## Today's topics

- Security
  - Demo from RSA Security (www.rsa.com)
  - Sildes taken from Tammy Bailey
  - Slides taken from Kevin Wayne & Robert Sedgewick at Princeton University
  - For further reference "Applied Cryptography" by Bruce Schneier
- Upcoming
  - Complexity
- Reading
  - ► Sections 3.5, 4.5 and 11 in *Brookshear*.
  - Chapters 11,13 in Great Ideas.

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## Cryptography

Cryptography: science of creating secret codes. Cryptanalysis : science of code breaking Cryptology: science of secret communication. Goal: Information Security in presence of malicious adversaries.

- Confidentiality...
- Integrity...
- Authentication...
- Authorization...
- Non-repudiation...
- **RSA PRESENTATION**

## Information security

- All measures taken to prevent unauthorized use of electronic data
  - unauthorized use includes disclosure, alteration, substitution, or destruction of the data concerned

Computer Security is the prevention of, or protection against:
 Access to information by unauthorized recipients

• Authentication: verifying the identity of a person or system

Physical security of the system is also important.

What is an example of a good password?

Change your password often. A particular

Intentional but unauthorized destruction or alteration of

implementation of this idea is ONE-TIME PASSWORDS.

- Provision of the following three services
  - ► Confidentiality
    - · concealment of data from unauthorized parties
  - ► Integrity
    - assurance that data is genuine
  - ► Availability
    - system still functions efficiently after security provisions are in place
- No single measure can ensure complete security

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**Security** 

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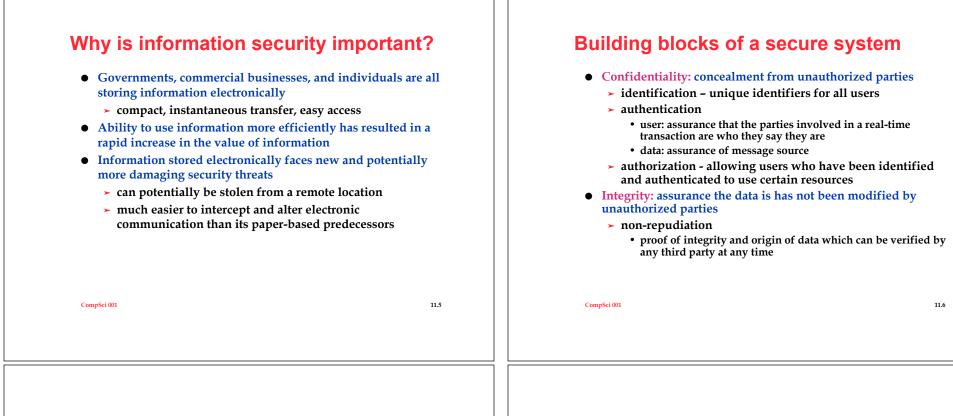
that information.

Username and Password

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## **Completing the security process**

- Confidentiality + integrity → system security
- However, it is not enough for system to be secure
- System must also be available
  - must allow guaranteed, efficient and continuous use of information
  - security measures should not prohibitively slow down or crash system or make it difficult to use
    - what good is a secure system if you can't use it?
- Cryptographic systems
  - high level of security and flexibility
  - can potentially provide all objectives of information security: confidentiality, integrity, and availability

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## Encryption

- Goal: information security in presence of malicious adversaries
  - ▶ confidentiality
  - ▶ integrity

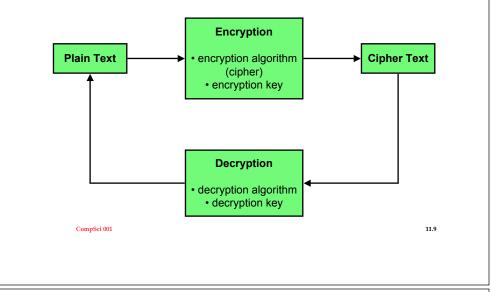
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- authentication
- authorization
- non-repudiation
- Encryption can be used to ...
  - prevent your kid sister from intercepting, reading, and/or altering your messages and files
  - prevent CIA or FBI from intercepting, reading, and/or altering your messages and files

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### **Process**



## **Algorithms & Keys**

**Restricted Algorithm** 

- If the security depends on keeping the working of the algorithm secret.
- Can't support a large or changing group of users...Why?
- No quality control.
- Modern cryptology solves this with a KEY (K).
- Key might be any of a large number of values.
- Range of possible values called a keyspace.
- Now security depends on the security of the Key.
- The algorithms for encrypting and decrypting can be mass produced and optimized.

