Due on January 27, 2020
30 points total

General directions:

All answers to non-programming questions must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a PDF file will also be accepted. To submit your file, upload your PDF on Gradescope for assignment "HW X (PDF)." Handwritten answers or PDF files that cannot be opened will not be graded and will not receive any credit.

Additionally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Point values: Every problem has a specified amount of points which are awarded for the correctness of your solutions. In addition, each proof-oriented problem has an additional **style point**. In the homework handout, this is signified by a "+1" in the point value. To earn this point, your solutions should be clear, well organized, and easy to follow. This is to encourage not only perfectly correct solutions, but well presented ones.

Problem 1 (10 points)
Express the negation of the following predicate formulas P:

(a) [5 points] Let $E(y) =$ "$y > 3$" and $O(y) =$ "y is an integer". Negate:

$$P = (\exists y \in \mathbb{R}. \quad \neg E(y) \vee \neg O(y)).$$

(b) [5 points] Let $A(x), B(x), C(x)$, and $D(x)$ be predicates that depend only on x. Negate:

$$P = (\forall x \in \mathbb{Q}. \quad A(x) \wedge ((\neg B(x) \vee \neg C(x)) \wedge D(x))).$$

Problem 2 (10 points)
Express the following as predicate formulas. Clearly state the meaning of each predicate you create, and the domains of the variables used in the quantifiers:

(a) [5 points] There exists an integer q such that $q + x = x$ for every real number x.

(b) [5 points] Every odd integer greater than 5 is the sum of three prime numbers.

Problem 3 (8+2 points)
For the following pairs of formulas F and G, is there a way to fill in the blanks of G with quantifiers ($\forall$ or $\exists$) so that: (i) G is equivialent to F, (ii) G implies F, and (iii) F implies G? For each of these, your answer should either give the quantifiers that satisfy the equivalence/implication and a short explanation for it, or state that this is impossible.

For part (a), your answers must pick at least one quantifier differently for the variables in G than in F. For part (b), there is no restriction on which quantifiers you may use.

(a) [4+1 points]

$$F = \forall x \exists y.\, P(x, y)$$
$$G = \_x\_y.\, P(x, y)$$

(b) [4+1 points]

$$F = \exists x \forall y.\, P(x, y)$$
$$G = \_y\_x.\, P(x, y)$$

1 Appendix

1.1 Useful Tex Commands

A lot of latex syntax can be learned using google. Overleaf can be a useful service. You might find the following link useful: Overleaf Latex Command Cheat Sheet. Specifically for the logic operations you will be doing in this homework, we have compiled a table of commands that are helpful.

LaTeX command	Result
$\backslash neg$	$\neg$
$\backslash land$	$\land$
$\backslash oplus$	$\oplus$
$\backslash lor$	$\lor$
$\backslash rightarrow$	$\rightarrow$
$\backslash leftrightarrow$	$\leftrightarrow$
$\backslash Leftrightarrow$	$\Leftrightarrow$
$\backslash implies$	$\implies$
$\backslash iff$	$\iff$
$\backslash exists$	$\exists$
$\backslash forall$	$\forall$
$\backslash neq$	$\neq$
$\backslash mathbb\{N\}$	$\mathbb{N}$