CPS296.2 Advance Topics in CPS: Mesh Generation

Homework # 2

Due date: September 25, Wednesday, the beginning of the class.

Credits: 10 full + 4 bonus

1. **(two credits)** A polygon P is *monotone* if there exists a line ℓ such that all the lines orthogonal to ℓ intersects the boundary of P at most twice.
   (a) Prove or disprove: Every simple polygon with 6 vertices is monotone.
   (b) Prove or disprove: Every simple polygon with 5 or less vertices is monotone.

2. **(four credits)** Let P be a polygon with n vertices; r of them are non-convex.
   (a) Prove that any subdivision of P into convex regions has at least $\lceil r/2 \rceil + 1$ regions.
   (b) Let T be a triangulation of P. As long as there remains an edge such that the union of the two regions it separates is convex, remove that edge. Prove that, in the end, there are at most $2r + 1$ regions.

3. **(two credits)** The degree of a vertex in a triangulation is the number of edges incident to it. Give an example of a non-degenerate set of n points in the plane such that, every triangulation of the point set includes a vertex whose degree is $n - 1$.

4. **(two credits)** Let p, q, and r be the vertices of a triangle in the the Delaunay triangulation T of a point set P. Let $a \notin P$ be a point inside the the triangle pqr. Prove that the edges pa, qa, and ra are edges of the Delaunay triangulation of $P \bigcup \{a\}$.

5. **(two credits)** The *weight* of a triangulation is the sum of the length of all edges of the triangulation. A *minimum weight triangulation* is one whose weight is minimal among all triangulations of a point set. Prove or Disprove: The Delaunay triangulation is a minimum weight triangulation.

6. **(two credits)** Prove that among all possible triangulations of a point set, Delaunay triangulation minimizes the largest circumcircle.