CompSci 201, L20: Binary Heaps

People in CS: Clarence “Skip” Ellis

- Born 1943 in Chicago. PhD in CS from U. Illinois UC in 1969
  - First African American in US to complete a PhD in CS
- Founding member of the CS department at U. Colorado, also worked in industry.
  - Developing original graphical user interfaces, object-oriented programming, collaboration tools.

“People put together an image of what I was supposed to be,” he recalled. “So I always tell my students to push.”

[Read more here]

Logistics, Coming up

- Today, Wednesday 3/29
  - APT 7 due
- Next Monday, 4/3
  - Nothing due, start on P5 Huffman
- Next Wednesday, 4/5
  - APT 8 due

Today's agenda

- Wrap up Huffman Coding Intro
- Priority Queue revisited: Implementations, especially binary heap

Huffman Compression

Representing data with bits: Preferably fewer bits

- Zip

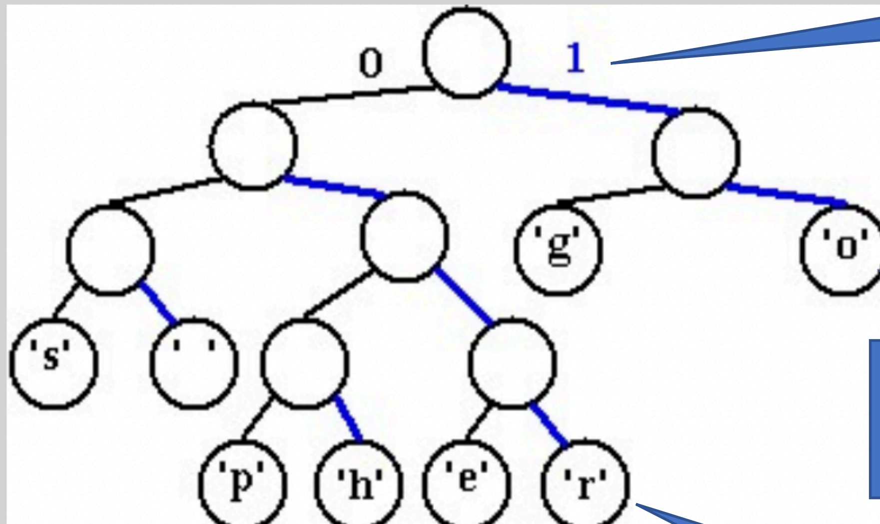
- Unicode
- JPEG

- MP3

Huffman compression used in all of these and more!

Prefix property encoding as a tree

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



Convention: 0 for left and 1 for right

Values you want to encode are leaves: Ensures prefix property.

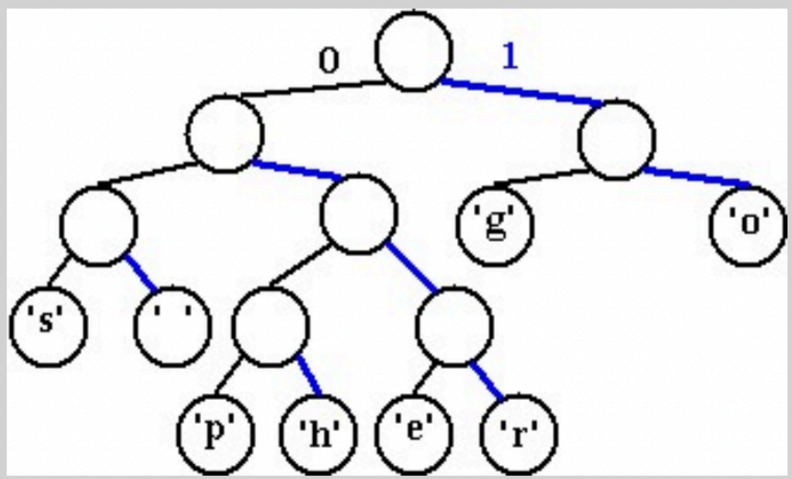
Encoding is the sequence of 0's and 1's on root to leaf path

Values deeper in tree encoded with more bits than those earlier in the tree.

Decoding bits using Huffman tree

Goal: Decode 10011011 assuming it was encoded with this tree.

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



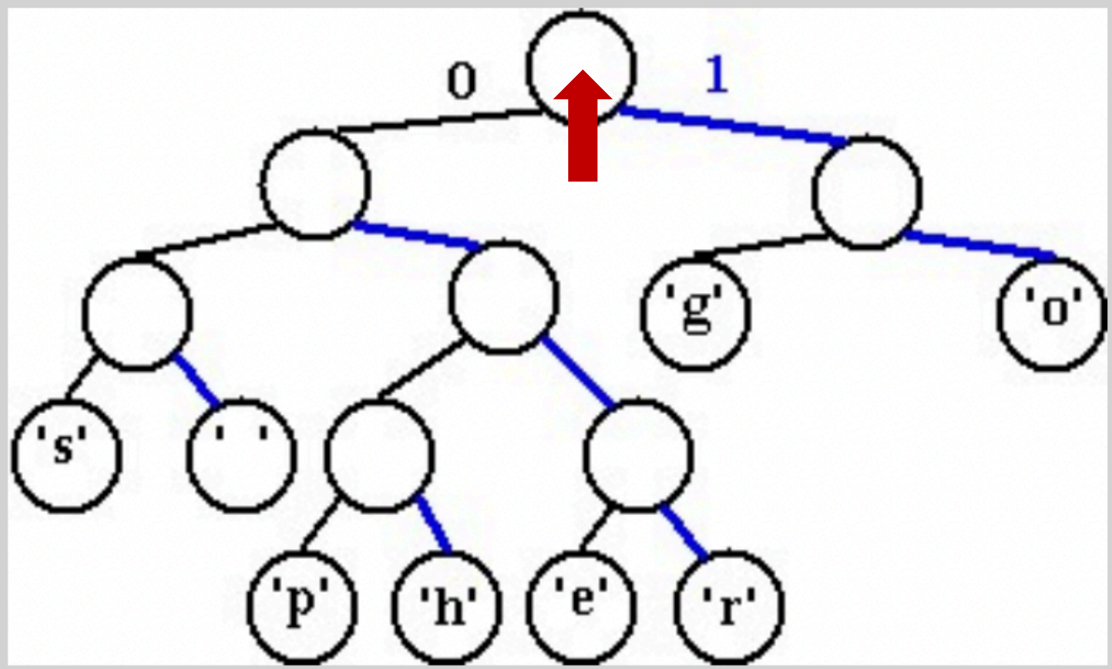
- Read bit at a time, traverse left or right edge.
- When you reach a leaf, decode the character, restart at root.

Decoding bits using Huffman tree

Decode 10011011

Initialize at root

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001

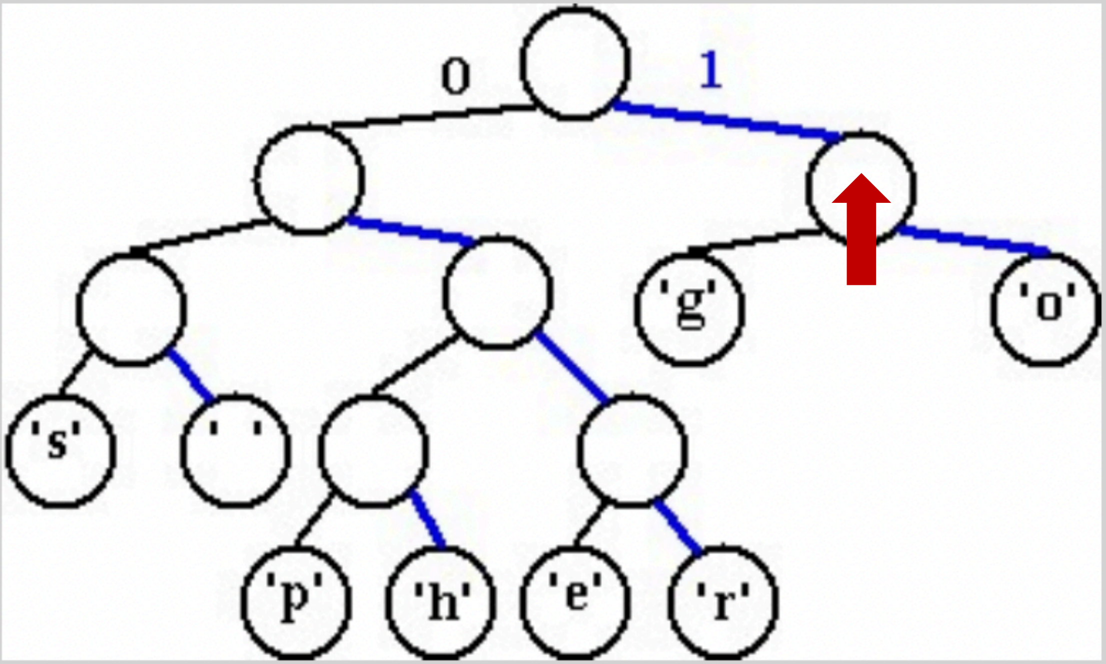


Decoding bits using Huffman tree

Decode 10011011

Read 1, go to right child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001

