Relational Model & Algebra

CPS 116

Introduction to Database Systems

Announcements (Thurs. September 1)

- Please sign up for mailing list and database (IBM DB2) accounts on the sign-up sheet (now circulating)
- Homework #1 will be assigned next Tuesday
- Office hours: see also course Web page
  - Jun: TTH afternoon
  - Ming: MW late afternoon
- Book update
  - $101 (new) / $75.75 (used) from Duke bookstore
    - Available possibly tomorrow and definitely by next Tuesday
  - $86.15 (new, free shipping) from Amazon

Relational data model

- A database is a collection of relations (or tables)
- Each relation has a list of attributes (or columns)
- Each attribute has a domain (or type)
  - Set-valued attributes not allowed
- Each relation contains a set of tuples (or rows)
  - Each tuple has a value for each attribute of the relation
  - Duplicate tuples are not allowed
    - Two tuples are identical if they agree on all attributes

☞ Simplicity is a virtue!

Example

Student

SID	*name*	*age*	*GPA*
142	Bart	10	2.3
123	Milhouse	10	3.1
857	Lisa	8	4.3
456	Ralph	8	2.3
...	...	...	...

Course

CID	*title*
CPS116	Intro. to Database Systems
CPS130	Analysis of Algorithms
CPS114	Computer Networks
...	...

Enroll

SID	*CID*
142	CPS116
142	CPS114
123	CPS116
857	CPS116
857	CPS130
456	CPS114
...	...

Ordering of rows doesn't matter (even though the output is always in *some* order)

Schema versus instance

- Schema (metadata)
  - Specification of how data is to be structured logically
  - Defined at set-up
  - Rarely changes
- Instance
  - Content
  - Changes rapidly, but always conforms to the schema

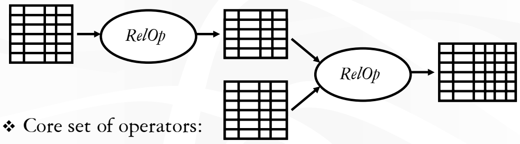
☞ Compare to type and objects of type in a programming language

Example

- Schema
  - *Student* (*SID* integer, *name* string, *age* integer, *GPA* float)
  - *Course* (*CID* string, *title* string)
  - *Enroll* (*SID* integer, *CID* integer)
- Instance
  - { ⟨142, Bart, 10, 2.3⟩, ⟨123, Milhouse, 10, 3.1⟩, ...}
  - { ⟨CPS116, Intro. to Database Systems⟩, ...}
  - { ⟨142, CPS116⟩, ⟨142, CPS114⟩, ...}

Relational algebra operators

A language for querying relational databases based on operators:



- Core set of operators:
  - Selection, projection, cross product, union, difference, and renaming
- Additional, derived operators:
  - Join, natural join, intersection, etc.
- Compose operators to make complex queries

Selection

- Input: a table R
- Notation: $\sigma_p\ R$
  - p is called a selection condition/predicate
- Purpose: filter rows according to some criteria
- Output: same columns as R, but only rows of R that satisfy p

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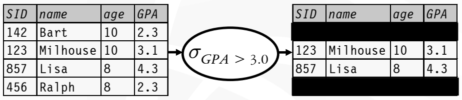
Selection example

- Students with GPA higher than 3.0

$\sigma_{GPA > 3.0}\ Student$



SID	name	age	GPA
142	Bart	10	2.3
123	Milhouse	10	3.1
857	Lisa	8	4.3
456	Ralph	8	2.3

SID	name	age	GPA
123	Milhouse	10	3.1
857	Lisa	8	4.3

More on selection

- Selection predicate in general can include any column of R, constants, comparisons ($=$, $\leq$, etc.), and Boolean connectives ($\wedge$: and, $\vee$: or, and $\neg$: not)
  - Example: straight A students under 18 or over 21

    $\sigma_{GPA \geq 4.0 \wedge (age < 18 \vee age > 21)}\ Student$
- But you must be able to evaluate the predicate over a single row of the input table
  - Example: student with the highest GPA

    $\sigma_{\cancel{GPA \geq \text{all } GPA \text{ in } Student \text{ table}}}\ Student$

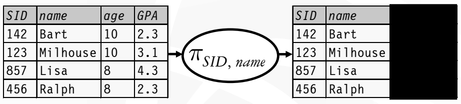
Projection

- Input: a table R
- Notation: $\pi_L\ R$
  - L is a list of columns in R
- Purpose: select columns to output
- Output: same rows, but only the columns in L

