

# Relational Database Design Theory

## Part II

CPS 116  
Introduction to Database Systems

## Announcements (October 13)

- ❖ Midterm graded; sample solution available
  - Please verify your grades on Blackboard
- ❖ Project milestone #1 due today

## Review

- ❖ Functional dependencies
  - $X \rightarrow Y$ : If two rows agree on  $X$ , they must agree on  $Y$
  - ☞ A generalization of the key concept
- ❖ Non-key functional dependencies: a source of redundancy
  - Non-trivial  $X \rightarrow Y$  where  $X$  is not a superkey
  - ☞ Called a BCNF violation
- ❖ BCNF decomposition: a method for removing redundancies
  - Given  $R(X, Y, Z)$  and a BCNF violation  $X \rightarrow Y$ , decompose  $R$  into  $R_1(X, Y)$  and  $R_2(X, Z)$
  - ☞ A lossless join decomposition
- ❖ Schema in BCNF has no redundancy due to FD's

## Next

- ❖ 3NF (BCNF is too much)
- ❖ Multivalued dependencies: another source of redundancy
- ❖ 4NF (BCNF is not enough)

## Motivation for 3NF

- ❖ *Address* (*street\_address*, *city*, *state*, *zip*)
  - $street\_address, city, state \rightarrow zip$
  - $zip \rightarrow city, state$
- ❖ Keys
  - $\{street\_address, city, state\}$
  - $\{street\_address, zip\}$
- ❖ BCNF?
  - Violation:  $zip \rightarrow city, state$

## To decompose or not to decompose

- Address*<sub>1</sub> (*zip*, *city*, *state*)  
*Address*<sub>2</sub> (*street\_address*, *zip*)
- ❖ FD's in *Address*<sub>1</sub>
    - $zip \rightarrow city, state$
  - ❖ FD's in *Address*<sub>2</sub>
    - None!
  - ❖ Hey, where is  $street\_address, city, state \rightarrow zip$ ?
    - Cannot check without joining *Address*<sub>1</sub> and *Address*<sub>2</sub> back together
  - ❖ Problem: Some lossless join decomposition is not dependency-preserving
  - ❖ Dilemma: Should we get rid of redundancy at the expense of making constraints harder to enforce?

## 3NF

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- ❖  $R$  is in Third Normal Form (3NF) if for every non-trivial FD  $X \rightarrow A$  (where  $A$  is single attribute), either
  - $X$  is a superkey of  $R$ , or
  - $A$  is a member of at least one key of  $R$
 ☞ Intuitively, BCNF decomposition on  $X \rightarrow A$  would “break” the key containing  $A$
- ❖ So *Address* is already in 3NF
- ❖ Tradeoff:
  - Can enforce all original FD’s on individual decomposed relations
  - Might have some redundancy due to FD’s

## BCNF = no redundancy?

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### ❖ *Student* ( $SID$ , $CID$ , $club$ )

- Suppose your classes have nothing to do with the clubs you join
- FD’s?
  - None
- BCNF?
  - Yes
- Redundancies?
  - Tons!

$SID$	$CID$	$club$
142	CPS116	ballet
142	CPS116	sumo
142	CPS114	ballet
142	CPS114	sumo
123	CPS116	chess
123	CPS116	golf
...	...	...

## Multivalued dependencies

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- ❖ A multivalued dependency (MVD) has the form  $X \twoheadrightarrow Y$ , where  $X$  and  $Y$  are sets of attributes in a relation  $R$
- ❖  $X \twoheadrightarrow Y$  means that whenever two rows in  $R$  agree on all the attributes of  $X$ , then we can swap their  $Y$  components and get two new rows that are also in  $R$

$X$	$Y$	$Z$
$a$	$b1$	$c1$
$a$	$b2$	$c2$
$a$	$b1$	$c2$
$a$	$b2$	$c1$
...	...	...

} Must be in  $R$  too

## MVD examples

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### *Student* ( $SID$ , $CID$ , $club$ )

- ❖  $SID \twoheadrightarrow CID$
- ❖  $SID \twoheadrightarrow club$ 
  - Intuition: given  $SID$ ,  $CID$  and club are “independent”
- ❖  $SID, CID \twoheadrightarrow club$ 
  - Trivial: LHS  $\cup$  RHS = all attributes of  $R$
- ❖  $SID, CID \twoheadrightarrow SID$ 
  - Trivial: LHS  $\supseteq$  RHS

