Due on Feb. 17, 2019
20 points total

General directions:

All answers to non-programming questions must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a PDF file will also be accepted. To submit your file, upload your PDF on Gradescope for assignment "HW X (PDF)." Handwritten answers or PDF files that cannot be opened will not be graded and will not receive any credit.

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Point values: Every problem has a specified amount of points which are awarded for the correctness of your solutions. In addition, each proof-oriented problem has an additional **style point**. In the homework handout, this is signified by a "+1" in the point value. To earn this point, your solutions should be clear, well organized, and easy to follow. This is to encourage not only perfectly correct solutions, but well presented ones.

Problem 1 (11+1 points)
Recall that F_n is the n^{th} number of the Fibonacci sequence, defined as follows:

$$F_0 = 0$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2} \quad \text{for } n \geq 2.$$

Prove by strong induction that every non-Fibonacci positive integer can be written as the sum of distinct non-consecutive Fibonacci numbers. For example, $7 = 5 + 2 = F_5 + F_3$, $12 = 8 + 3 + 1 = F_6 + F_4 + F_1$.

(**Hint:** *Consider how to use your inductive hypothesis in your inductive steps.*)

Problem 2 (7+1 points)
For any node u in a binary tree, let $A[u]$ denote the value stored at u. A *binary min-heap* is a rooted binary tree with the following property: if u is the parent of v, then $A[u] \leq A[v]$.

Prove by induction that the minimum value in a binary min-heap is stored at the root.