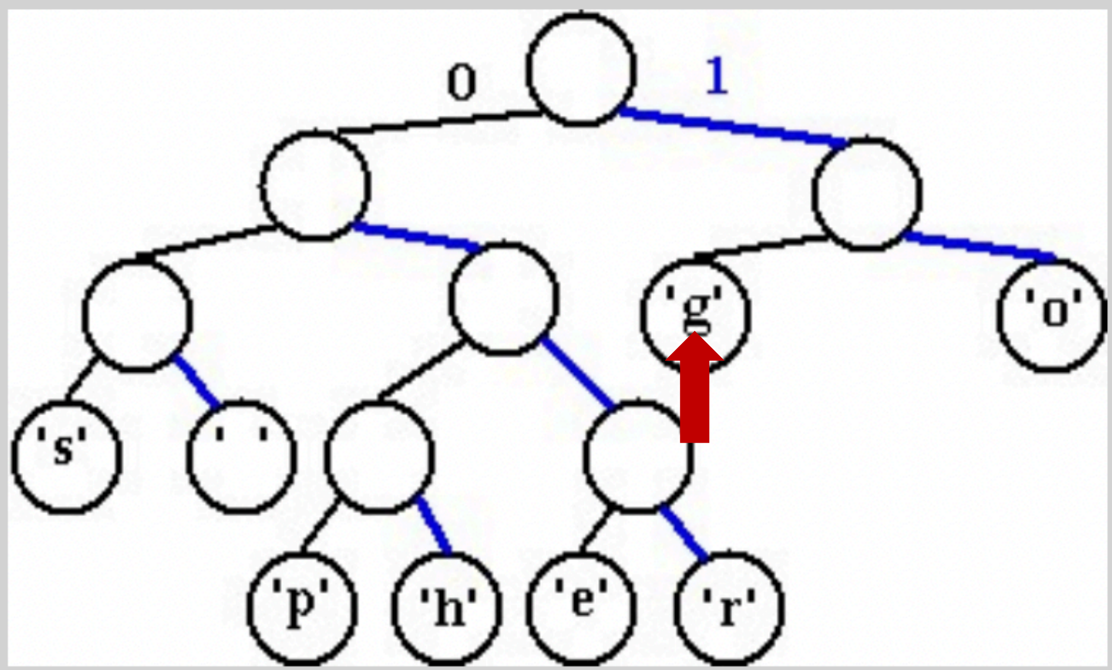


Decoding bits using Huffman tree

Decode 10011011

Read 0, go to left child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



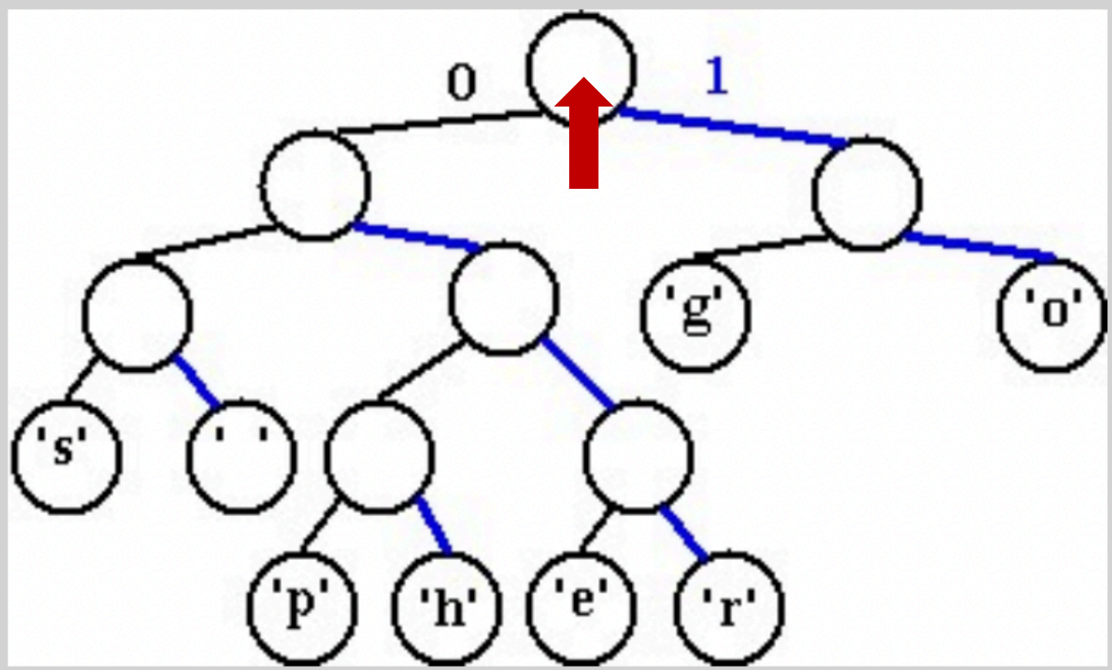
Decoding bits using Huffman tree

Decode 10011011

g

Leaf, decode ‘g’, restart at root

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



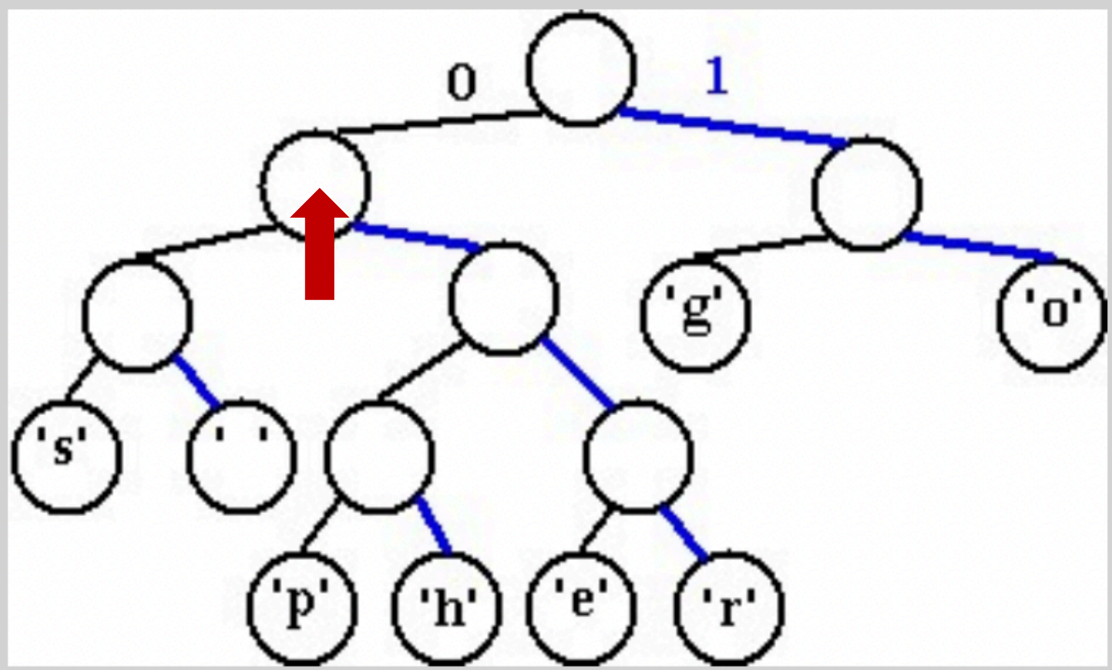
Decoding bits using Huffman tree

Decode 10011011

g

Read 0, go to left child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



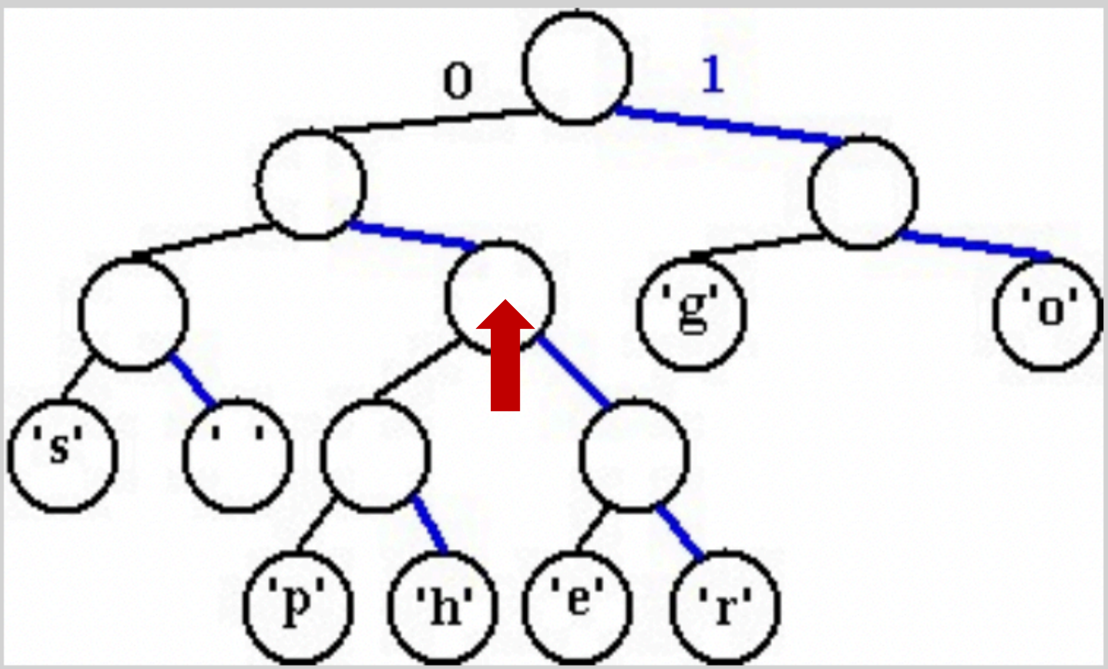
Decoding bits using Huffman tree

Decode 10011011

g

Read 1, go to right child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