## Complete MVD + FD rules

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- ❖ FD reflexivity, augmentation, and transitivity
- ❖ MVD complementation:
  - If  $X \twoheadrightarrow Y$ , then  $X \twoheadrightarrow attr(R) - X - Y$
- ❖ MVD augmentation:
  - If  $X \twoheadrightarrow Y$  and  $V \subseteq W$ , then  $XW \twoheadrightarrow YV$
- ❖ MVD transitivity:
  - If  $X \twoheadrightarrow Y$  and  $Y \twoheadrightarrow Z$ , then  $X \twoheadrightarrow Z - Y$
- ❖ Replication (FD is MVD):
  - If  $X \rightarrow Y$ , then  $X \twoheadrightarrow Y$  Try proving things using these!
- ❖ Coalescence:
  - If  $X \twoheadrightarrow Y$  and  $Z \subseteq Y$  and there is some  $W$  disjoint from  $Y$  such that  $W \rightarrow Z$ , then  $X \rightarrow Z$

## An elegant solution: chase

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- ❖ Given a set of FD’s and MVD’s  $\mathcal{D}$ , does another dependency  $d$  (FD or MVD) follow from  $\mathcal{D}$ ?
- ❖ Procedure
  - Start with the hypothesis of  $d$ , and treat them as “seed” tuples in a relation
  - Apply the given dependencies in  $\mathcal{D}$  repeatedly
    - If we apply an FD, we infer equality of two symbols
    - If we apply an MVD, we infer more tuples
  - If we infer the conclusion of  $d$ , we have a proof
  - Otherwise, if nothing more can be inferred, we have a counterexample

## Proof by chase

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❖ In  $R(A, B, C, D)$ , does  $A \twoheadrightarrow B$  and  $B \twoheadrightarrow C$  imply that  $A \twoheadrightarrow C$ ?

	Have	Need																								
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## Another proof by chase

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❖ In  $R(A, B, C, D)$ , does  $A \rightarrow B$  and  $B \rightarrow C$  imply that  $A \rightarrow C$ ?

	Have	Need												
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a	b2	c2	d2											
$A \rightarrow B$	$b1 = b2$													
$B \rightarrow C$	$c1 = c2$													

In general, both new tuples and new equalities may be generated

## Counterexample by chase

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❖ In  $R(A, B, C, D)$ , does  $A \twoheadrightarrow BC$  and  $CD \rightarrow B$  imply that  $A \rightarrow B$ ?

	Have	Need												
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Counterexample!

## 4NF

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- ❖ A relation  $R$  is in Fourth Normal Form (4NF) if
  - For every non-trivial MVD  $X \twoheadrightarrow Y$  in  $R$ ,  $X$  is a superkey
  - That is, all FD's and MVD's follow from "key  $\rightarrow$  other attributes" (i.e., no MVD's, and no FD's besides key functional dependencies)
- ❖ 4NF is stronger than BCNF
  - Because every FD is also a MVD

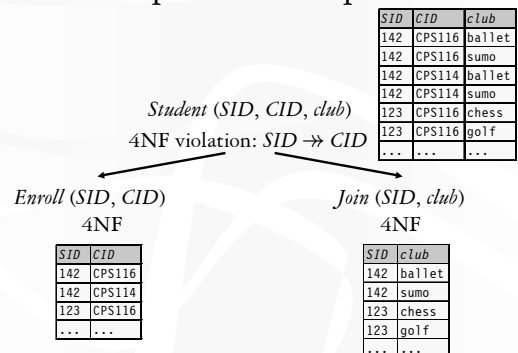
## 4NF decomposition algorithm

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- ❖ Find a 4NF violation
  - A non-trivial MVD  $X \twoheadrightarrow Y$  in  $R$  where  $X$  is not a superkey
- ❖ Decompose  $R$  into  $R_1$  and  $R_2$ , where
  - $R_1$  has attributes  $X \cup Y$
  - $R_2$  has attributes  $X \cup Z$  ( $Z$  contains attributes not in  $X$  or  $Y$ )
- ❖ Repeat until all relations are in 4NF
- ❖ Almost identical to BCNF decomposition algorithm
- ❖ Any decomposition on a 4NF violation is lossless

## 4NF decomposition example

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## 3NF, BCNF, 4NF, and beyond

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Anomaly/normal form	3NF	BCNF	4NF
Lose FD's?	No	Possible	Possible
Redundancy due to FD's	Possible	No	No
Redundancy due to MVD's	Possible	Possible	No

### ❖ Of historical interests

- 1NF: All column values must be atomic
- 2NF: Slightly more relaxed than 3NF

## Summary

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- ❖ Philosophy behind BCNF, 4NF:  
Data should depend on the key, the whole key, and nothing but the key!
- ❖ Philosophy behind 3NF:  
... But not at the expense of more expensive constraint enforcement!