Relational Model & Algebra

CPS 196.3

Introduction to Database Systems

Announcements

- Lectures slides on Web
  - "Notes" version available an hour before lecture
  - Complete version available after the lecture
  - No hardcopy handout
- Reading assignment posted on Web (under "Tentative Syllabus")
- The first homework will be assigned next Tuesday
- Office hours
  - After lectures on Tuesdays and Thursday, in D327
  - Or by appointment

Relational data model

- A database is a collection of relations (or tables)
- Each relation has a list of attributes (or columns)
  - Set-valued attributes not allowed
- Each attribute has a domain (or type)
- Each relation contains a set of tuples (or rows)
  - Duplicates not allowed
- Simplicity is a virtue!

Example

Student

SID	*name*	*age*	*GPA*
142	Bart	10	2.3
123	Milhouse	10	3.1
857	Lisa	8	4.3
456	Ralph	8	2.3
...	...	...	...

Course

CID	*title*
CPS196	Intro. to Database Systems
CPS130	Analysis of Algorithms
CPS114	Computer Networks
...	...

Enroll

SID	*CID*
142	CPS196
142	CPS114
123	CPS196
857	CPS196
857	CPS130
456	CPS114
...	...

Ordering of rows doesn't matter (even though the output is always in *some* order)

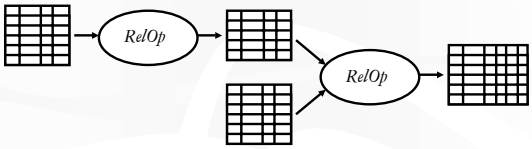
Schema versus instance

- Schema (metadata)
  - Specification of how data is to be structured logically
  - Defined at set-up
  - Rarely changes
- Instance
  - Content
  - Changes rapidly, but always conforms to the schema
- Compare to types and variables in a programming language

Example

- Schema
  - *Student* (*SID* integer, *name* string, *age* integer, *GPA* float)
  - *Course* (*CID* string, *title* string)
  - *Enroll* (*SID* integer, *CID* integer)
- Instance
  - { ⟨142, Bart, 10, 2.3⟩, ⟨123, Milhouse, 10, 3.1⟩, ...}
  - { ⟨CPS196, Intro. to Database Systems⟩, ...}
  - { ⟨142, CPS196⟩, ⟨142, CPS114⟩, ...}

Relational algebra operators



- Core set of operators:
  - Selection, projection, cross product, union, difference, and renaming
- Additional, derived operators:
  - Join, natural join, intersection, etc.

Selection

- Input: a table R
- Notation: $\sigma_p(R)$
  - p is called a selection condition/predicate
- Purpose: filter rows according to some criteria
- Output: same columns as R, but only rows of R that satisfy p

Selection example

- Students with GPA higher than 3.0

  $\sigma_{GPA > 3.0}$ (*Student*)

SID	name	age	GPA
142	Bart	10	2.3
123	Milhouse	10	3.1
857	Lisa	8	4.3
456	Ralph	8	2.3

$\sigma_{GPA > 3.0}$

SID	name	age	GPA
123	Milhouse	10	3.1
857	Lisa	8	4.3

More on selection

- Selection predicate in general can include any column of R, constants, comparisons such as $=$, $\leq$, etc., and Boolean connectives $\wedge$, $\vee$, and $\neg$
  - Example: straight A students under 18 or over 21

    $\sigma_{GPA \geq 4.0 \wedge (age < 18 \vee age > 21)}$ (*Student*)
- But you must be able to evaluate the predicate over a single row
  - Example: student with the highest GPA

    $\sigma_{\cancel{GPA \geq \text{ all GPA in } Student \text{ table}}}$ (*Student*)

Projection

- Input: a table R
- Notation: π_L (R)
  - L is a list of columns in R
- Purpose: select columns to output
- Output: same rows, but only the columns in L

Projection example

- ID's and names of all students

  $\pi_{SID,\ name}$ (*Student*)

SID	name	age	GPA
142	Bart	10	2.3
123	Milhouse	10	3.1
857	Lisa	8	4.3
456	Ralph	8	2.3

$\pi_{SID,\ name}$

SID	name
142	Bart
123	Milhouse
857	Lisa
456	Ralph

More on projection

- Duplicate output rows must be removed
  - Example: student ages

π_{age} (*Student*)

SID	name	age	GPA
142	Bart	10	2.3
123	Milhouse	10	3.1
857	Lisa	8	4.3
456	Ralph	8	2.3

π_{age}

age
10
8

Cross product

- Input: two tables R and S
- Notation: $R \times S$
- Purpose: pairs rows from two tables
- Output: for each row r in R and each row s in S, output a row rs (concatenation of r and s)

Cross product example

- *Student* × *Enroll*

SID	name	age	GPA
142	Bart	10	2.3
123	Milhouse	10	3.1
...	...	...	...

