SQL: Recursion

CPS 116

Introduction to Database Systems

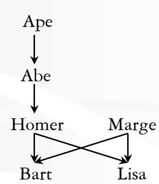
Announcements (October 4)

- Midterm this Thursday in class
  - Format similar to the sample midterm; covers everything up to the lecture today; emphasizes on materials in homeworks
- Midterm review this Tuesday 7-8pm in Room D344
  - For those of you who cannot attend, Ming will make notes (some in hardcopies) from the session available during office hours
- Available: solutions to Homework #2 and sample midterm
  - ☞Handouts you missed can be found online or in the handout box outside my office (D327)
- Watch for email from Ming regarding graded Homework #2 (hopefully you will get them back on Wednesday)
- Project milestone #1 due next Thursday

A motivating example

Parent (*parent*, *child*)

parent	*child*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe



- Example: find Bart's ancestors
- "Ancestor" has a recursive definition
  - *X* is *Y*'s ancestor if
    - *X* is *Y*'s parent, or
    - *X* is *Z*'s ancestor and *Z* is *Y*'s ancestor

Recursion in SQL

- SQL2 had no recursion
  - You can find Bart's parents, grandparents, great grandparents, etc.

```
SELECT p1.parent AS grandparent
FROM Parent p1, Parent p2
WHERE p1.child = p2.parent
AND p2.child = 'Bart';
```

  - But you cannot find all his ancestors with a single query
- SQL3 introduces recursion
  - `WITH` clause
  - Implemented in DB2 (called common table expressions)

Ancestor query in SQL3

```
WITH Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
SELECT anc
FROM Ancestor
WHERE desc = 'Bart';
```

base case

recursion step

Define a relation recursively

Query using the relation defined in `WITH` clause

How do we compute such a recursive query?

Fixed point of a function

- If f: $T \rightarrow T$ is a function from a type T to itself, a fixed point of f is a value x such that $f(x) = x$
- Example: What is the fixed point of $f(x) = x / 2$?
  - 0, because $f(0) = 0 / 2 = 0$
- To compute a fixed point of f
  - Start with a "seed": $x \leftarrow x_0$
  - Compute $f(x)$
    - If $f(x) = x$, stop; x is fixed point of f
    - Otherwise, $x \leftarrow f(x)$; repeat
- Example: compute the fixed point of $f(x) = x / 2$
  - With seed 1: 1, 1/2, 1/4, 1/8, 1/16, … $\rightarrow$ 0

Fixed point of a query

- A query q is just a function that maps an input table to an output table, so a fixed point of q is a table T such that $q(T) = T$
- To compute fixed point of q
  - Start with an empty table: $T \leftarrow \varnothing$
  - Evaluate q over T
    - If the result is identical to T, stop; T is a fixed point
    - Otherwise, let T be the new result; repeat
  - ☞Starting from ∅ produces the unique minimal fixed point (assuming q is monotone)

Finding ancestors

```
WITH Ancestor(anc, desc) AS
  ((SELECT parent, child FROM Parent)
   UNION
   (SELECT a1.anc, a2.desc
    FROM Ancestor a1, Ancestor a2
    WHERE a1.desc = a2.anc))
```

Parent (*parent*, *child*)

parent	child
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

- Think of it as *Ancestor* = *q*(*Ancestor*)

anc	desc

→

anc	desc
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

→

anc	desc
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer

→

anc	desc
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Ape	Bart
Ape	Lisa

Intuition behind fixed-point iteration

- Initially, we know nothing about ancestor-descendent relationships
- In the first step, we deduce that parents and children form ancestor-descendent relationships
- In each subsequent steps, we use the facts deduced in previous steps to get more ancestor-descendent relationships
- We stop when no new facts can be proven

Linear recursion

- With linear recursion, a recursive definition can make only one reference to itself
- Non-linear:

```
WITH Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
```

- Linear:

```
WITH Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT anc, child
  FROM Ancestor, Parent
  WHERE desc = parent))
```