Projection example

- ID's and names of all students

$\pi_{SID,\ name}\ Student$



SID	name	age	GPA
142	Bart	10	2.3
123	Milhouse	10	3.1
857	Lisa	8	4.3
456	Ralph	8	2.3

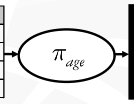
SID	name
142	Bart
123	Milhouse
857	Lisa
456	Ralph

More on projection

- ❖ Duplicate output rows are removed (by definition)
  - Example: student ages

π_{age} *Student*

SID	*name*	*age*	*GPA*
142	Bart	10	2.3
123	Milhouse	10	3.1
857	Lisa	8	4.3
456	Ralph	8	2.3



[redacted]

age
10
[redacted]
8
[redacted]

Cross product

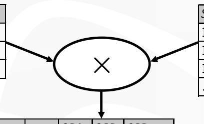
- ❖ Input: two tables R and S
- ❖ Notation: $R \times S$
- ❖ Purpose: pairs rows from two tables
- ❖ Output: for each row r in R and each row s in S, output a row rs (concatenation of r and s)

Cross product example

- ❖ *Student* × *Enroll*

SID	*name*	*age*	*GPA*
142	Bart	10	2.3
123	Milhouse	10	3.1
...	...	...	...

SID	*CID*
142	CPS116
142	CPS114
123	CPS116
...	...



SID	*name*	*age*	*GPA*	*SID*	*CID*
142	Bart	10	2.3	142	CPS116
142	Bart	10	2.3	142	CPS114
142	Bart	10	2.3	123	CPS116
123	Milhouse	10	3.1	142	CPS116
123	Milhouse	10	3.1	142	CPS114
123	Milhouse	10	3.1	123	CPS116
...	...	...	...	...	...

A note on column ordering

- ❖ The ordering of columns in a table is considered unimportant (as is the ordering of rows)

SID	*name*	*age*	*GPA*	*SID*	*CID*
142	Bart	10	2.3	142	CPS116
142	Bart	10	2.3	142	CPS114
142	Bart	10	2.3	123	CPS116
123	Milhouse	10	3.1	142	CPS116
123	Milhouse	10	3.1	142	CPS114
123	Milhouse	10	3.1	123	CPS116
...	...	...	...	...	...

=

SID	*CID*	*SID*	*name*	*age*	*GPA*
142	CPS116	142	Bart	10	2.3
142	CPS114	142	Bart	10	2.3
123	CPS116	142	Bart	10	2.3
142	CPS116	123	Milhouse	10	3.1
142	CPS114	123	Milhouse	10	3.1
123	CPS116	123	Milhouse	10	3.1
...	...	...	...	...	...

- ❖ That means cross product is commutative, i.e., $R \times S = S \times R$ for any R and S

Derived operator: join

- ❖ Input: two tables R and S
- ❖ Notation: $R \square_p S$
  - p is called a join condition/predicate
- ❖ Purpose: relate rows from two tables according to some criteria
- ❖ Output: for each row r in R and each row s in S, output a row rs if r and s satisfy p
- ❖ Shorthand for $\sigma_p\ (R \times S)$

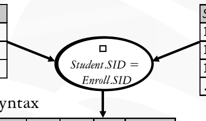
Join example

- ❖ Info about students, plus CID's of their courses

Student $\square_{Student.SID\ =\ Enroll.SID}$ *Enroll*

SID	*name*	*age*	*GPA*
142	Bart	10	2.3
123	Milhouse	10	3.1
...	...	...	...

SID	*CID*
142	CPS116
142	CPS114
123	CPS116
...	...



Use *table_name. column_name* syntax to disambiguate identically named columns from different input tables

SID	*name*	*age*	*GPA*	*SID*	*CID*
142	Bart	10	2.3	142	CPS116
142	Bart	10	2.3	142	CPS114
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
123	Milhouse	10	3.1	123	CPS116
...	...	...	...	...	...

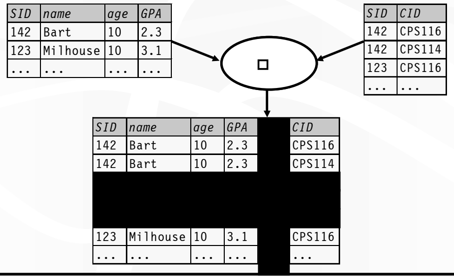
Derived operator: natural join

- Input: two tables R and S
- Notation: $R \bowtie S$
- Purpose: relate rows from two tables, and
  - Enforce equality on all common attributes
  - Eliminate one copy of common attributes
- Shorthand for π_L ($R \bowtie_p S$), where
  - p equates all attributes common to R and S
  - L is the union of all attributes from R and S, with duplicate attributes removed

Natural join example

- $Student \bowtie Enroll = \pi_? (Student \bowtie_? Enroll)$

$= \pi_{SID,\ name,\ age,\ GPA,\ CID} (Student \bowtie_{Student.SID\ =\ Enroll.SID} Enroll)$



SID	name	age	GPA
142	Bart	10	2.3
123	Milhouse	10	3.1
...	...	...	...

