| **COMPSCI 230: Discrete Mathematics for Computer Science** | January 1 |
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# Recitation CPS 230

*Lecturer: Spring 2020* *Scribe: TA Team*

In this recitation, we will implicitly assume all numbers are positive integers. This recitation is to review the material covered in lecture 1. Specifically, we will talk about logical equivalences. First, some reminders:

**1:** Everyone should have access to Sakai, Piazza, and Gradescope. Please contact the course staff TODAY if you do not have access to one of these platforms.

**2:** You will recieve an invitation to MyDigitalHand this weekend

**3:** Lecture notes will be available on the course website.

**4:** Homework 1 will be released on Monday, 1/13.

**5:** Office hours will also start on Monday, 1/13.

**6:** All feedback is welcome and appreciated. Let us know via email or Piazza!

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Recall the two approaches described in lecture to prove an implication — a proposition of the form "If $P$, then $Q$," "$P$ implies $Q$," or "$Q$ if $P$." $P$ is called the *antecedent*, and $Q$ is called the *consequent*.

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## 1 Show that $\neg(\neg P)$ is equivalent to $P$.

**Proof:**

| $P$ | $\neg P$ | $\neg(\neg P)$ |
|---|---|---|
| $T$ | $F$ | $T$ |
| $F$ | $T$ | $F$ |

Since the first and last columns are identical, the two propositions $P$ and $\neg\neg P$ are equivalent. □

## 2 Show that $P \to Q$ is equivalent to $\neg P \vee Q$ by comparing the statements' truth tables.

**Proof:**

| $P$ | $Q$ | $P \to Q$ | $\neg Q$ | $\neg P \vee Q$ |
|---|---|---|---|---|
| $T$ | $T$ | $T$ | $F$ | $T$ |
| $T$ | $F$ | $F$ | $T$ | $F$ |
| $F$ | $T$ | $T$ | $F$ | $T$ |
| $F$ | $F$ | $T$ | $T$ | $T$ |

Since the third and last columns are identical, the respective propositions are equivalent. □

## 3 Recall the truth table for $P \oplus Q$. Find another logical statement equivalent to $P \oplus Q$ and show this equivalence by comparing the statements' truth tables

**Proof:** For example, $P \oplus Q$ is equivalent to $(P \wedge \neg Q) \vee (\neg P \wedge Q)$.

| $P$ | $Q$ | $P \oplus Q$ | $\neg P$ | $\neg Q$ | $P \wedge \neg Q$ | $\neg P \wedge Q$ | $(P \wedge \neg Q) \vee (\neg P \wedge Q)$ |
|---|---|---|---|---|---|---|---|
| $T$ | $T$ | $F$ | $F$ | $F$ | $F$ | $F$ | $F$ |
| $T$ | $F$ | $T$ | $F$ | $T$ | $T$ | $F$ | $T$ |
| $F$ | $T$ | $T$ | $T$ | $F$ | $F$ | $T$ | $T$ |
| $F$ | $F$ | $F$ | $T$ | $T$ | $F$ | $F$ | $F$ |

□

## 4 Show that the following proposition is True with a direct proof: If $x$ is an even and square integer, then $x$ is divisible by 4.

**Proof:** Let $x$ be an arbitrary even and square integer. Since $x$ is square, then $x = k^2$ for some positive integer $k$. Since $x$ is even, $k^2$ is even, which implies $k$ is even. This implies $k = 2\ell$ for some positive integer $\ell$, so $x = k^2 = (2\ell)^2 = 4\ell^2$ which is divisible by 4. We conclude that the proposition is TRUE.

□

## 5 Show that the following proposition is True: If $x$ an even and prime integer, then $x = 2$.

**Proof:** *Direct proof:* Assume $x$ is a prime number. Since $x$ is prime, $x$ is not divisible by a number smaller than $x$ (other than 1). Since $x$ is even, $x$ is divisible by 2. It follows that $x$ is a number divisible by only 2 and 1, and thus $x = 2$. Therefore, the implication is TRUE. □

## 6 Show that the following proposition is False: If $x$ is even, square, and greater than $4$, then $x$ is divisible by $8$.

**Discussion:** Recall that to prove an implication is false, we must exhibit an $x$ for which the antecedent is True, but the consequent is False. Such an $x$ is called a *counterexample*. For this particular problem, this means we must identify $x$ such that $x$ that is even, square, and greater than 4, but not divisible by 8.

**Proof:** Let $x = 36$. $x$ is even, square, and greater than 4, but $x$ is not divisible by 8. Thus, the proposition is False. □

## 7 Show that the following proposition is False: If $x^2$ is divisible by $1323$, then $x$ is divisible by $1323$.

**Discussion:** For this proposition to be false, there must exist an $x$ such that $x^2$ is divisible by 1323, but $x$ is *not* divisible by 1323. Our goal is to identify such an $x$. Consider the prime factorization of $1323 = 3^3 \cdot 7^2$. Any counterexample $x$ must be such that $x$ has less than three multiples of 3 in its prime factorization or less than two multiples of 7 in its prime factorization (otherwise $x$ would be divisible by 1323). Furthermore, a counterexample $x$ must be so that the prime factorization of $x^2$ has at least three multiples of 3 and two multiples of 7 (so that $x^2$ is divisible by 1323).

**Proof:** Let $x = 3^2 \cdot 7^1 = 63$. Then $x^2 = 3969$ which is divisible by 1323. However, $x$ is not divisible by 1323. Thus, $x = 63$ is a counterexample to the given proposition, so the proposition is False. □

(Note that in the previous problem, 63 is only one of infinitely many counterexamples, but it is the smallest positive counterexample. Can you prove these two propositions?)