Linear vs. non-linear recursion

- Linear recursion is easier to implement
  - For linear recursion, just keep joining newly generated *Ancestor* rows with *Parent*
  - For non-linear recursion, need to join newly generated *Ancestor* rows with all existing *Ancestor* rows
- Non-linear recursion may take fewer steps to converge
  - Example: $a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$
  - Linear recursion takes 4 steps
  - Non-linear recursion takes 3 steps

Mutual recursion example

- Table *Natural* (*n*) contains 1, 2, …, 100
- Which numbers are even/odd?
  - An odd number plus 1 is an even number
  - An even number plus 1 is an odd number
  - 1 is an odd number

```
WITH Even(n) AS
     (SELECT n FROM Natural
      WHERE n = ANY(SELECT n+1 FROM Odd)),
     Odd(n) AS
     ((SELECT n FROM Natural WHERE n = 1)
      UNION
      (SELECT n FROM Natural
       WHERE n = ANY(SELECT n+1 FROM Even)))
```

Operational semantics of WITH

- WITH R_1 AS Q_1, …,
  R_n AS Q_n
  Q;
  - $Q_1, \ldots, Q_n$ may refer to $R_1, \ldots, R_n$
- Operational semantics
  1. $R_1 \leftarrow \varnothing, \ldots, R_n \leftarrow \varnothing$
  2. Evaluate $Q_1, \ldots, Q_n$ using the current contents of $R_1, \ldots, R_n$: $R_1^{new} \leftarrow Q_1, \ldots, R_n^{new} \leftarrow Q_n$
  3. If $R_i^{new} \neq R_i$ for any i
     3.1. $R_1 \leftarrow R_1^{new}, \ldots, R_n \leftarrow R_n^{new}$
     3.2. Go to 2.
  4. Compute Q using the current contents of $R_1, \ldots, R_n$ and output the result

Computing mutual recursion


```
WITH Even(n) AS
     (SELECT n FROM Natural
      WHERE n = ANY(SELECT n+1 FROM Odd)),
     Odd(n) AS
     ((SELECT n FROM Natural WHERE n = 1)
      UNION
      (SELECT n FROM Natural
       WHERE n = ANY(SELECT n+1 FROM Even)))
```

- $Even = \varnothing, Odd = \varnothing$
- $Even = \varnothing, Odd = \{1\}$
- $Even = \{2\}, Odd = \{1\}$
- $Even = \{2\}, Odd = \{1, 3\}$
- $Even = \{2, 4\}, Odd = \{1, 3\}$
- $Even = \{2, 4\}, Odd = \{1, 3, 5\}$
- …

Fixed points are not unique


```
WITH Ancestor(anc, desc) AS
   ((SELECT parent, child FROM Parent)
    UNION
    (SELECT a1.anc, a2.desc
     FROM Ancestor a1, Ancestor a2
     WHERE a1.desc = a2.anc))
```

Parent (*parent*, *child*)

parent	*child*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Ape	Bart
Ape	Lisa
bogus	bogus

- There may be many other fixed points
- But if q is monotone, then all these fixed points must contain the fixed point we computed from fixed-point iteration starting with $\varnothing$
  - Thus the unique minimal fixed point is the "natural" answer to the query

Note that the bogus tuple reinforces itself!

Mixing negation with recursion

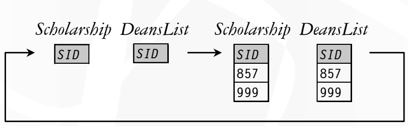
- If q is non-monotone
  - The fixed-point iteration may flip-flop and never converge
  - There could be multiple minimal fixed points—so which one is the right answer?
- Example: reward students with GPA higher than 3.9
  - Those not on the Dean's List should get a scholarship
  - Those without scholarships should be on the Dean's List
  - ```
    WITH Scholarship(SID) AS
         (SELECT SID FROM Student WHERE GPA > 3.9
          AND SID NOT IN (SELECT SID FROM DeansList)),
         DeansList(SID) AS
         (SELECT SID FROM Student WHERE GPA > 3.9
          AND SID NOT IN (SELECT SID FROM Scholarship))
    ```

Fixed-point iteration does not converge


