Due on January 27, 2020
60 points total

General directions: All answers to non-programming questions must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a PDF file will also be accepted. To submit your file, upload your PDF on Gradescope for assignment "HW X (PDF)." Handwritten answers or PDF files that cannot be opened will not be graded and will not receive any credit.

Additionally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Point values: Every problem has a specified amount of points which are awarded for the correctness of your solutions. In addition, each proof-oriented problem has an additional **style point**. In the homework handout, this is signified by a "+1" in the point value. To earn this point, your solutions should be clear, well organized, and easy to follow. This is to encourage not only perfectly correct solutions, but well presented ones.

Problem 1 (4+1 points)
Socrates says, "If I am guilty, then I must be punished. I am not guilty. Thus, I must not be punished." Formally speaking, is this statement logically sound? Justify your answer. (Hint: put the statements in propositional form.)

Problem 2 (11+3 points)
Consider the operator $\oplus$ defined so that $P \oplus Q$ is **True** if P and Q have different truth values, and **False** otherwise. Prove or disprove each of the following.

(a) **(2+1 points)** $\oplus$ is associative.

(b) **(2+1 points)** $\oplus$ is commutative.

(c) **(2+1 points)** $\oplus$ is idempotent.

(d) **(5 points)** $P \oplus Q = \neg(Q \to P) \vee (P \wedge \neg Q)$.

Problem 3 (20 points)
Determine if the following propositional formulas are equivalent, and give a truth table to back up your answer.

(a) **3 points** $\neg(P \wedge Q)$ and $(\neg P \vee \neg Q)$

(b) **4 points** $\neg(P \to Q)$ and $(\neg P \vee \neg Q) \wedge (\neg Q \to P) \wedge \neg Q$

(c) **5 points** $(P \wedge Q) \to (\neg P \vee Q)$ and $(P \to (\neg P \vee Q)) \wedge (Q \to (\neg P \vee Q))$

(d) **4 points** $((P \to Q) \to Q) \to Q$ and $P \to Q$

(e) **4 points** $P \vee (\neg Q \wedge R)$ and $Q \vee \neg R \to P$

Problem 4 (9 points)
Consider the operator $\#$ defined so that $A \# B$ is **True** if and only if A is **False** and B is **True**. For each of the following expressions, give an equivalent expression which uses only the $\#$ and $\neg$ operators. That is, your expression should have only P, Q, $\neg$, $\#$, and parentheses (where each can be used any number of times).

(a) **(3 points)** $P \wedge Q$.

(b) **(3 points)** $P \vee Q$.

(c) **(3 points)** $P \to Q$.

Problem 5 (12 points)
Write each of the following in both conjunctive normal form, and disjunctive normal form. Show your work (see below for truth table formatting).

- (a) **(4 points)** $(\neg A \vee \neg B) \to \neg((A \wedge C) \wedge (B \wedge \neg C))$
- (b) **(4 points)** $(\neg A \wedge (\neg B \to C)) \leftrightarrow C$
- (c) **(4 points)** $\neg((A \vee B) \wedge C) \wedge (\neg(A \vee B) \vee (C \wedge \neg B)) \wedge \neg(A \wedge B)$

1 Appendix

1.1 Useful Tex Commands

A lot of latex syntax can be learned using google. Overleaf can be a useful service. You might find the following link useful: Overleaf Latex Command Cheat Sheet. Specifically for the logic operations you will be doing in this homework, we have compiled a table of commands that are helpful.

LaTeX command	*Result*
$\neg$	$\neg$
$\land$	$\land$
$\oplus$	$\oplus$
$\lor$	$\lor$
$\rightarrow$	$\rightarrow$
$\leftrightarrow$	$\leftrightarrow$
$\implies$	$\implies$
$\iff$	$\iff$
$\exists$	$\exists$
$\forall$	$\forall$
$\neq$	$\neq$
$\array$	this array of commands uses this!

