Algorithmic mechanism design

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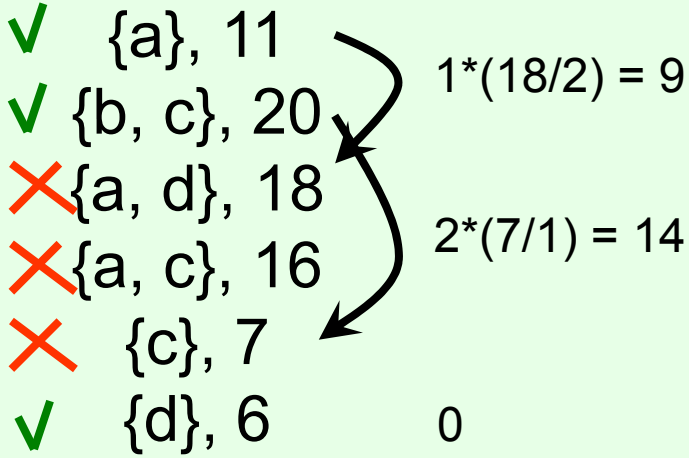
Algorithmic mechanism design

- Mechanisms should be accompanied by an efficient algorithm for computing the outcome
- May not be easy
  - E.g., using the Clarke (VCG) mechanism in combinatorial auctions requires solving the winner determination problem optimally
- If the mechanism's outcomes are too hard to compute, we may need a different mechanism
- Algorithmic mechanism design [Nisan & Ronen STOC 99] = simultaneous design of mechanism and algorithm for computing its outcomes
  - Given a mechanism, is there an efficient algorithm for computing its outcomes?
  - Given an algorithm for choosing outcomes, can we make it incentive compatible (e.g., using payments)?

Combinatorial auctions: mechanisms that solve the winner determination problem approximately

- Running Clarke mechanism using approximation algorithms for WDP is generally not strategy-proof
- Assume bidders are single-minded (only want a single bundle)
- A greedy strategy-proof mechanism [Lehmann, O'Callaghan, Shoham JACM 03]:

1. Sort bids by (value/bundle size)
2. Accept greedily starting from top



3. Winning bid pays bundle size times (value/bundle size) of first bid forced out by the winning bid

Worst-case approximation ratio = (#items)

Can get a better approximation ratio, √(#items), by sorting by value/√(bundle size)

Clarke-type payments with same approximation algorithm do not work

✓ {a}, 11
✓ {b, c}, 20
✗ {a, d}, 18
✗ {a, c}, 16
✗ {c}, 7
✓ {d}, 6

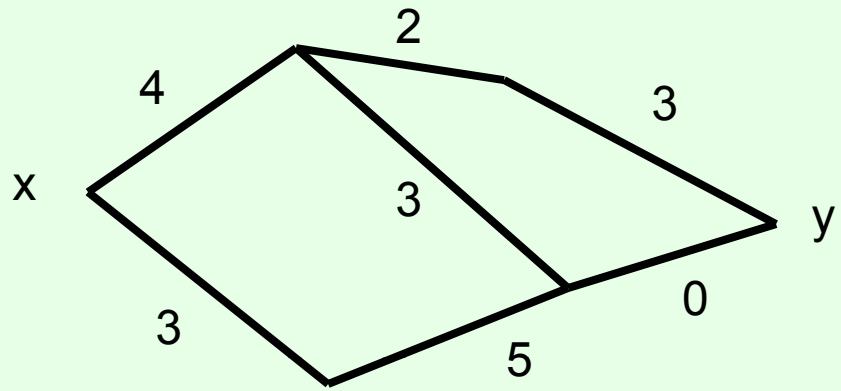
Total value to bidders other than the {a} bidder: 26

✓ {b, c}, 20
✓ {a, d}, 18
✗ {a, c}, 16
✗ {c}, 7
✗ {d}, 6

Total value: 38

{a} bidder should pay 38 - 26 = 12, more than her valuation!