## Terminology

- Encryption
  - process of obscuring or scrambling data to render it incomprehensible to unauthorized viewers.
- Cipher text
  - encrypted data or "code"
- Plain text
  - original, readable data prior to encryption
- Cipher or encryption algorithm
  - particular method for encrypting or scrambling data
- Key
  - data required by the encryption algorithm to process the plain text and convert it to cipher text
- Decryption
  - > process of converting cipher text back into plain text
  - requires a key and a decryption algorithm

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### **Attacks**

- Compromise systems in ways that affect services of information security
  - attack on confidentiality:
    - unauthorized disclosure of information
  - ► attack on integrity:
    - destruction or corruption of information
  - ► attack on availability:
    - disruption or denial of services

### Prevention, detection, response

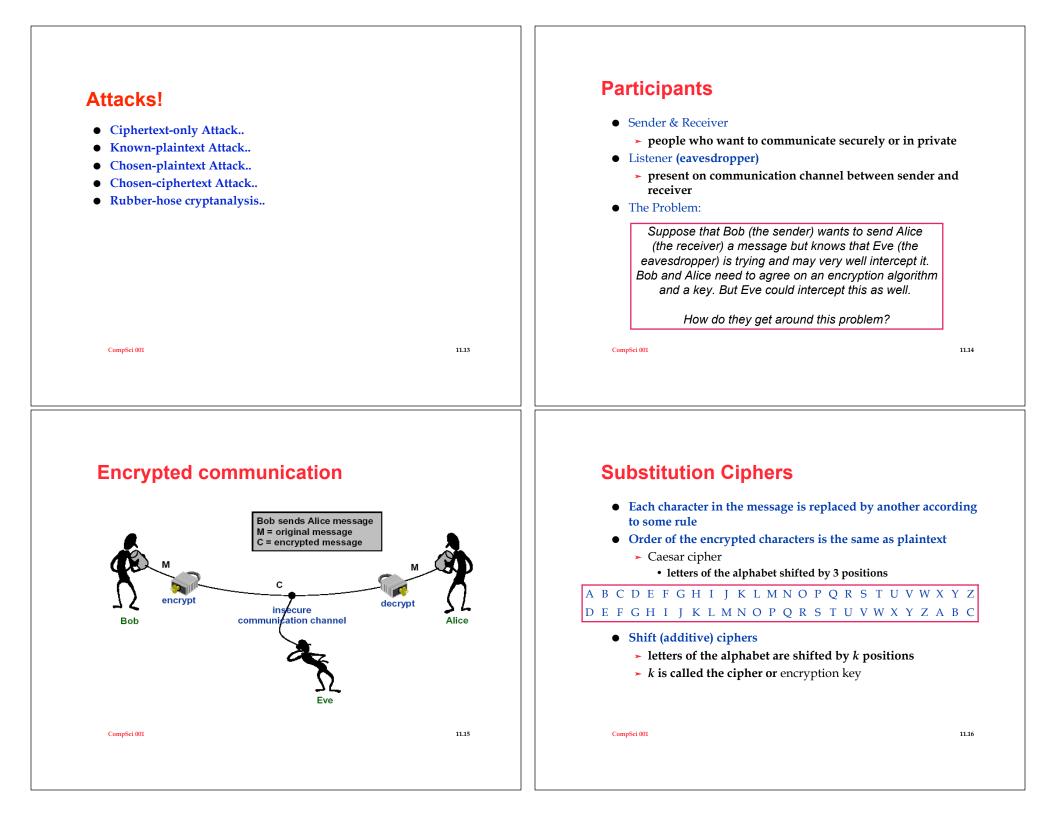
 proper planning reduces risk of attack and increases capabilities of detection and response if an attack does occur

