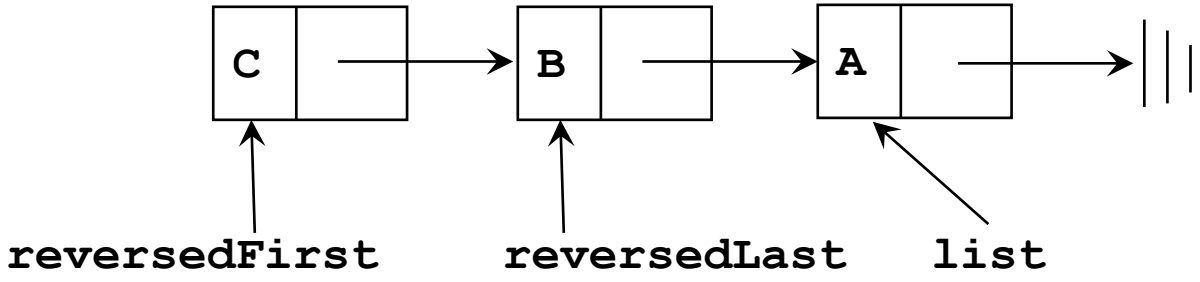



```
 9        reversedLast.next = list;
10        list.next = null;
11        return reversedFirst;
```

Make **B** point to **A**

Make **A** point to **`null`**

Return overall reversed list



Putting it all together

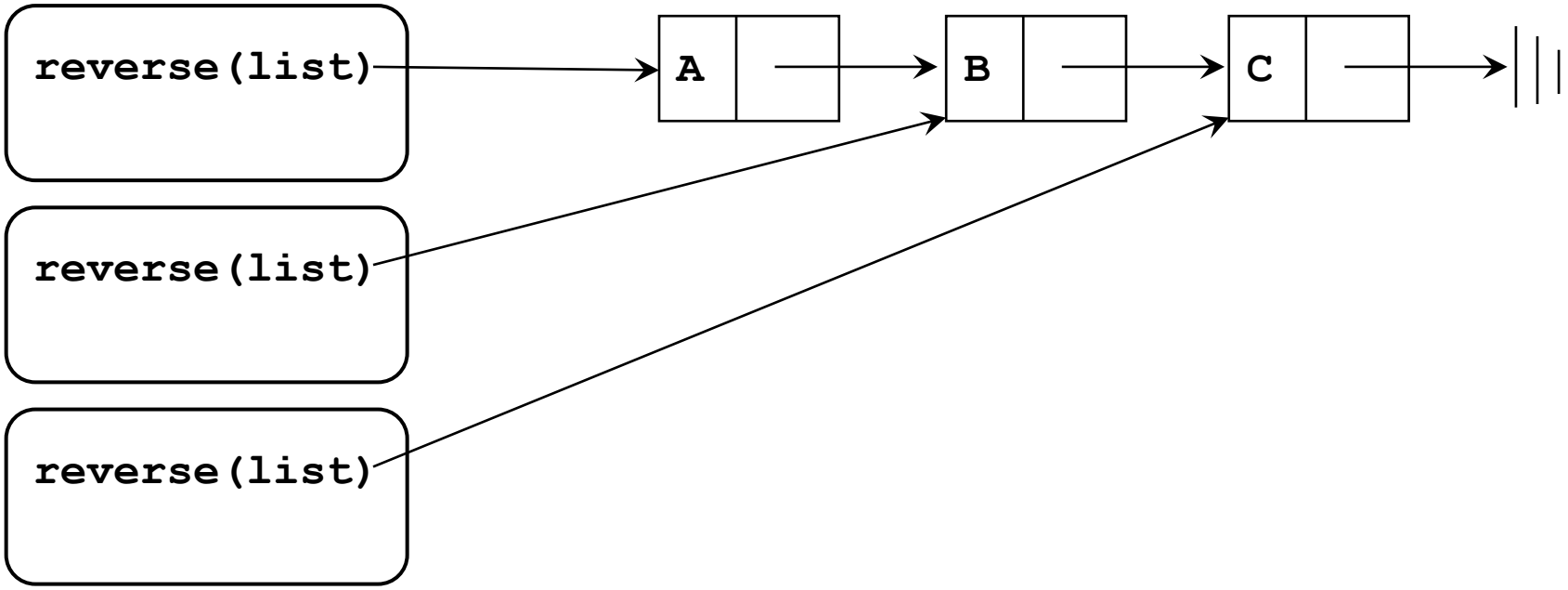
```
3   public static ListNode reverse(ListNode list) {
4       if (list == null || list.next == null) {
5           return list;
6       }
7       ListNode reversedLast = list.next;
8       ListNode reversedFirst = reverse(list.next);
9       reversedLast.next = list;
10      list.next = null;
11      return reversedFirst;
12  }
```