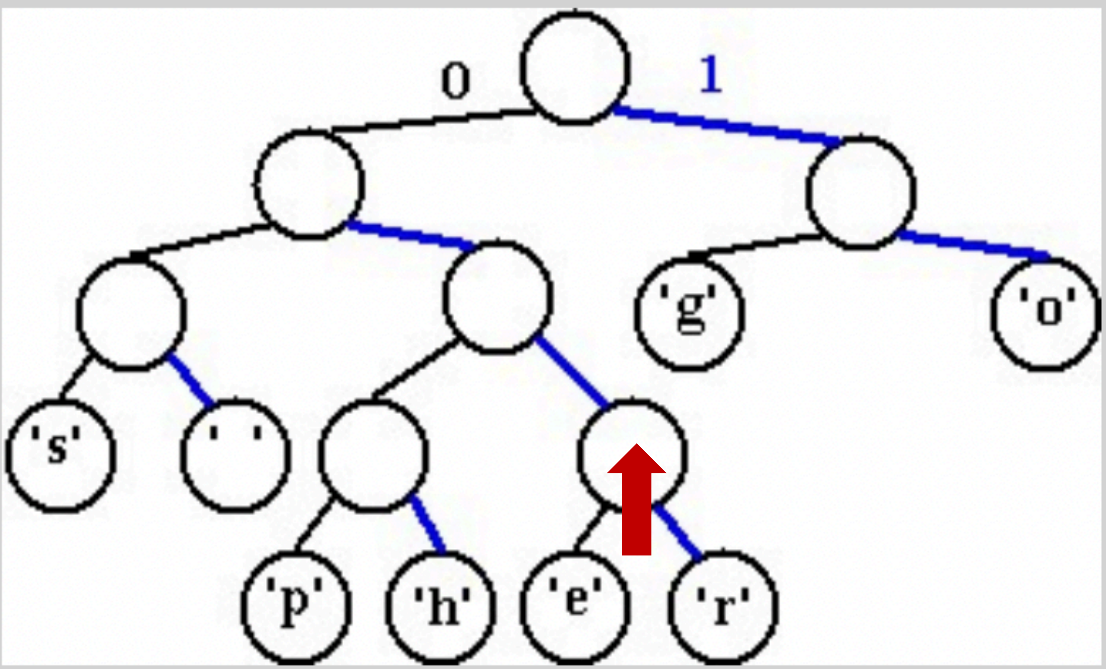
Decoding bits using Huffman tree

Decode 10011011

g

Read 1, go to right child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



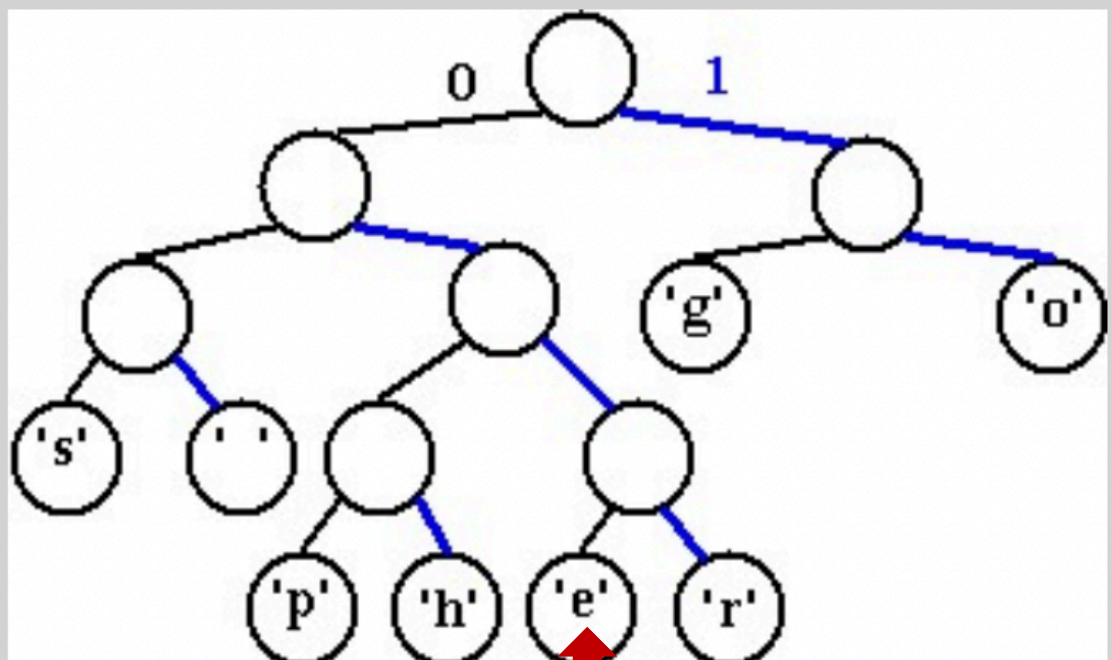
Decoding bits using Huffman tree

Decode 10011011

g

Read 0, go to left child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



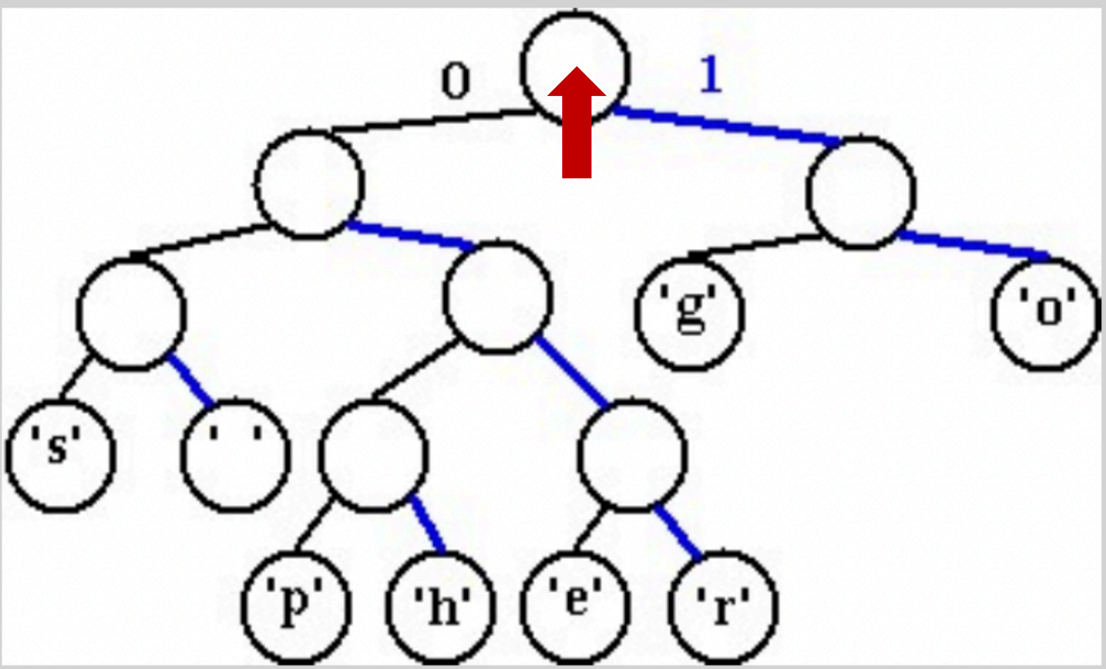
Decoding bits using Huffman tree

Decode 10011011

ge

Leaf, decode ‘e’, restart at root

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



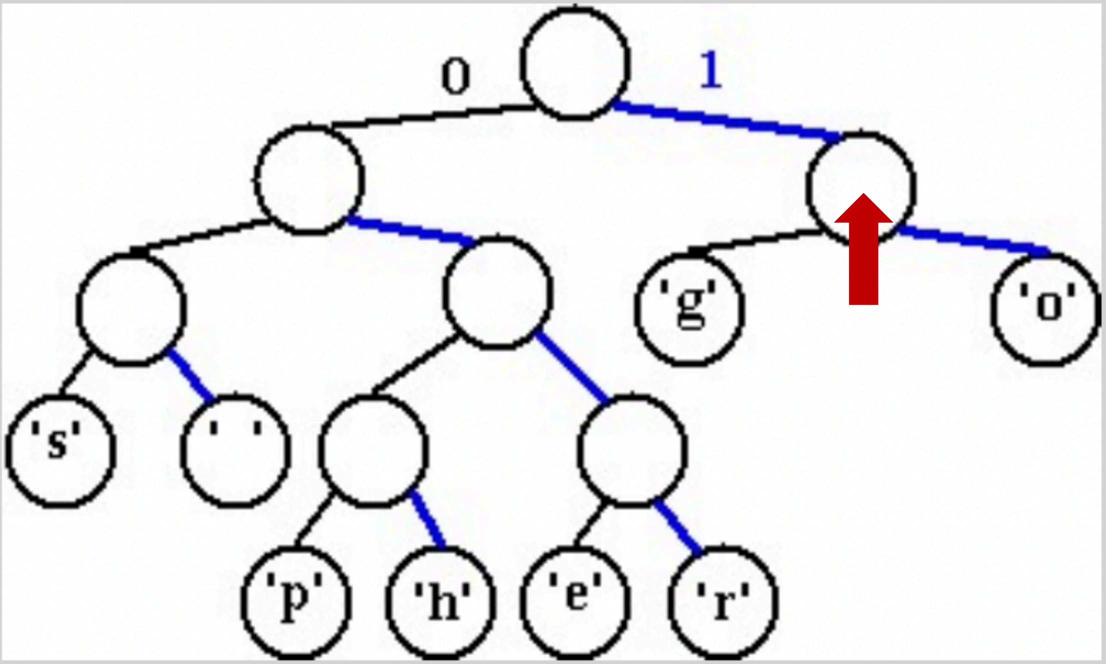
Decoding bits using Huffman tree

Decode 10011011

ge

Read 1, go to right child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