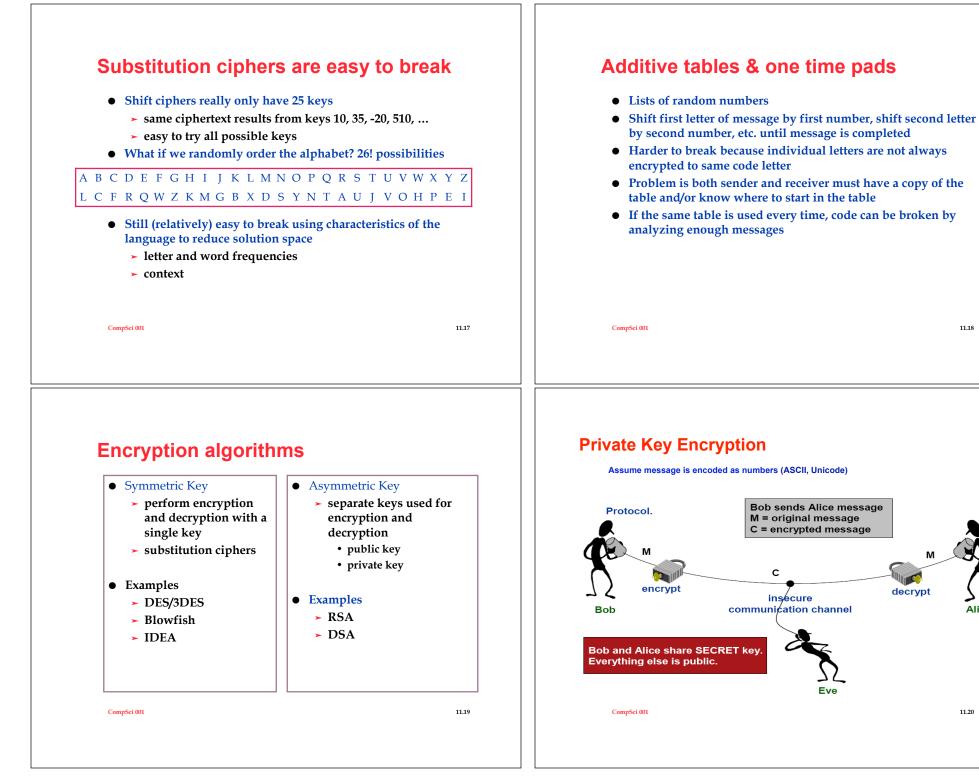
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Alice

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## Symmetric key algorithms (private key)

- Perform encryption and decryption with a single key
- Advantages

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- ► algorithms are very fast
- computationally less intensive
- Security of system determined by protecting the secret key from disclosure
- Applicable only in situations where the distribution of the key can occur in a secure manner
- If every user is going to communicate with every other user, how many keys are required for a system with 1000 users?

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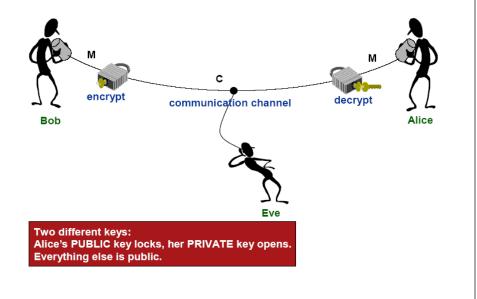
## Asymmetric algorithms (public key)

- Two separate keys used for encryption and decryption
  - public key
    - used for encryption, not secret, available for widespread dissemination
  - ► private key
    - used for decryption
    - private to the individual who owns it
- Plain text encrypted with one key can be decrypted with the other key only
  - ► similar to a mailbox
- Computationally infeasible to derive the private key from the known public key
- If every user is going to communicate with every other user, how many keys are required for a system with 1000 users?

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### **Public Key Encryption**



## **Padlock problem**

- Al and Sue are not allowed to directly communicate with each other in any way. Al has a box, a padlock for the box, a key for that padlock, and a diamond. Sue has a different padlock, and a key for that padlock. The only way Al and Sue can communicate, or send things to each other is through Ted, who will steal everything except a locked box, or an empty box. Ted will not try to pry open any locks with any tools, etc. But if a box is unlocked, and not empty, then Ted will steal its contents.
- *Question*: How does Al get the diamond to Sue using Ted?