SID	CID
142	CPS116
142	CPS114
123	CPS116
...	...

SID	name	age	GPA	[redacted]	CID
142	Bart	10	2.3	[redacted]	CPS116
142	Bart	10	2.3	[redacted]	CPS114
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
123	Milhouse	10	3.1	[redacted]	CPS116
...	...	...	...	[redacted]	...

Union

- Input: two tables R and S
- Notation: $R \cup S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R and all rows in S, with duplicate rows eliminated

Difference

- Input: two tables R and S
- Notation: $R - S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R that are not found in S

Derived operator: intersection

- Input: two tables R and S
- Notation: $R \cap S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows that are in both R and S
- Shorthand for $R - (R - S)$
- Also equivalent to $S - (S - R)$
- And to $R \bowtie S$

Renaming

- Input: a table R
- Notation: $\rho_S\ R$, or $\rho_{S(A_1, A_2, \ldots)}\ R$
- Purpose: rename a table and/or its columns
- Output: a renamed table with the same rows as R
- Used to
  - Avoid confusion caused by identical column names
  - Create identical columns names for natural joins

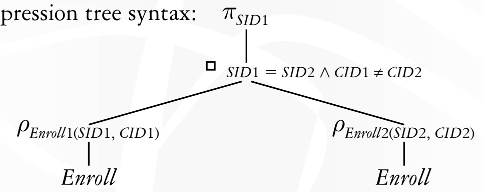
Renaming example

- ❖ SID's of students who take at least two courses

$Enroll \bowtie_{?} Enroll$

$\pi_{SID}\,(Enroll \bowtie_{\cancel{Enroll.SID = Enroll.SID \wedge Enroll.CID \neq Enroll.CID}} Enroll)$

Expression tree syntax:



π_{SID1}

$\bowtie_{SID1 = SID2 \wedge CID1 \neq CID2}$

$\rho_{Enroll1(SID1,\ CID1)}$ $\rho_{Enroll2(SID2,\ CID2)}$

$Enroll$ $Enroll$

Summary of core operators

- ❖ Selection: $\sigma_p\ R$
- ❖ Projection: $\pi_L\ R$
- ❖ Cross product: $R \times S$
- ❖ Union: $R \cup S$
- ❖ Difference: $R - S$
- ❖ Renaming: $\rho_{\ S(A_1,\ A_2,\ \ldots)}\ R$
  - Does not really add to processing power

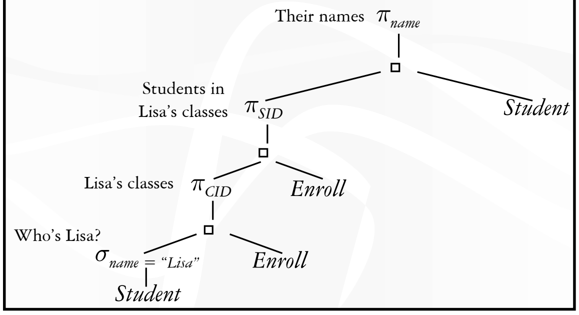
Summary of derived operators

- ❖ Join: $R \bowtie_p S$
- ❖ Natural join: $R \bowtie S$
- ❖ Intersection: $R \cap S$

- ❖ Many more
  - Semijoin, anti-semijoin, quotient, …

An exercise

- ❖ Names of students in Lisa's classes



Their names π_{name}

$\bowtie$

Students in Lisa's classes π_{SID}

$Student$

$\bowtie$

Lisa's classes π_{CID}

$Enroll$

$\bowtie$

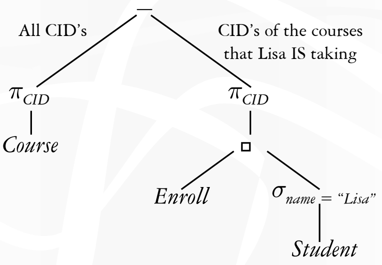
Who's Lisa? $\sigma_{name\ =\ \text{“Lisa”}}$