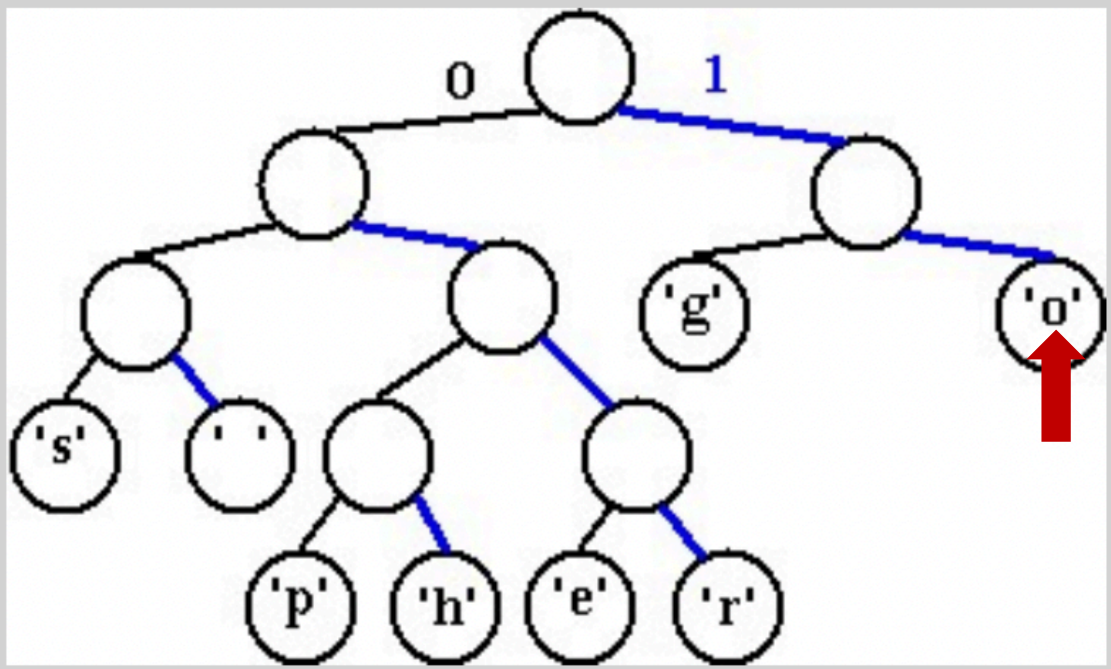
Decoding bits using Huffman tree

Decode 10011011

ge

Read 1, go to right child

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



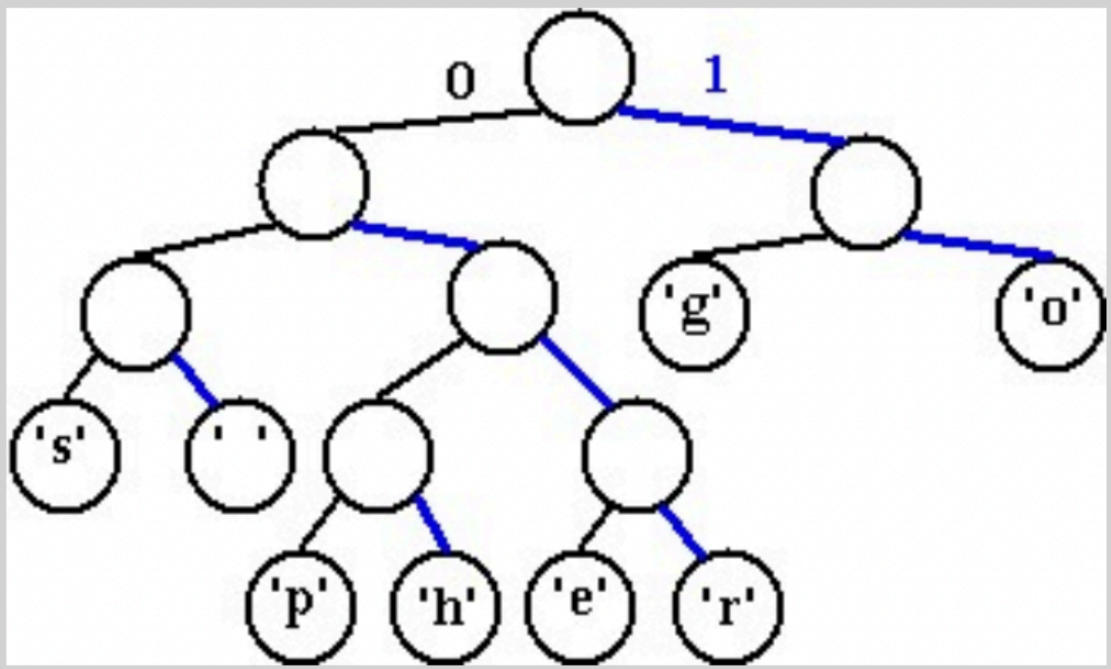
Decoding bits using Huffman tree

Decode 1001 1011

geo

Leaf, decode ‘o’

char	binary
'g'	10
'o'	11
'p'	0100
'h'	0101
'e'	0110
'r'	0111
's'	000
' '	001



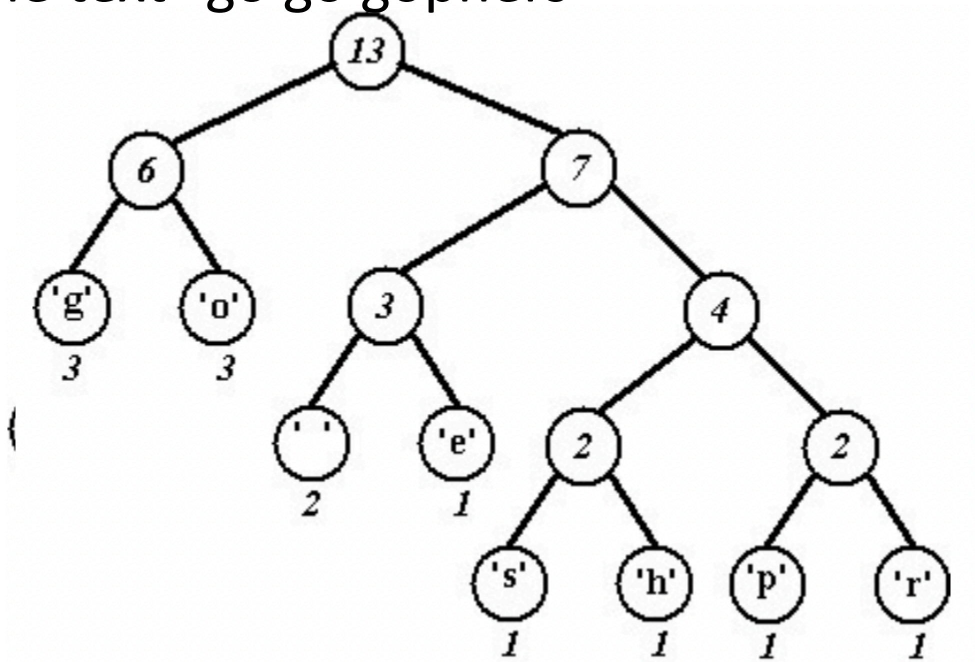
Huffman Coding

- Greedy algorithm for building an optimal variable length encoding tree.
  - Start with the leaf nodes containing values you want to encode with weights = frequency.
  - Iteratively choose the ***lowest weight nodes*** to connect “up” to a new node with weight = sum of children.

