Due on March 2, 2020
45 points total

General directions:

All answers to non-programming questions must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a PDF file will also be accepted. To submit your file, upload your PDF on Gradescope for assignment "HW X (PDF)." Handwritten answers or PDF files that cannot be opened will not be graded and will not receive any credit.

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Point values: Every problem has a specified amount of points which are awarded for the correctness of your solutions. In addition, each proof-oriented problem has an additional **style point**. In the homework handout, this is signified by a "+1" in the point value. To earn this point, your solutions should be clear, well organized, and easy to follow. This is to encourage not only perfectly correct solutions, but well presented ones.

Problem 1 (24+1 points)
For each relation defined on the set of real numbers $\mathbb{R}$ below, prove if the relation is reflexive, symmetric, and/or transitive.

If $A, B \subseteq \mathbb{R}^2$, then $B \circ A = \{(x, z) \in \mathbb{R}^2 : \exists y \in \mathbb{R}$ such that $(x, y) \in A$ and $(y, z) \in B\}$.

If A and B are sets, then $A \Delta B = (A \setminus B) \cup (B \setminus A)$.

(a) (2 points) $R_1 = \{(x, y) : x < y\}$.

(b) (4 points) $\mathbb{R}^2 \Delta R_2$ where $R_2 = \{(x, y) : x > y\}$.

(c) (4 points) $R_3 \circ R_3$ where $R_3 = \{(x, y) : x \neq y\}$.

(d) (4 points) $R_4 \circ R_5$ where $R_4 = \{(x, y) : x \leq y\}$ and $R_5 = \{(x, y) : x > y\}$.

(e) (4 points) $R_4 \setminus R_1$.

(f) (6 points) $R_7 = \{(x, y) : x - y \in \mathbb{Q}\}$, where $\mathbb{Q}$ is the set of rational numbers.

Problem 2 (19+1 points)
For each function $f : A \to B$ below, prove if the function is surjective and/or injective.

(a) (5 points) $f(x) = 2^x$, $A = \mathbb{Z}$, the set of integers, and $B = \mathbb{R}^+$, the set of positive real numbers.

(b) (5 points) $f((x, y)) = x - y$, $A = \mathbb{Z} \times \mathbb{Z}^+$, where $\mathbb{Z}^+$ is the set of positive integers and $B = \mathbb{Z}$.

(c) (9 points) $f(x) = \begin{cases} 2 - x & \text{if } x \leq 1 \\ 1/x & \text{otherwise} \end{cases}$. Here, $A = B = \mathbb{R}$, the set of real numbers.