

CPS296.2 Advance Topics in CPS: Mesh Generation

Homework # 4

Due date: October 30, Wednesday, the beginning of the class.

Credits: 10 full + 5 bonus

1. **(five credits)** A set B of unit disks in \mathbb{R}^2 is a packing if their interiors are pairwise disjoint. Let S be the set of centers. The density of B is

$$\rho(B) = \frac{\text{area}(\text{convHull}(S) \cap \bigcup B)}{\text{area}(\text{convHull}(S))}$$

- (a) Prove that each unit disk touches at most 6 other unit disks in a packing.
(b) What is the density of the hexagonal grid, where each disk touches 6 others?
(c) Argue that no packing has higher density than that of the hexagonal grid.
2. **(five credits)** Prove that in a packing of unit balls in \mathbb{R}^3 any one ball can touch at most 14 others. [Actual upper bound is 12, which you can work on proving for extra credits.]
3. **(five credits)** Let A , B , C , and D be four non-overlapping disks such that A and B , B and C , C and D , and D and A are tangent to each other. Prove that these four tangency points are co-circular.