- Implementation? Priority queue!

Visualizing the greedy algorithm

Encoding the text “go go gophers”



char	binary
'g'	00
'o'	01
'p'	1110
'h'	1101
'e'	101
'r'	1111
's'	1100
' '	100

P5 Outline

1. Write Decompress first
   - Takes a compressed file (we give you some)
   - Reads Huffman tree from bits
   - Uses tree to decode bits to text

2. Write Compress second
   - Count frequencies of values/characters
   - Greedy algorithm to build Huffman tree
   - Save tree and file encoded as bits

Priority Queues Revisited
Binary Heaps

java.util.PriorityQueue Class

- Kept in sorted order, smallest out first
  - Objects must be Comparable OR provide Comparator to priority queue

```java
PriorityQueue<String> pq = new PriorityQueue<>();
pq.add("is");
pq.add("Compsci 201");
pq.add("wonderful");
while (! pq.isEmpty()) {
    System.out.println(pq.remove());
}
```

```
Compsci 201
is
wonderful
```

```java
PriorityQueue<String> pq = new PriorityQueue<>(
        Comparator.comparing(String::length));
pq.add("is");
pq.add("Compsci 201");
pq.add("wonderful");
while (! pq.isEmpty()) {
    System.out.println(pq.remove());
}
```

```
is
wonderful
Compsci 201
```

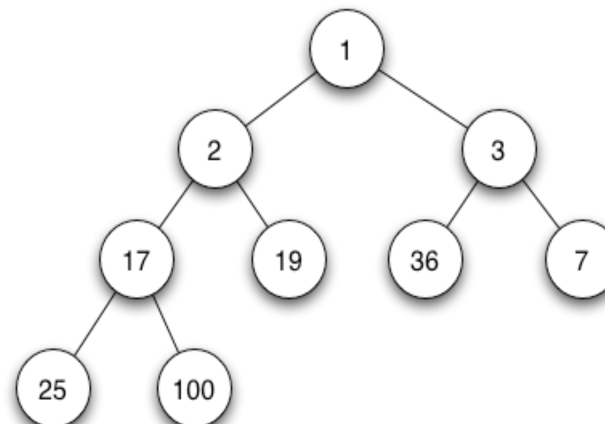
java.util PriorityQueue basic methods

Method	Behavior	Runtime Complexity
`add(element)`	Add an element to the priority queue	O(log(N)) comparisons
`remove()`	Remove and return the minimal element	O(log(N)) comparisons
`peek()`	Return (do *not* remove) the minimal element	O(1)
`size()`	Return number of elements	O(1)

Binary Heap at a high level

A **binary heap** is a binary tree satisfying the following structural invairants:

- Maintain the **heap property** *that every node is less than or equal to its successors*, and
- The **shape property** *that the tree is complete* (full except perhaps last level, in which case it should be filled from left to right*)*



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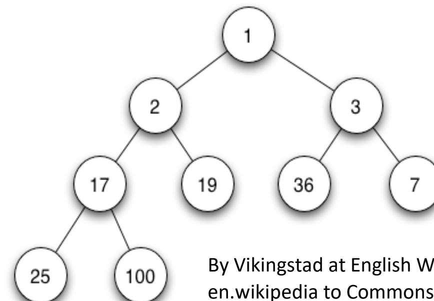
How are binary heaps typically implemented?

- Normally think about a conceptual binary tree underlying the binary heap.



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- Usually implement with an array
  - minimizes storage (no explicit points/nodes)
  - simpler to code, no explicit tree traversal
  - faster too (constant factor, not asymptotically)---children are located by index/position in array

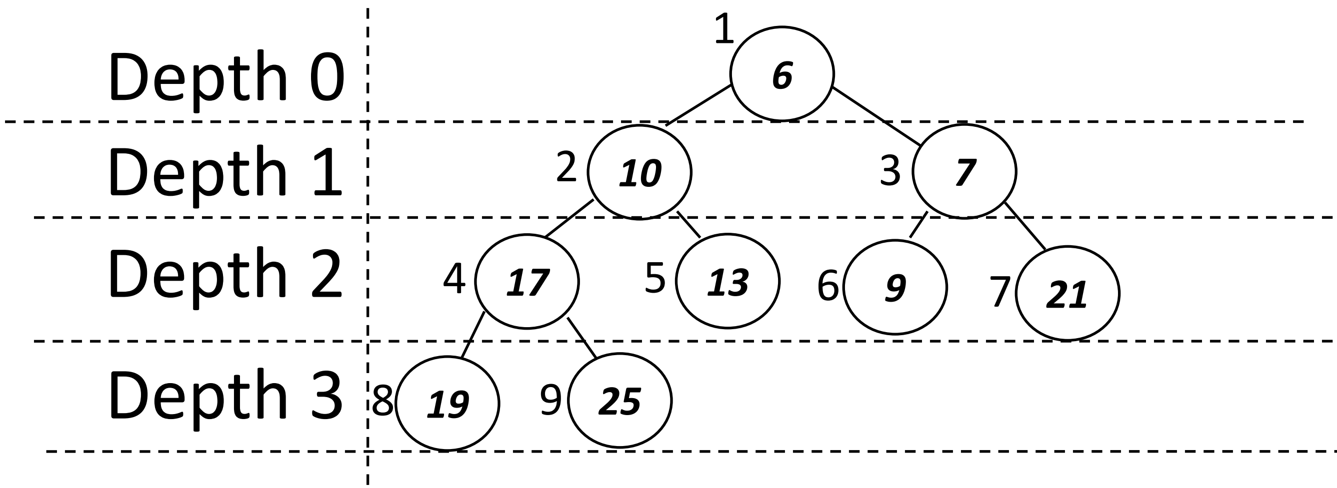
Aside: How much less memory?

- Storing an int takes 4 bytes = 32 bits on most machines.
- Storing one *reference to an object* (a memory location) takes 8 bytes = 64 bits on most machines.

- For a heap storing N integers…
  - Array of N integers takes ~ 4N bytes.
  - Binary tree where each node has an int, left, and right reference takes ~20N bytes.
  - So maybe a 5x savings in memory (just an estimate). Not an asymptotic improvement.

Using an array for a Heap

- Makes it easy to keep track of last "node" in "tree"
- Index positions in the tree level by level, left to right:



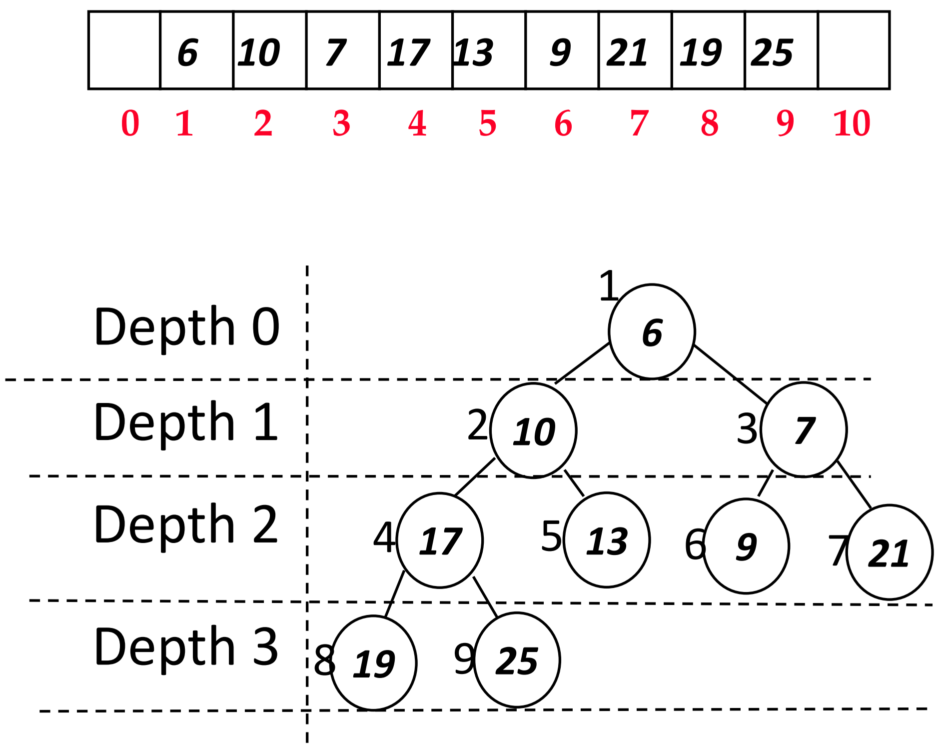
- Last node in the heap is always just the largest index
- Can use indices to represent as an array!