SID	CID
142	CPS196
142	CPS114
123	CPS196
...	...

$\times$

SID	name	age	GPA	SID	CID
142	Bart	10	2.3	142	CPS196
142	Bart	10	2.3	142	CPS114
142	Bart	10	2.3	123	CPS196
123	Milhouse	10	3.1	142	CPS196
123	Milhouse	10	3.1	142	CPS114
123	Milhouse	10	3.1	123	CPS196
...	...	...	...	...	...

A note on column ordering

- The ordering of columns in a table is considered unimportant (so is the ordering of rows)

SID	name	age	GPA	SID	CID
142	Bart	10	2.3	142	CPS196
142	Bart	10	2.3	142	CPS114
142	Bart	10	2.3	123	CPS196
123	Milhouse	10	3.1	142	CPS196
123	Milhouse	10	3.1	142	CPS114
123	Milhouse	10	3.1	123	CPS196
...	...	...	...	...	...

=

SID	CID	SID	name	age	GPA
142	CPS196	142	Bart	10	2.3
142	CPS114	142	Bart	10	2.3
123	CPS196	142	Bart	10	2.3
142	CPS196	123	Milhouse	10	3.1
142	CPS114	123	Milhouse	10	3.1
123	CPS196	123	Milhouse	10	3.1
...	...	...	...	...	...

- That means cross product is commutative, i.e., $R \times S = S \times R$ for any R and S

Derived operator: join

- Input: two tables R and S
- Notation: $R \bowtie_p S$
  - p is called a join condition/predicate
- Purpose: relate rows from two tables according to some criteria
- Output: for each row r in R and each row s in S, output a row rs if r and s satisfy p
- Shorthand for σ_p ($R \times S$)

Join example

- Info about students, plus CID's of their courses

Student $\bowtie_{Student.SID\ =\ Enroll.SID}$ *Enroll*

SID	name	age	GPA
142	Bart	10	2.3
123	Milhouse	10	3.1
...	...	...	...

SID	CID
142	CPS196
142	CPS114
123	CPS196
...	...

$\bowtie_{Student.SID\ =\ Enroll.SID}$

SID	name	age	GPA	SID	CID
142	Bart	10	2.3	142	CPS196
142	Bart	10	2.3	142	CPS114
123	Milhouse	10	3.1	123	CPS196
...	...	...	...	...	...

Derived operator: natural join

- Input: two tables R and S
- Notation: $R \bowtie S$
- Purpose: relate rows from two tables, and
  - Enforce equality on all common attributes
  - Eliminate one copy of common attributes
- Shorthand for $\pi_L (R \bowtie_p S)$
  - L is the union of all attributes from R and S, with duplicates removed
  - p equates all attributes common to R and S

Natural join example

- $Student \bowtie Enroll = \pi_? (Student \bowtie_? Enroll)$

  $= \pi_{SID,\, name,\, age,\, GPA,\, CID} (Student \bowtie_{Student.SID\, =\, Enroll.SID} Enroll)$

SID	name	age	GPA
142	Bart	10	2.3
123	Milhouse	10	3.1
...	...	...	...

SID	CID
142	CPS196
142	CPS114
123	CPS196
...	...

SID	name	age	GPA		CID
142	Bart	10	2.3		CPS196
142	Bart	10	2.3		CPS114
123	Milhouse	10	3.1		CPS196
...	...	...	...		...

Union

- Input: two tables R and S
- Notation: $R \cup S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R and all rows in S, with duplicates eliminated

Difference

- Input: two tables R and S
- Notation: $R - S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R that are not found in S

Derived operator: intersection

- Input: two tables R and S
- Notation: $R \cap S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows that are in both R and S
- Shorthand for $R - (R - S)$
- Also equivalent to $S - (S - R)$
- And to $R \bowtie S$

Renaming

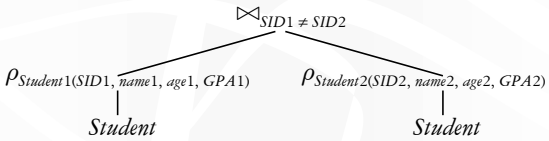
- Input: a table R
- Notation: $\rho_S (R)$, or $\rho_{S(A_1,\, A_2,\, \ldots)} (R)$
- Purpose: rename a table and/or its columns
- Output: a renamed table with the same rows as R
- Used to
  - Avoid confusion caused by identical column names
  - Create identical columns names for natural joins