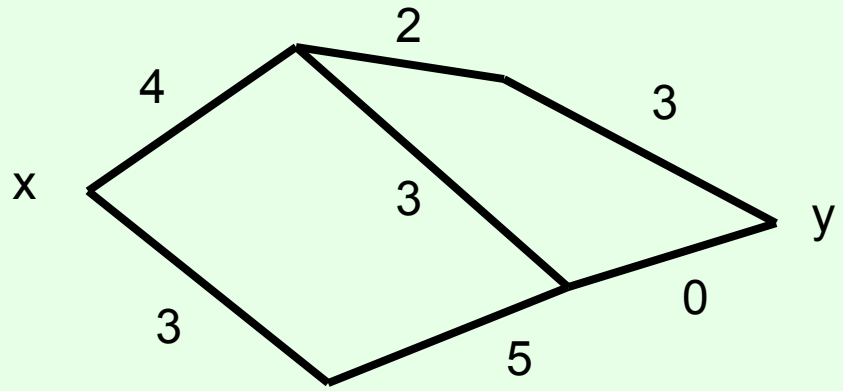
A shortest path/combinatorial reverse auction problem [Nisan & Ronen STOC 99]



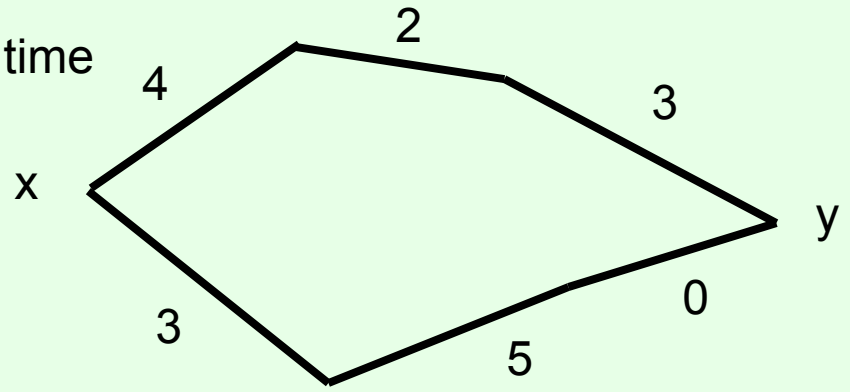
- Someone wants to buy edges that constitute a path from x to y
- Each edge e has a separate owner, and that owner submits (bids) a cost c_e for it
- Goal:
  - buy the shortest path (= path with minimum total weight),
  - pay every edge according to Clarke mechanism
    - no incentive to misreport costs
- That is, an edge e on the shortest path is paid

  (cost of shortest path without e) - (cost of shortest path with e) + c_e

Computing Clarke payments



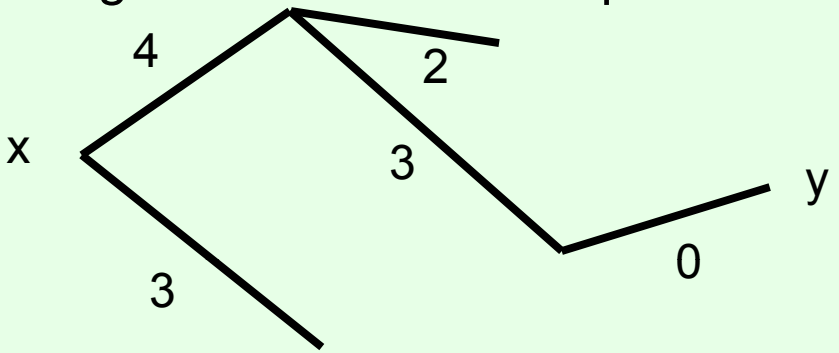
- One strategy:
  - Compute shortest path (e.g., using Dijkstra's algorithm)
  - For each edge on the shortest path, remove that edge, solve the problem again
    - $O(nm + n^2 \log n)$ total time