0	1	2	3	4	5	6	7	8	9	10
	6	*10*	*7*	*17*	*13*	*9*	*21*	*19*	*25*	

(`ArrayList` if you want it to be growable)

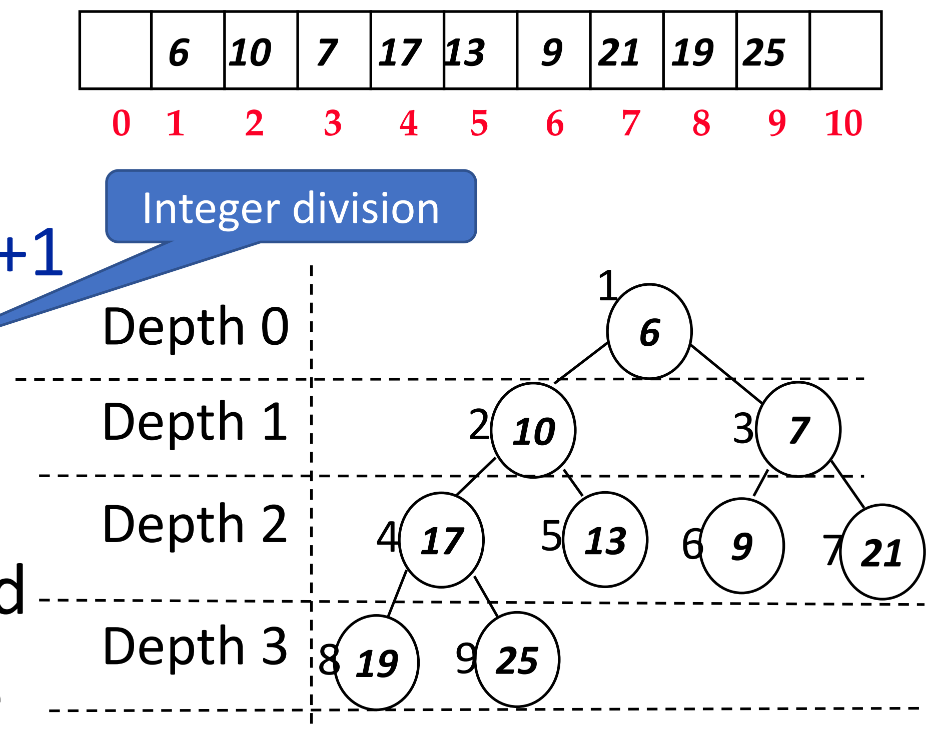
Properties of the Heap Array

- Store "node values" in array beginning at index 1
  - Could 0-index, Zybook does this
- Last "node" is always at the max index
- Minimum "node" is always at index 1
- peek is easy, return first value.
  - How about add?
  - Remove?



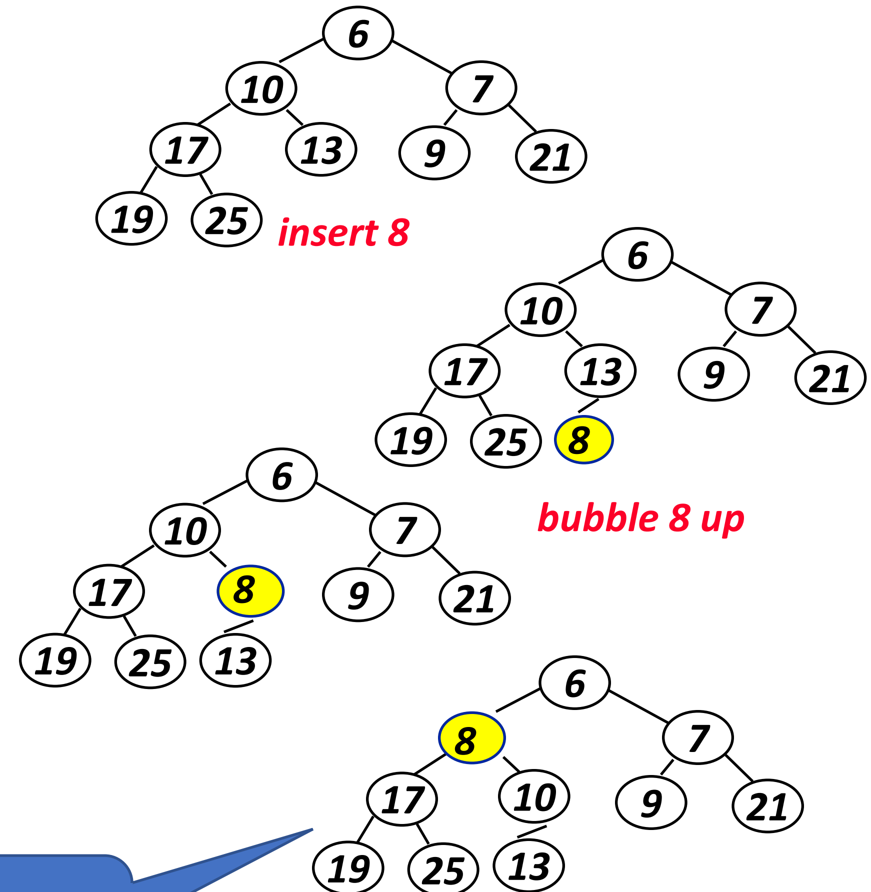
Relating Nodes in Heap Array

- When 1-indexing: For node with index k
  - left child: index 2*k
  - right child: index 2*k+1
  - parent: index k/2
- Why? Follows from:
  - Heap is *complete*, and
  - Complete binary tree has 2^d nodes at depth d (except last)



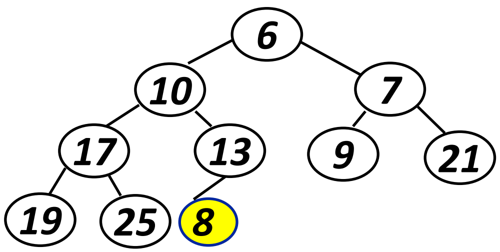
Adding values to heap in pictures

- Add to first open position in last level of the tree
  - (really, add to end of array)
- Swap with parent if heap property violated
  - stop when parent is smaller
  - Or you reach the root



Heap property re-established

Heap add implementation




```java
public void add(Integer value) {
    heap.add(value); // add to last position
    size++;

    int index = size; // note we are 1-indexing
    int parent = index / 2;

    while(parent >= 1 && heap.get(parent) > heap.get(index)) {
        swap(index, parent);
        index = parent;
        parent /= 2;
    }
}
```

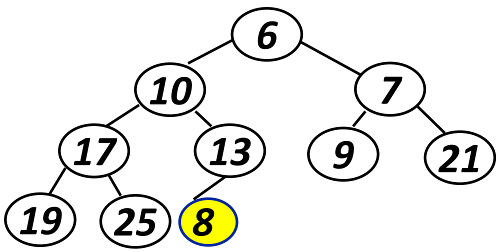
	6	10	7	17	13	9	21	19	25	8
0	1	2	3	4	5	6	7	8	9	10

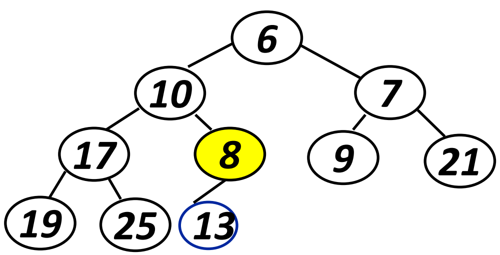
parent=5

index=10

ArrayList<Integer> heap

Heap add implementation






```java
public void add(Integer value) {
    heap.add(value); // add to last position
    size++;

    int index = size; // note we are 1-indexing
    int parent = index / 2;

    while(parent >= 1 && heap.get(parent) > heap.get(index)) {
        swap(index, parent);
        index = parent;
        parent /= 2;
    }
}
```

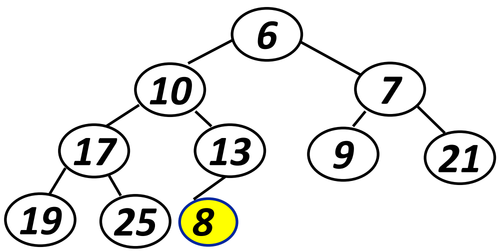
	6	10	7	17	8	9	21	19	25	13
0	1	2	3	4	5	6	7	8	9	10

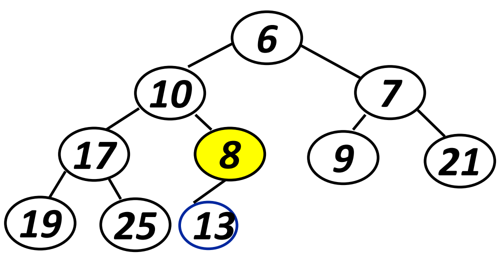
parent=2

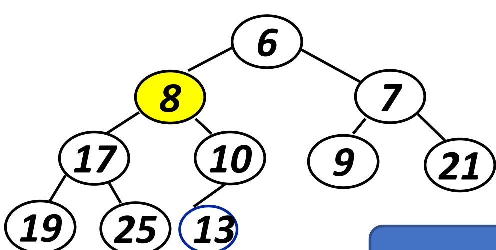
index=5

ArrayList<Integer> heap

Heap add implementation








```java
public void add(Integer value) {
    heap.add(value); // add to last position
    size++;

    int index = size; // note we are 1-indexing
    int parent = index / 2;

    while(parent >= 1 && heap.get(parent) > heap.get(index)) {
        swap(index, parent);
        index = parent;
        parent /= 2;
    }
}
```