Renaming example

- All pairs of (different) students

  $Student \bowtie_{?} Student$

  $Student \cancel{\bowtie_{Student.SID \neq Student.SID}} Student$



Summary of core operators

- Selection: $\sigma_p\ (\ R\ )$
- Projection: $\pi_L\ (\ R\ )$
- Cross product: $R \times S$
- Union: $R \cup S$
- Difference: $R - S$
- Renaming: $\rho_{\ S(A_1,\ A_2,\ \ldots)}\ (\ R\ )$
  - Does not really add to expressive power

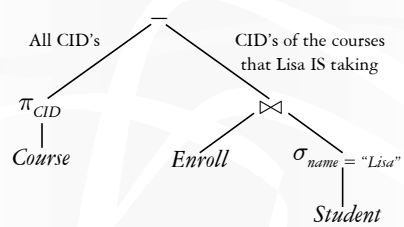
Summary of derived operators

- Join: $R \bowtie_p S$
- Natural join: $R \bowtie S$
- Intersection: $R \cap S$

- Many more
  - Semijoin, anti-semijoin, quotient, …

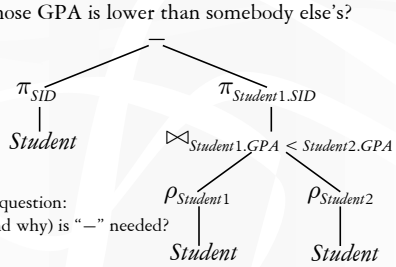
An exercise

- CID's of the courses that Lisa is NOT taking



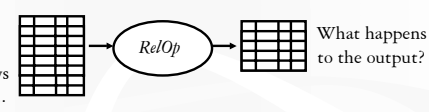
A trickier exercise

- Who has the highest GPA?
  - Who does NOT have the highest GPA?
  - Whose GPA is lower than somebody else's?



A deeper question:
When (and why) is "–" needed?

Monotone operators



- If some old output rows must be removed
  - Then the operator is non-monotone
- Otherwise the operator is monotone
  - That is, old output rows remain "correct" when more rows are added to the input
  - Formally, $R \subseteq R'$ implies $RelOp(\ R\ ) \subseteq RelOp(\ R'\ )$

Classification of relational operators

- Selection: $\sigma_p(R)$ — Monotone
- Projection: $\pi_L(R)$ — Monotone
- Cross product: $R \times S$ — Monotone
- Join: $R \bowtie_p S$ — Monotone
- Natural join: $R \bowtie S$ — Monotone
- Union: $R \cup S$ — Monotone
- Difference: $R - S$ — Non-monotone (not w.r.t. S)
- Intersection: $R \cap S$ — Monotone

Why is "−" needed for highest GPA?

- Composition of monotone operators produces a monotone query
  - Old output rows remain "correct" when more rows are added to the input
- Highest-GPA query is non-monotone
  - Current highest GPA is 4.3
  - Add another GPA 4.5
  - Old answer is invalidated
- So it must use difference!

Why do we need core operator X?

- Difference
  - The only non-monotone operator
- Cross product
  - The only operator that adds columns
- Union
  - The only operator that allows you to add rows?
  - A more rigorous proof?
- Selection? Projection?
  - Homework problem ☺

Why is r.a. a good query language?

- Declarative?
  - Yes, compared with older languages like CODASYL
  - But operators are inherently procedural
- Simple
  - A small set of core operators who semantics are easy to grasp
- Complete?
  - With respect to what?

Relational calculus

- $\{ s.SID \mid s \in Student \wedge \neg(\exists s' \in Student: s.GPA < s'.GPA) \}$, or
  $\{ s.SID \mid s \in Student \wedge (\forall s' \in Student: s.GPA \geq s'.GPA) \}$
- Relational algebra = "safe" relational calculus
  - Every query expressible as a safe relational calculus query is also expressible as a relational algebra query
  - And vice versa
- Example of an unsafe relational calculus query
  - $\{ s.name \mid \neg(s \in Student) \}$
  - Cannot evaluate this query just by looking at the database

Turing machine?

- Relational algebra has no recursion
  - Example of something not expressible in relational algebra: Given relation Parent(parent, child), who are Bart's ancestors?
- Why not recursion?
  - Optimization becomes undecidable
  - You can always implement it at the application level
  - Recursion is added to SQL nevertheless