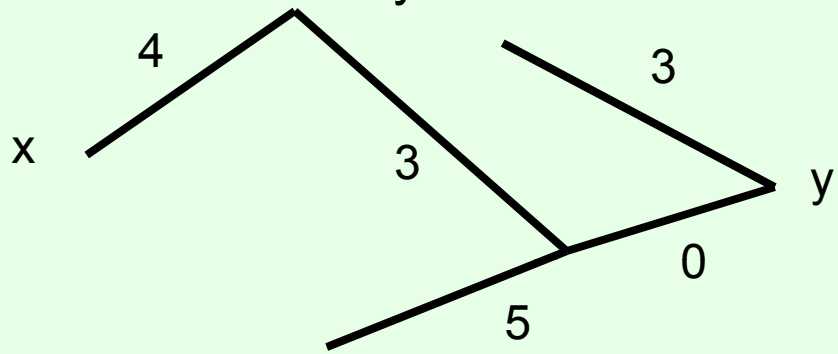


- Is there a more efficient algorithm?

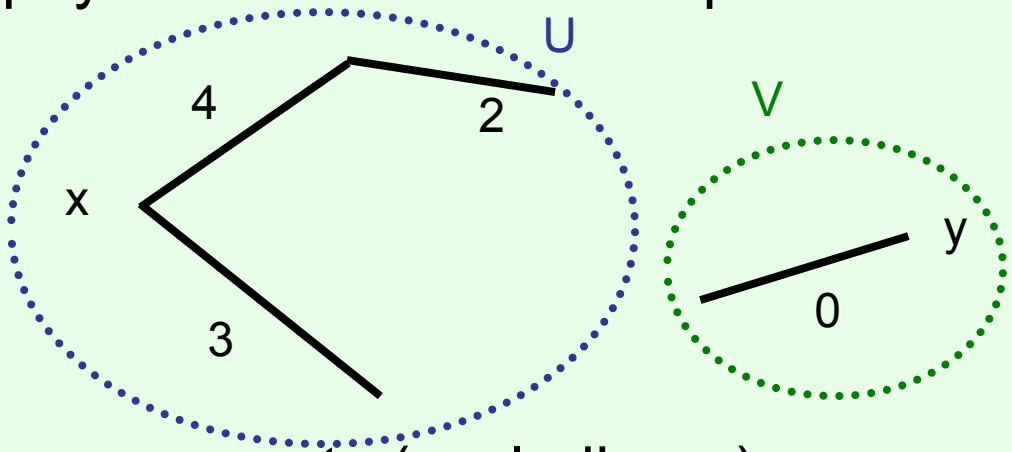
Hershberger-Suri [FOCS 01] algorithm

- Compute the shortest path trees from x and from y
  - using Dijkstra
  - gives us the shortest path from any vertex to x and to y





- Remove the edge e whose payment we wish to compute from the first (x) tree
  - Cuts the graph into U and V



- Over all edges (u, v) across components (excluding e), minimize d(x, u) + c(u, v) + d(v, y)
  - Using data structures, can be done for all edges in O(n log n + m)

A make-span/reverse auction problem

[Nisan & Ronen STOC 99]

- There are m jobs that need to be scheduled on (say) 2 machines
- Each machine is owned by a separate agent
- c_{ij} is the time that machine i would take on job j
  - also the cost that machine i has for doing j
  - private information
- The objective is to minimize the make-span
  - = highest total cost for an agent, = completion time of last job
- One possibility: just use Clarke mechanism
  - Award job j to the machine that can do it faster (minimize total work),
  - Pay that machine the cost of the other machine for j
- Gives a 2-approximation to the make-span
  - Theorem: No deterministic mechanism does better

A bad instance for the Clarke mechanism

- Two jobs
- Machine 1: $c_{11} = 1$, $c_{12} = 1$
- Machine 2: $c_{21} = 1+\varepsilon$, $c_{22} = 1+\varepsilon$