	6	8	7	17	10	9	21	19	25	13
0	1	2	3	4	5	6	7	8	9	10

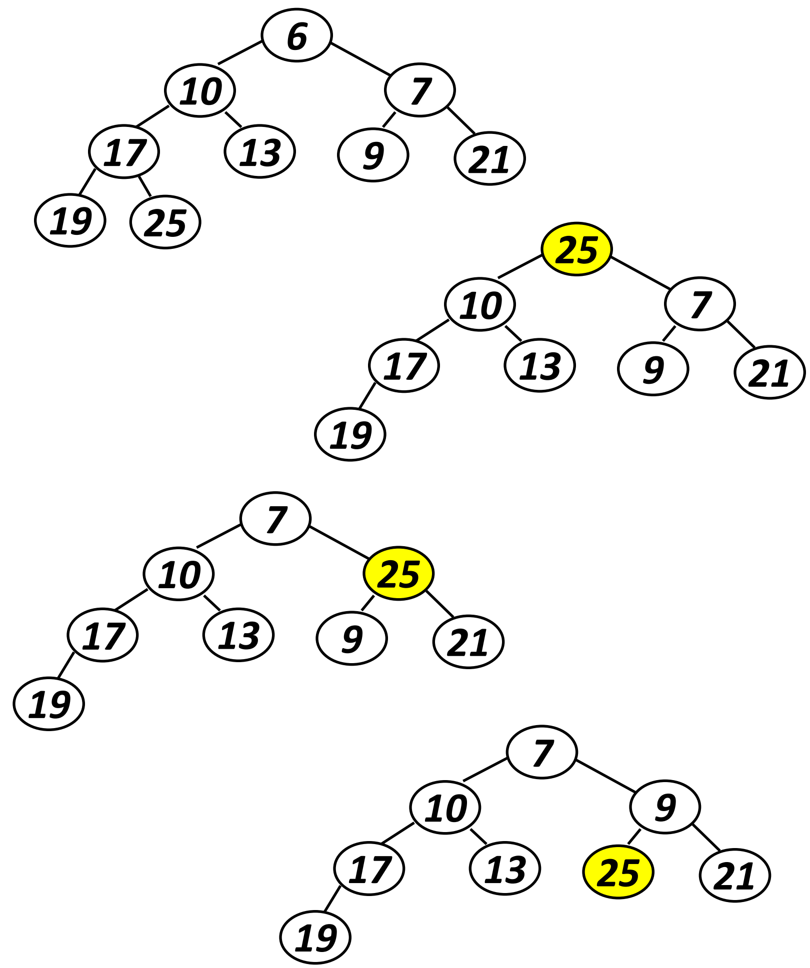
parent=1

index=2

ArrayList<Integer> heap

Heap remove in pictures

- Always return root value
- Replace root with last node in the heap
- While heap property violated, swap with ***smaller*** child.



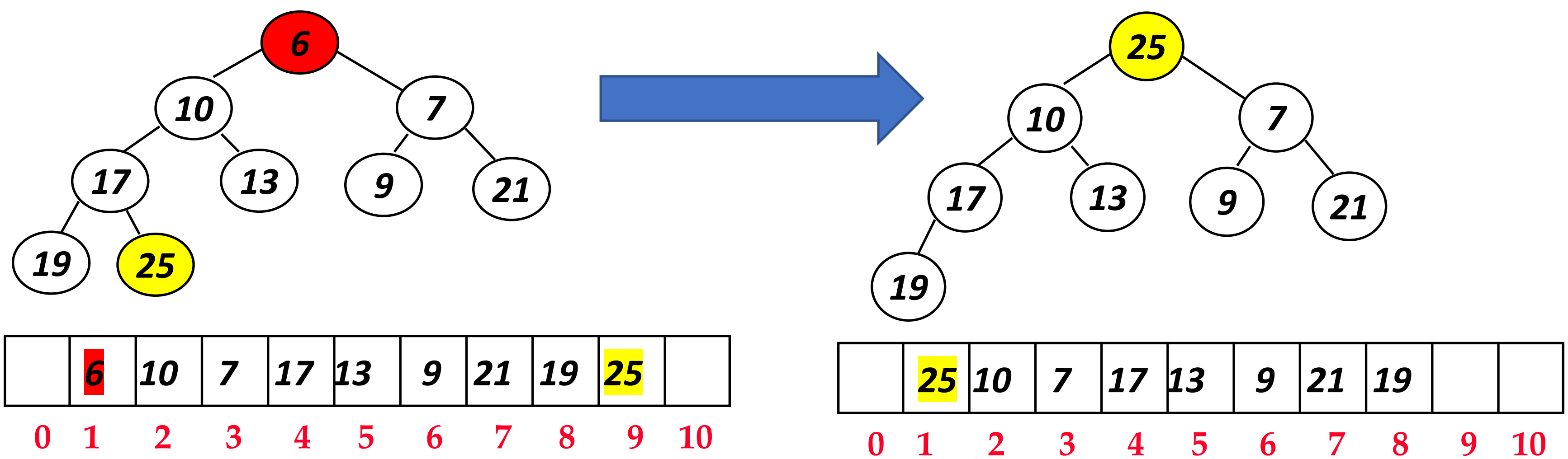
Heap remove implementation

```java
public Integer remove() {
    if (size < 1) { return null; }
    Integer retVal = heap.get(index:1);
    heap.set(index:1, heap.get(size));
    heap.remove(size);
    size--;
    if (size == 0) { return retVal; }
```

Get the minimal value

Replace ”root” with “last node”

Delete “last node”



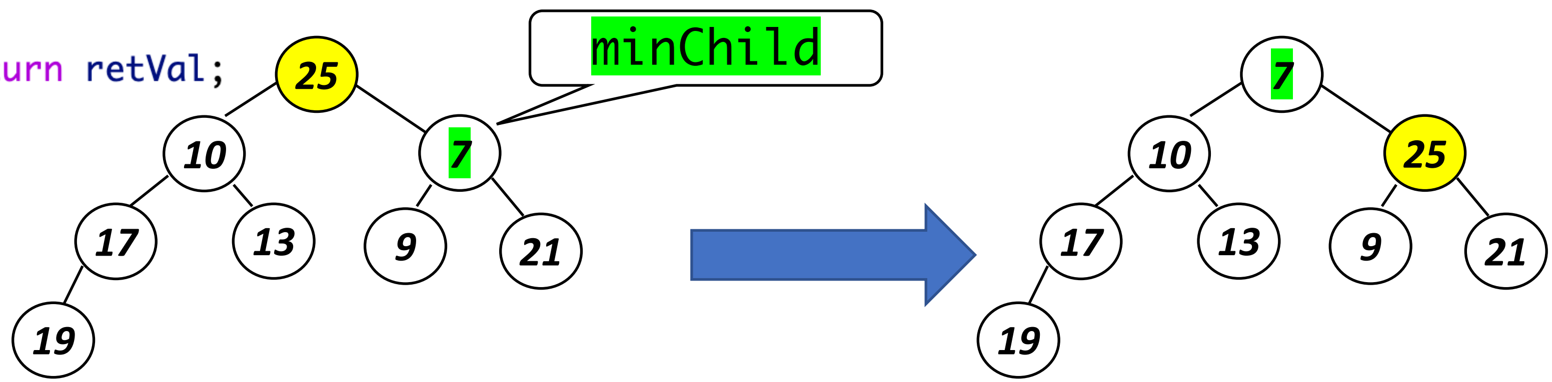
Heap remove implementation

```java
int index = 1;
int minChild = 2;
if (size > 2 && heap.get(index:3) < heap.get(index:2)) { minChild = 3; }
while (minChild <= size && heap.get(index) > heap.get(minChild)) {
    swap(index, minChild);
    index = minChild;
    minChild = minChild * 2;
    if (size > minChild && heap.get(minChild + 1) < heap.get(minChild)) { minChild++; }
}
return retVal;
```