1.2 Truth Table Format

There are two valid formats for truth tables in this course. First, the verbose method, uses a separate column for each operator. For the proposition $(A \land B) \lor \neg((C \lor A) \rightarrow B)$, this looks as follows.

A	B	C	$(A \land B)$	$(C \lor A)$	$(C \lor A) \rightarrow B$	$\neg((C \lor A) \rightarrow B)$	$(A \land B) \lor \neg((C \lor A) \rightarrow B)$
True	**True**	**True**	**True**	**True**	**True**	**False**	**True**
True	**True**	**False**	**True**	**True**	**True**	**False**	**True**
True	**False**	**True**	**False**	**True**	**False**	**True**	**True**
True	**False**	**False**	**False**	**True**	**False**	**True**	**True**
False	**True**	**True**	**False**	**True**	**True**	**False**	**False**
False	**True**	**False**	**False**	**False**	**True**	**False**	**False**
False	**False**	**True**	**False**	**True**	**False**	**True**	**True**
False	**False**	**False**	**False**	**False**	**True**	**False**	**False**

Here is the latex code for this format:

```
\[
\begin{array}{c|c|c|c|c|c|c|c}
     A&B&C&(A \land B) & (C\lor A)& (C\lor A) \rightarrow B & \neg

 ((C\lor A) \rightarrow B) & (A \land B) \lor \neg ((C \lor A) \rightarrow B)\\
     \hline
     \True & \True & \True&\True & \True & \True & \False&\True\\
     \True&\True&\False&\True&\True&\True&\False&\True\\
     \True&\False&\True&\False&\True&\False&\True&\True\\
     \True&\False&\False&\False&\True&\False&\True&\True\\
     \False&\True&\True&\False&\True&\True&\False&\False\\
     \False&\True&\False&\False&\False&\True&\False&\False\\
     \False&\False&\True&\False&\True&\False&\True&\True\\
     \False&\False&\False&\False&\False&\True&\False&\False
\end{array}
\]
```

The following, while less clear, will also be accepted. The structure is to place a truth column beneath each literal and operator of the original formula as in the example below, with a row on the bottom to indicate the order in which columns were calculated.

A	B	C	$(A$	$\land$	$B)$	$\lor$	$\neg$	$((C$	$\lor$	$A)$	$\rightarrow$	$B)$
True	**True**	**True**		**True**		**True**	**False**		**True**		**True**	
True	**False**	**True**		**False**		**True**	**True**		**True**		**False**	
True	**True**	**False**		**True**		**True**	**False**		**True**		**True**	
True	**False**	**False**		**False**		**True**	**True**		**True**		**False**	
False	**True**	**True**		**False**		**False**	**False**		**True**		**True**	
False	**False**	**True**		**False**		**True**	**True**		**True**		**False**	
False	**True**	**False**		**False**		**False**	**False**		**False**		**True**	
False	**False**	**False**		**False**		**False**	**False**		**False**		**True**	
				1		5	4		2		3	

Here is the latex code for the second format:

```
\[
\begin{array}{ccc|cccccccccc}
     A&B&C&(A&\land &B) &\lor & \neg & ((C&\lor&  A) &\rightarrow& B)  \\
     \hline
     \True&\True & \True && \True&&\True&\False&&\True&&\True\\
     \True & \False & \True&&\False&&\True&\True&&\True&&\False\\
     \True & \True & \False & &\True&&\True&\False&&\True&&\True\\
     \True & \False & \False & &\False&&\True&\True&&\True&&\False\\
     \False & \True & \True & &\False&&\False&\False&& \True&&\True\\
     \False & \False & \True & &\False&&\True&\True&&\True&&\False\\
     \False & \True & \False & &\False&&\False&\False&&\False&&\True\\
     \False & \False & \False & &\False&&\False&\False&&\False&&\True\\
     \hline
     &&&&1&&5&4&&2&&3
\end{array}
\]
```