- Clarke mechanism will give both jobs to 1
- Make-span: 2
- Can get $1+\varepsilon$ by giving one job to each (ignoring mechanism design considerations)

Weighted Groves mechanisms

- Recall a Groves mechanism
  - chooses an allocation o that maximizes the sum of reported utilities,
  - pays agent i: $\Sigma_{j \neq i} u_j(\theta_j', o) + h(\theta_{-i}')$ for some function h
- A weighted Groves mechanism
  - has a weight w_i for each agent,
  - chooses an allocation o that maximizes $\Sigma w_i u_i(\theta_i', o)$,
  - pays agent i: $(1/w_i)\Sigma_{j \neq i} w_j u_j(\theta_j', o) + h(\theta_{-i}')$ for some function h
- Weighted Groves mechanisms are strategy-proof [Roberts 1979]

A biased mechanism based on a weighted Groves mechanism

[Nisan & Ronen STOC 99]

- For each job j, bias the mechanism towards accepting one of the two agents i
  - For some b > 1, award job j to i if and only if $c_{ij} < bc_{(-i)j}$
  - If so, i gets payment $bc_{(-i)j}$
  - Otherwise, -i gets payment c_{ij}/b
- Weighted Groves mechanism, so strategy-proof
- A randomized mechanism:
  - set b = 4/3,
  - for each job independently, randomly choose the agent to which the mechanism is biased
- Gives a 7/4 approximation

Characterizing allocation rules that can be made incentive compatible

- We saw that we may be interested in choosing allocations that do not maximize social welfare (sum of utilities)
  - Different objectives (e.g., make-span)
  - Social welfare maximizing allocation may be computationally too hard to find
- Some (not all) allocation rules can be made incentive compatible with the right payment rule
- What are necessary and sufficient conditions on an allocation rule for this to be possible?

Weak monotonicity

[Bikhchandani et al. Econometrica 06]

- Consider the case of a single type reporting agent
  - Equivalently, fix the types of the other players
- $o(\theta)$ is the allocation chosen when the agent reports θ
- $u(\theta, o)$ is the agent's utility for allocation o given true type θ
- Rule $o(\cdot)$ is said to be weakly monotone if the following condition holds for every θ, θ':

  $u(\theta, o(\theta)) - u(\theta, o(\theta')) \geq u(\theta', o(\theta)) - u(\theta', o(\theta'))$
- In words: if there are no payments, then
  - the utility loss from misreporting θ' when the true type is θ
  - is at least as great as
  - the utility gain from misreporting θ when the true type is θ'

Necessity of weak monotonicity

- Suppose an allocation rule $o(\cdot)$, together with a payment rule $\pi(\cdot)$, incentivizes the agent to tell the truth
- Then, for any θ, θ', the following must hold:
- $u(\theta, o(\theta)) + \pi(\theta) - u(\theta, o(\theta')) - \pi(\theta') \geq 0$
- $u(\theta', o(\theta')) + \pi(\theta') - u(\theta', o(\theta)) - \pi(\theta) \geq 0$
- Adding these two together gives
- $u(\theta, o(\theta)) - u(\theta, o(\theta')) + u(\theta', o(\theta')) - u(\theta', o(\theta)) \geq 0$
- Equivalently
- $u(\theta, o(\theta)) - u(\theta, o(\theta')) \geq u(\theta', o(\theta)) - u(\theta', o(\theta'))$
- But this is the weak monotonicity condition!

Sufficiency of weak monotonicity

- Suppose the agent has a partial order ≥ over the allocations
- Here o ≥ o' indicates that the agent prefers o to o' for every type that she may have
  - E.g., in o, she is allocated a superset of what she is allocated in o', and free disposal holds
- The set of types is said to be rich if every utility function consistent with ≥ corresponds to some type
- **Theorem.** If preferences are rich, weak monotonicity is sufficient for incentive compatibility
  - I.e., for any weakly monotone allocation rule, a payment function making this rule incentive compatible exists
- With more restricted type spaces, weak monotonicity is not always sufficient