Find the smaller of 2 child nodes

Swap

Violating heap property



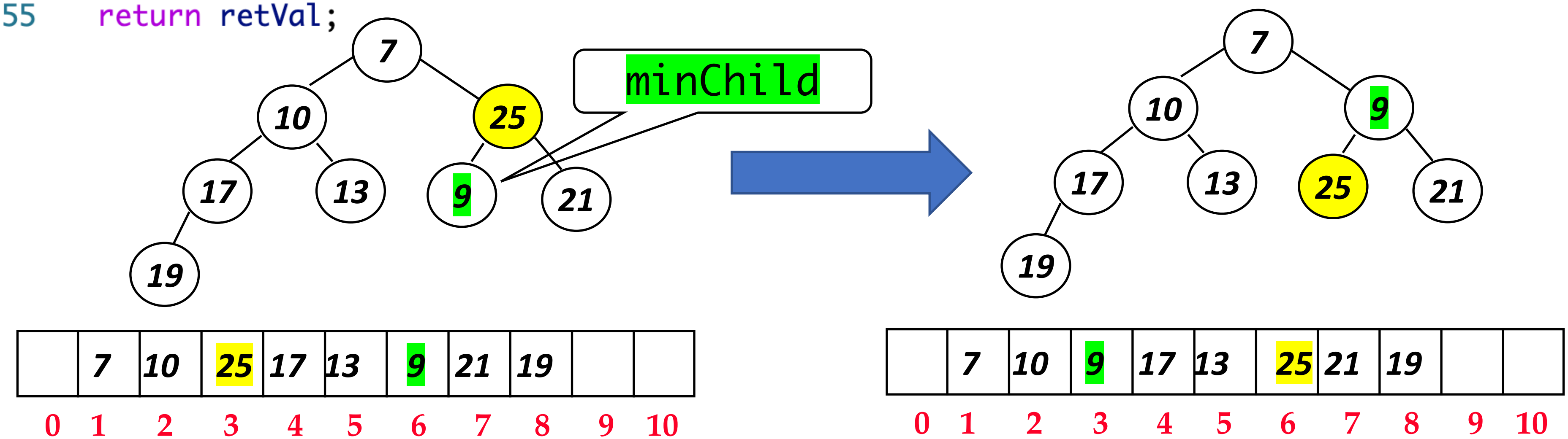
	25	10	7	17	13	9	21	19		
0	1	2	3	4	5	6	7	8	9	10

	7	10	25	17	13	9	21	19		
0	1	2	3	4	5	6	7	8	9	10

Heap remove implementation

```java
int index = 1;
int minChild = 2;
if (size > 2 && heap.get(index:3) < heap.get(index:2)) { minChild = 3; }
while (minChild <= size && heap.get(index) > heap.get(minChild)) {
    swap(index, minChild);
    index = minChild;
    minChild = minChild * 2;
    if (size > minChild && heap.get(minChild + 1) < heap.get(minChild)) { minChild++; }
}
return retVal;
```

Update minChild

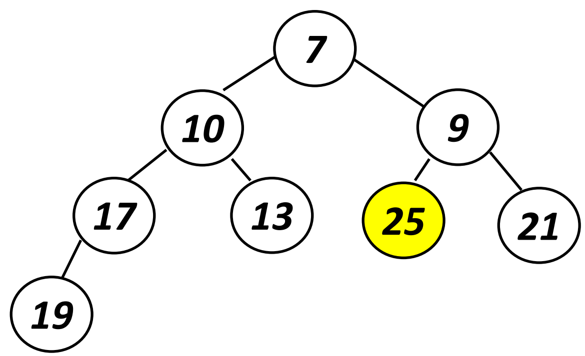


Heap remove implementation

```
int index = 1;
int minChild = 2;
if (size > 2 && heap.get(index:3) < heap.get(index:2)) { minChild = 3; }
while (minChild <= size && heap.get(index) > heap.get(minChild)) {
    swap(index, minChild);
    index = minChild;
    minChild = minChild * 2;
    if (size > minChild && heap.get(minChild + 1) < heap.get(minChild)) { minChild++; }
}
return retVal;
```

2*6 = 12 > size

Return retVal (6)



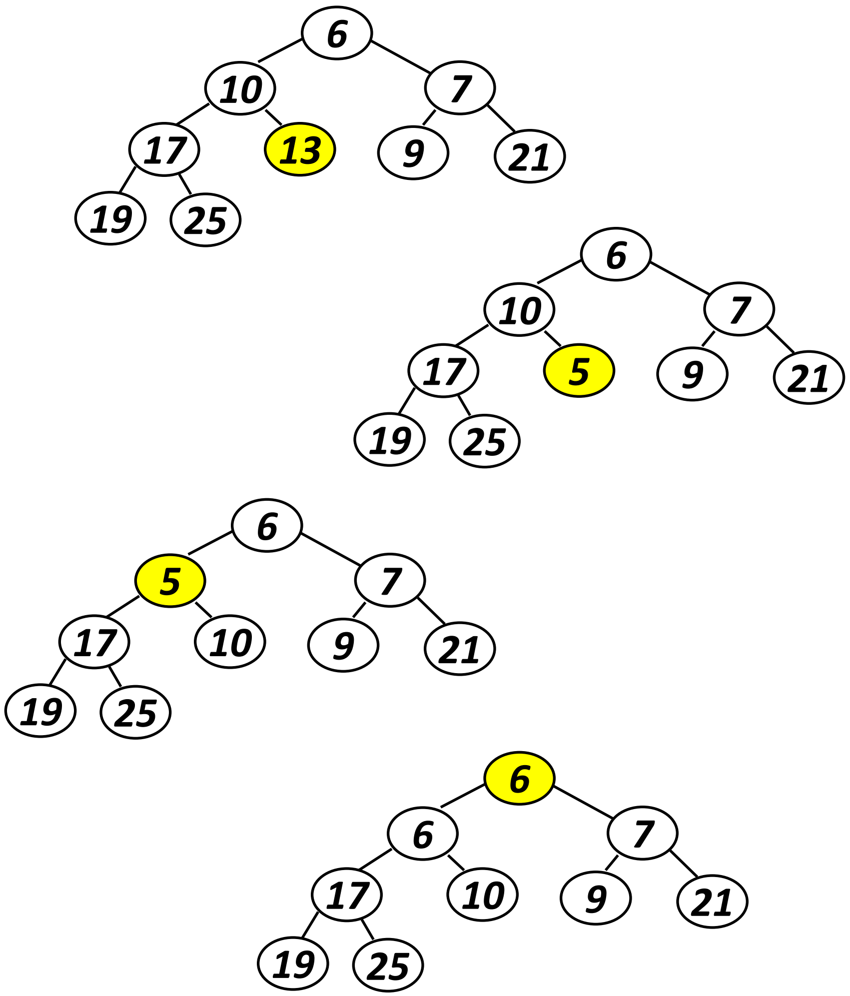
	7	10	9	17	13	25	21	19		
0	1	2	3	4	5	6	7	8	9	10

Heap Complexity

- Claimed that:
  - Peek: O(1)
  - Add: O(log(N))
  - Remove: O(log(N))
- On a heap with N values. Why?
  - Peek: Easy, return first value in an Array
  - Complete binary tree always has height O(log(N)).
  - add and remove “traverse” ***one*** root-leaf path, at most O(log(N)).

decreaseKey Operation?

- Suppose we decrease the 13 to 5.
- Violates heap property
- Fix like in the add operation: While violating heap property:
  - Swap with parent



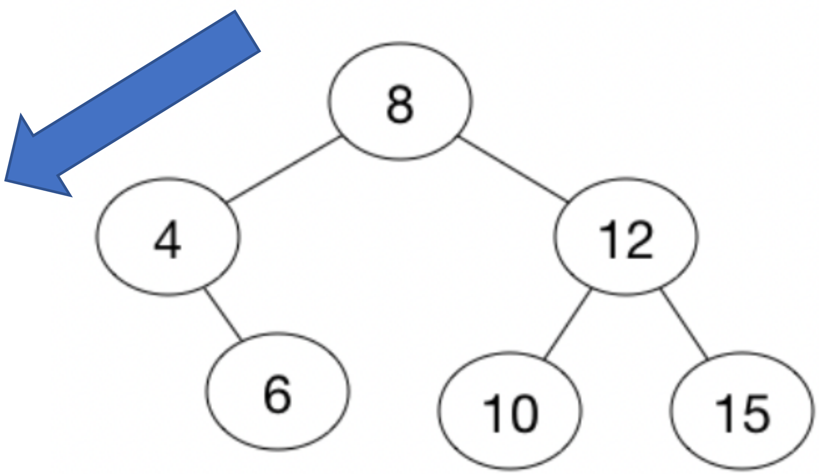
decreaseKey NOT in java.util

- decreaseKey is important for some algorithms, but not supported in many standard libraries (including the java.util PriorityQueue)

- Why not?
  - Note that binary heap does not support efficient *search*
  - In order to do decreaseKey in O(log(n)) time, need to store *references/indices* of all the "nodes."
  - Adds overhead, not done in java.util

Alternative Implementation: Binary Search Tree

- If your keys happen to be unique…
- Can support O(log(n)) add & remove (smallest) using a binary search tree!
- Smallest is leftmost child



PriorityQueue (with unique keys) using a java.util TreeSet

```
import java.util.TreeSet;

public class BSTPQ<T extends Comparable<T>> {
    private TreeSet<T> bst;

    public BSTPQ() { bst = new TreeSet<>(); }
    public void add(T element) { bst.add(element); }
    public int size() { return bst.size(); }
    public T peek() { return bst.first(); }

    public T remove() {
        T returnValue = bst.first();
        bst.remove(returnValue);
        return returnValue;
    }

    public void decreaseKey(T oldKey, T newKey) {
        bst.remove(oldKey);
        bst.add(newKey);
    }
}
```

`first` gives smallest element in TreeSet in O(log(n)) time

Can `decreaseKey` by removing and then re-adding, both O(log(n)) time for a TreeSet

Disadvantages to using a Binary Search Tree for your priority queue?

1. All elements must be unique

2. Not array-based, uses more memory and has higher constant factors on runtime

3. Much harder to implement with guarantees that the tree will be balanced.