$Enroll$

$Student$

Another exercise

- ❖ CID's of the courses that Lisa is NOT taking



$-$

All CID's

CID's of the courses that Lisa IS taking

π_{CID} π_{CID}

$Course$

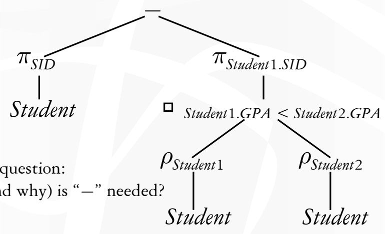
$\bowtie$

$Enroll$ $\sigma_{name\ =\ \text{“Lisa”}}$

$Student$

A trickier exercise

- ❖ Who has the highest GPA?
  - Who does NOT have the highest GPA?
  - Whose GPA is lower than somebody else's?



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π_{SID} $\pi_{Student1.SID}$

$Student$

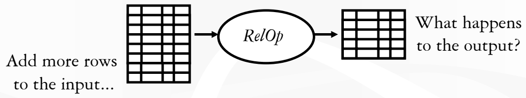
$\bowtie_{Student1.GPA\ <\ Student2.GPA}$

$\rho_{Student1}$ $\rho_{Student2}$

$Student$ $Student$

A deeper question:
When (and why) is "$-$" needed?

Monotone operators



- If some old output rows may need to be removed
  - Then the operator is non-monotone
- Otherwise the operator is monotone
  - That is, old output rows always remain "correct" when more rows are added to the input
  - Formally, for a monotone operator *RelOp*: $R \subseteq R'$ implies $RelOp(R) \subseteq RelOp(R')$

Classification of relational operators

Selection: $\sigma_p R$	Monotone
Projection: $\pi_L R$	Monotone
Cross product: $R \times S$	Monotone
Join: $R \bowtie_p S$	Monotone
Natural join: $R \bowtie S$	Monotone
Union: $R \cup S$	Monotone
Difference: $R - S$	Monotone w.r.t. R; non-monotone w.r.t S
Intersection: $R \cap S$	Monotone

Why is "–" needed for highest GPA?

- Composition of monotone operators produces a monotone query
  - Old output rows remain "correct" when more rows are added to the input
- Highest-GPA query is non-monotone
  - Current highest GPA is 4.1
  - Add another GPA 4.2
  - Old answer is invalidated

☞ So it must use difference!

Why do we need core operator *X*?

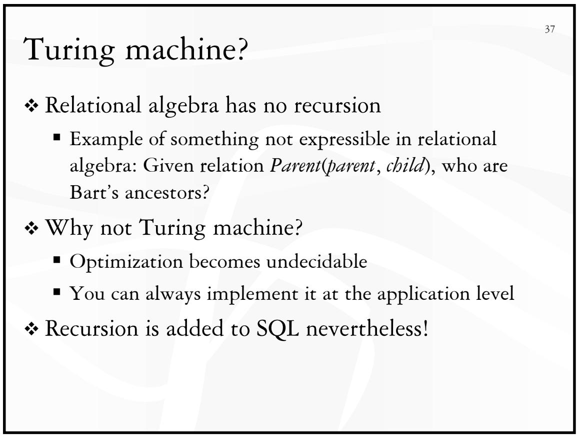
- Difference
  - The only non-monotone operator
- Cross product
  - The only operator that adds columns
- Union
  - The only operator that allows you to add rows?
  - A more rigorous argument?
- Selection? Projection?
  - Homework problem ☺

Why is r.a. a good query language?

- Simple
  - A small set of core operators who semantics are easy to grasp
- Declarative?
  - Yes, compared with older languages like CODASYL
  - Though operators do look somewhat "procedural"
- Complete?
  - With respect to what?

Relational calculus

- $\{ s.SID \mid s \in Student \wedge \neg(\exists s' \in Student: s.GPA < s'.GPA) \}$, or
  $\{ s.SID \mid s \in Student \wedge (\forall s' \in Student: s.GPA \geq s'.GPA) \}$
- Relational algebra = "safe" relational calculus
  - Every query expressible as a safe relational calculus query is also expressible as a relational algebra query
  - And vice versa
- Example of an unsafe relational calculus query
  - $\{ s.name \mid \neg(s \in Student) \}$
  - Cannot evaluate this query just by looking at the database



Turing machine?

- Relational algebra has no recursion
  - Example of something not expressible in relational algebra: Given relation *Parent*(*parent*, *child*), who are Bart's ancestors?
- Why not Turing machine?
  - Optimization becomes undecidable
  - You can always implement it at the application level
- Recursion is added to SQL nevertheless!