

CPS296.2 Advance Topics in CPS: Mesh Generation

Homework # 1

Due date: September 9, Monday, the beginning of the class.

Credits: 10 full + 4 bonus

1. **(two credits)** Show that the following procedure returns twice the signed area of a given triangle $\triangle(a b c)$.

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AREA ( $a, b, c \in \mathbb{R}^2$ )  
  return  $(c_y - a_y)(b_x - a_x) - (b_y - a_y)(c_x - a_x)$ ;
```

2. **(two credits)** Prove or disprove: The dual graph of the triangulation of a monotone polygon is always a chain, that is any node in this graph has degree at most two.
3. **(four credits)** Let K be a triangulation of a set of n points in the plane. Let ℓ be a line that avoids all points. Prove that ℓ intersects at most $2n - 4$ edges of K and that this upper bound is tight for every $n \geq 3$.
4. **(four credits)** A k -coloring of a graph $G(V, E)$ is a function $\gamma : V \rightarrow \{1, 2, \dots, k\}$ such that $\gamma(u) \neq \gamma(v)$ if $(u, v) \in E$. Prove that a planar triangulation has a 6-coloring.
5. **(two credits)** An *ear* is a triangle bounded by a diagonal and two polygon edges. Prove that every triangulation of an n -gon has to have at least one ear, provided $n \geq 4$.