```
WITH Scholarship(SID) AS
     (SELECT SID FROM Student WHERE GPA > 3.9
      AND SID NOT IN (SELECT SID FROM DeansList)),
     DeansList(SID) AS
     (SELECT SID FROM Student WHERE GPA > 3.9
      AND SID NOT IN (SELECT SID FROM Scholarship))
```

Student

SID	*name*	*age*	*GPA*
857	Lisa	8	4.3
999	Jessica	10	4.2

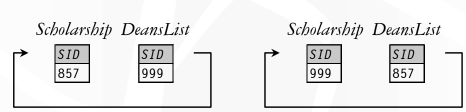


Multiple minimal fixed points


```
WITH Scholarship(SID) AS
     (SELECT SID FROM Student WHERE GPA > 3.9
      AND SID NOT IN (SELECT SID FROM DeansList)),
     DeansList(SID) AS
     (SELECT SID FROM Student WHERE GPA > 3.9
      AND SID NOT IN (SELECT SID FROM Scholarship))
```

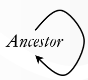
Student

SID	*name*	*age*	*GPA*
857	Lisa	8	4.3
999	Jessica	10	4.2

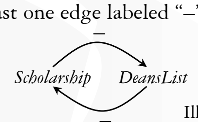

```

Legal mix of negation and recursion

- Construct a dependency graph
  - One node for each table defined in `WITH`
  - A directed edge $R \rightarrow S$ if R is defined in terms of S
  - Label the directed edge "–" if the query defining R is not monotone with respect to S
- Legal SQL3 recursion: no cycle containing a "–" edge
  - Called stratified negation
- Bad mix: a cycle with at least one edge labeled "–"



Legal!

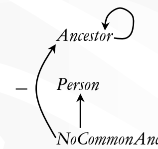


Illegal!

Stratified negation example

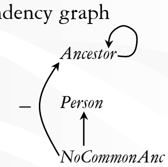
- Find pairs of persons with no common ancestors

```
WITH Ancestor(anc, desc) AS
     ((SELECT parent, child FROM Parent) UNION
      (SELECT a1.anc, a2.desc
       FROM Ancestor a1, Ancestor a2
       WHERE a1.desc = a2.anc)),
     Person(person) AS
     ((SELECT parent FROM Parent) UNION
      (SELECT child FROM Parent)),
     NoCommonAnc(person1, person2) AS
     ((SELECT p1.person, p2.person
       FROM Person p1, Person p2
       WHERE p1.person <> p2.person)
      EXCEPT
      (SELECT a1.desc, a2.desc
       FROM Ancestor a1, Ancestor a2
       WHERE a1.anc = a2.anc))
SELECT * FROM NoCommonAnc;
```



Evaluating stratified negation

- The stratum of a node R is the maximum number of "–" edges on any path from R in the dependency graph
  - *Ancestor*: stratum 0
  - *Person*: stratum 0
  - *NoCommonAnc*: stratum 1



- Evaluation strategy
  - Compute tables lowest-stratum first
  - For each stratum, use fixed-point iteration on all nodes in that stratum
    - Stratum 0: *Ancestor* and *Person*
    - Stratum 1: *NoCommonAnc*

  ☞Intuitively, there is no negation within each stratum

Summary

- SQL3 `WITH` recursive queries
- Solution to a recursive query (with no negation): unique minimal fixed point
- Computing unique minimal fixed point: fixed-point iteration starting from $\varnothing$
- Mixing negation and recursion is tricky
  - Illegal mix: fixed-point iteration may not converge; there may be multiple minimal fixed points
  - Legal mix: stratified negation (compute by fixed-point iteration stratum by stratum)