CPS 130
Homework 2
Due February 12 at the beginning of class

Homework

• Use a separate sheet for each problem.
• Make sure you write your name on every sheet.
• Collaboration is allowed, even encouraged, provided that the names of collaborators are
listed on solutions - you must write up your solutions on your own. No credit is given for
solutions received late. For special situations contact Lars Arge.
• Make sure to justify your answers!

1. Solve the recurrence: \( T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(n - 1) + n(n - 1) & \text{if } n \geq 2 \end{cases} \)
   
   Hint: use \( \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \).

2. Give asymptotic upper and lower bounds for the following recurrences. Assume \( T(n) \) is
   constant for \( n \leq 8 \). Make your bounds as tight as possible, and justify your answers.
   (a) \( T(n) = 2T(n/2) + n^3 \)
   (b) \( T(n) = 2T(n/4) + \sqrt{n} \)
   (c) \( T(n) = 7T(n/2) + n^2 \)

3. Give asymptotic upper and lower bounds for the following recurrences. Assume \( T(n) \) is
   constant for \( n \leq 8 \). Make your bounds as tight as possible, and justify your answers.
   (a) \( T(n) = T(n - 1) + n \)
   (b) \( T(n) = T(\sqrt{n}) + 1 \)
   (c) \( T(n) = 2T(n/2) + n/\log n \)
   (d) \( T(n) = T(n - 1) + 1/n \)

4. 7.1-2 CLRS

5. 7.4-5 CLRS

6. 7-3 CLRS
Practice problems for recitation sessions

1. [From CPS130 spring 2000 midterm]
   Solve the recurrence: \( T(n) = \begin{cases} 
   1 & \text{if } n = 1 \\
   T(n - 1) + n(n - 1)(n + 1) & \text{if } n \geq 2
   \end{cases} \)
   
   **Hint:** use \( \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} \)

2. [From CPS130 spring 2000 final]
   Show by induction that the recurrence \( T(n) = \begin{cases} 
   1 & \text{if } n = 1 \\
   2T\left(\frac{n}{2}\right) + b\log n & \text{if } n \geq 2
   \end{cases} \)
   where \( b \) is a positive constant has solution \( T(n) = O(n) \).
   
   **Hint:** Show that there exist positive constants \( a \) and \( c \) such that \( T(n) \leq an - b\log n - c \).

3. 7-2 CLR

4. 7-4 CLR