Base case

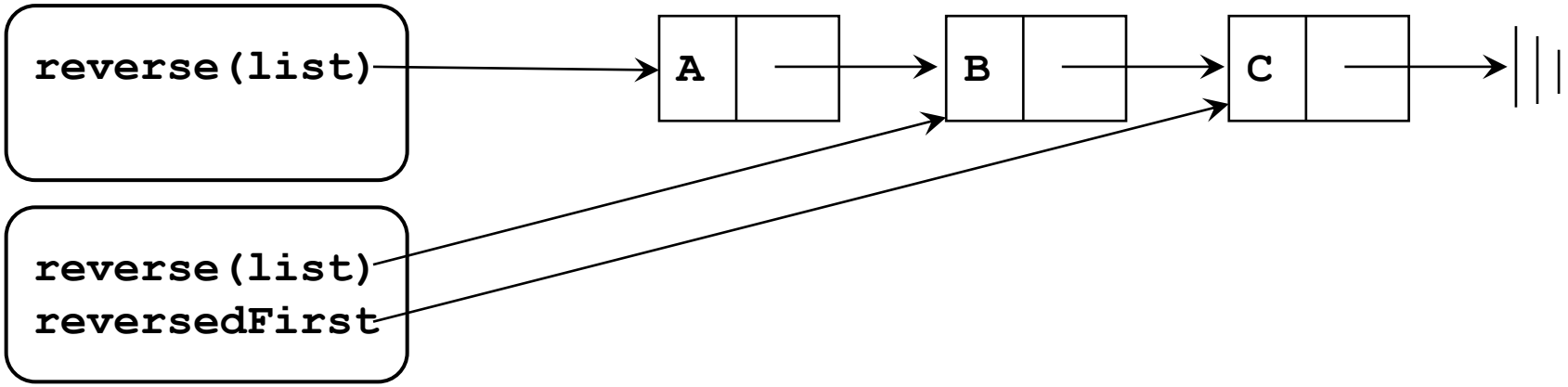
Recurse

Use result

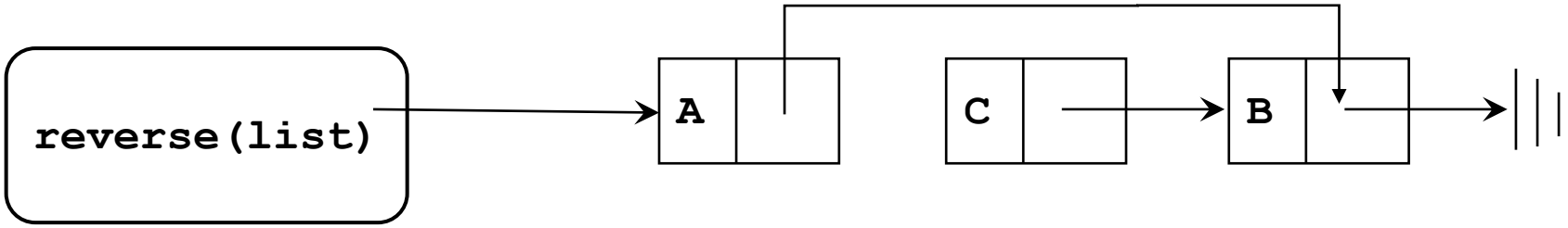
Revisiting the call stack: How it *really* works



Revisiting the call stack: How it *really* works



Revisiting the call stack: How it *really* works



Back to the case we considered first

WOTO

Go to duke.is/6effd

Not graded for correctness, just participation.

Try to answer *without* looking back at slides and notes.

But do talk to your neighbors!

2

Consider the rec method. If the input list is ['A', 'B', 'C'], what will be returned by rec(list)? *

```
3   public static ListNode rec(ListNode list) {
4       if (list == null || list.next == null) {
5           return list;
6       }
7       ListNode after = rec(list.next);
8       if (list.info <= after.info) {
9           list.next = after;
10          return list;
11      }
12      return after;
13  }
```

- ◯ ['A']
- ◯ ['C']
- ◯ ['A', 'B']
- ◯ ['C', 'B']
- ◯ ['A', 'B', 'C']
- ◯ ['C', 'B', 'A']

3

Same rec method. If the input list is ['C', 'B', 'A'], what will be returned by rec(list)? *

```
3   public static ListNode rec(ListNode list) {
4       if (list == null || list.next == null) {
5           return list;
6       }
7       ListNode after = rec(list.next);
8       if (list.info <= after.info) {
9           list.next = after;
10          return list;
11      }
12      return after;
13  }
```

- ['A']
- ['C']
- ['A', 'B']
- ['C', 'B']
- ['A', 'B', 'C']
- ['C', 'B', 'A']

4

Same rec method. For an input list with N nodes, the best characterization of the runtime complexity of rec(list) is... *

```
3   public static ListNode rec(ListNode list) {
4       if (list == null || list.next == null) {
5           return list;
6       }
7       ListNode after = rec(list.next);
8       if (list.info <= after.info) {
9           list.next = after;
10          return list;
11      }
12      return after;
13  }
```

- ◯ O(1)
- ◯ O(N)
- ◯ O(N^2)
- ◯ O(N^3)

5

Consider the mystery method. Note that it is the same as rec except for lines 24-29. If the input list is ['C', 'B', 'A'], what will be returned by mystery(list)? *

```
15  public static ListNode mystery(ListNode list) {
16      if (list == null || list.next == null) {
17          return list;
18      }
19      ListNode after = mystery(list.next);
20      if (list.info <= after.info) {
21          list.next = after;
22          return list;
23      }
24      ListNode current = after;
25      while (current.next != null && list.info > current.next.info) {
26          current = current.next;
27      }
28      list.next = current.next;
29      current.next = list;
30      return after;
31  }
```

- ['A']
- ['C']
- ['A', 'B']
- ['B', 'C']
- ['A', 'B', 'C']
- ['C', 'B', 'A']

6

Same mystery method. For an input list with N nodes, the best characterization of the runtime complexity of mystery(list) is... *

```
15  public static ListNode mystery(ListNode list) {
16      if (list == null || list.next == null) {
17          return list;
18      }
19      ListNode after = mystery(list.next);
20      if (list.info <= after.info) {
21          list.next = after;
22          return list;
23      }
24      ListNode current = after;
25      while (current.next != null && list.info > current.next.info) {
26          current = current.next;
27      }
28      list.next = current.next;
29      current.next = list;
30      return after;
31  }
```

- ◯ O(1)
- ◯ O(N)
- ◯ O(N^2)
- ◯ O(N^3)