Due on Feb. 3, 2020
30 points total

General directions:

All answers to non-programming questions must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a PDF file will also be accepted. To submit your file, upload your PDF on Gradescope for assignment "HW X (PDF)." Handwritten answers or PDF files that cannot be opened will not be graded and will not receive any credit.

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Point values: Every problem has a specified amount of points which are awarded for the correctness of your solutions. In addition, each proof-oriented problem has an additional **style point**. In the homework handout, this is signified by a "+1" in the point value. To earn this point, your solutions should be clear, well organized, and easy to follow. This is to encourage not only perfectly correct solutions, but well presented ones.

Problem 1 (9+1 points)
Prove by contradiction:

Any non-prime positive integer n that is greater than 1 must be divisible by an integer f such that $2 \leq f \leq \sqrt{n}$.

Problem 2 (9+1 points)
Prove the following:

For any integer n, the integer $n^3 - n$ is divisible by 6.

(**Hint:** *Examine the factors of the expression $n^3 - n$, similar to the technique shown in class to prove that $4x - x^3 + 1$ is positive for any $x \in [0, 2]$.*)

Problem 3 (9+1 points)
Prove the following statement by any suitable method:

For integers a, b and c, if $a^2 + b^2 = c^2$, then at least one of a or b is even.

(**Hint:** *Prove that the sum of squares of two odd numbers is even but not divisible by 4, and use this fact to solve the problem.*)