CPS102 Recitation

February 5, 2007

Problem 1

How many subsets of a set with 10 elements a) have fewer than 5 elements? b) have more than 7 elements? c) have an odd number of elements?

Solution: a) find the number of r-element subsets for r =0,1,2,3,4 and add. $C(10,0) + C(10,1) + C(10,2) + C(10,3) + C(10,4) = 1 + 10 + 45 + 120 + 210 = 386$

b) $C(10,8) + C(10,9) + C(10,10) = C(10,2) + C(10,1) + C(10,0) = 45 + 10 + 1 = 56$

c) $C(10,1) + C(10,3) + C(10,5) + C(10,7) + C(10,9) = 512$

Problem 2

A shelf holds 12 books in a row. How many ways are there to choose 5 books so that no two adjacent books are chosen? Hint: represent the books that are chosen by bars and the books not chosen by stars. Count the number of sequences of 5 bars and 7 stars so that no 2 bars are adjacent.

Solution: There are 5 bars, 6 places where 7 stars can fit. Each of the 2nd through 5th of these slots must have at least 1 star in it. Once we have placed these 4 stars, there are 3 stars to be placed in 6 slots. The number of ways to do this is therefore C(6+3-1,3) = C(8,3) = 56

Problem 3

How many solutions does the equation

$$x_1 + x_2 + x_3 = 8$$

have, where x_1, x_2, x_3 are nonnegative integers?

Solution: A way to select 8 items from a set with 3 elements, so that x_1 number of type 1, etc. The solution is equal to the number of 8-combinations with repetitions allowed from a set with 3 elements. C(3+8-1,8)

Problem 4

How many solutions are there to the inequality

$$x_1 + x_2 + x_3 \leq 11$$

where x_1, x_2, x_3 are nonnegative integers? Hint: introduce a variable x_4 such that $x_1 + x_2 + x_3 + x_4 = 11$.

Solution: x_4 is nonnegative, same as count the number of nonnegative solutions to the equality. It is C(4+11-1,11) = C(14,3) = 364. Theorem: there are C(n+r-1,r) r-combinations from a set with n elements when repetition of elements is allowed.

Problem 5

How many ways are there to put n identical objects into m distinct containers so that no container is empty?

Solution: solve the equation $x_1+x_2+...x_m = n$ such that $x_i \geq 1$, same as $y_1+...y_m = n-m$ such that $y_j = x_i + 1$. $C(m + (n - m) - 1, n - m) = C(n - 1, n - m)$. Assume $n \geq m$