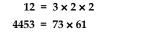
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## **RSA** encryption

- Rivest, Shamir, and Adleman, MIT, 1977
- Most widely-used cryptosystem
- Security relies on the on the difficulty of factoring very large integers into prime factors
  - primes are positive integers that are divisible only by 1 and themselves
  - for example, first 50 prime numbers are ...
    2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229

## Prime factorization

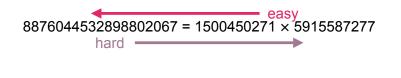
• A prime factorization is the expression of a positive integer as a product of prime numbers



### $10584 = 7 \times 7 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2$

### $124937125 = 2003 \times 499 \times 5 \times 5 \times 5$

- Large primes are easy to multiply
- Factoring large integers is hard



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## **Encrypting and decrypting**

- Alice and Bob would like to communicate with each other in private
- Alice uses RSA algorithm to generate public & private keys
  - Alice makes key (k, n) publicly available to Bob and anyone else wanting to send her private messages
- Bob uses Alice's public key (k, n) to encrypt message M:
  - ➤ compute E(M) =(M<sup>k</sup>)%n
  - ► Bob sends encrypted message E(M) to Alice
- Alice receives E(M) and uses private key (d, n) to decrypt it:
  - ► compute  $D(M) = (E(M)^d)%n$
  - decrypted message D(M) is original message M

# RSA algorithm

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•	Select two large prime numbers p, q
٠	Compute
	$n = p \times q$
	$\mathbf{v} = (\mathbf{p}-1) \mathbf{\times} (\mathbf{q}-1)$
•	Select small odd integer <b>k</b> relatively prime to (not a factor of) to <b>v</b>
•	Compute d such that $(d \times k)\%v = (k \times d)\%v = 1$

- Public key is (k, n)
- Private key is (d, n)
  How large should n be?
  - ► Number Theory

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-  $n/\ln n$  prime numbers between 2 and *n*.

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• example

p = 11

q = 29

n = 319

v = 280

d = 187

(3, 319)

(187, 319)

k = 3

• public key

• private key

### **RSA Attacks**

### Factoring.

- Factor n = pq.
- Then compute f.
- Then compute e.

### Timing attacks.

 Reconstruct d by sending C and monitoring how long it takes to compute Cd(mod n).

### Other means?

Long-standing open research question.

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## **Certification authority**

- A third party trusted by all users that creates, distributes, revokes, & manages certificates
- Certificates bind users to their public keys
- For example, if Alice wants to obtain Bob's public key
  - she retrieves Bob's certificate from a public directory
  - she verifies the CA's signature on the certificate itself
  - ► if signature verifies correctly, she has assurance from the trusted CA this really is Bob's public key
  - she can use Bob's public key to send confidential information to Bob or to verify Bob's signatures, protected by the assurance of the certificate
- Integrity is provided by the certification authority

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## **Digital Signature**

### Alice sends Bob a response.

 Bob wants to be really sure Alice really sent it, and not some imposter.

#### Alice wants to send Bob a response S.

- Alice uses private key d and computes: S'= Sd (mod
- Alice sends (S, S').

Bob receives digital signed response (S, S').

- Bob uses Alice's public key e
- Checks if S = (S')<sup>e</sup> (mod n ).
- If yes, then Bob concludes S sent by Alice.
- If no, then Bob concludes S or S' corrupted in trans or message is forgery.

#### Third party.

- Bob verifies Alice's signature on digitally signed m (e.g. electronic check).
- Bob forwards digitally signed message to bank.
- Bank re-verifies Alice's signature.

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## **Bad Cryptology.**

Good introductory explanation & details on Gregory Kesden's site (CMU) http://www-2.cs.cmu.edu/~dst/DeCSS/Kesden/

### Content Scrambling System (CSS).

• Use to encrypt DVD's.

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- Each disc has 3 40-bit keys.
- Each DVD decoder (software/hardware) has unique 40-bit key. •
- "Not possible" to play back on computer without disc. •

#### DeCSS. (Canman and SoupaFrog, 1999).

- Decryption algorithm written by two Norwegians.
- Used "in-circuit emulator" to monitor hardware activity.

Why CSS is fatally flawed. (Policy and Legal issues..)

### **Prevention**

- Establishment of policy and access control
  - who: identification, authentication, authorization
  - what: granted on "need-to-know" basis
- Implementation of hardware, software, and services
  - users cannot override, unalterable (attackers cannot defeat security mechanisms by changing them)
  - examples of preventative mechanisms
    - passwords prevent unauthorized system access
    - firewalls prevent unauthorized network access
    - encryption prevents breaches of confidentiality
    - physical security devices prevent theft
- Maintenance

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## Detection

- Determine that either an attack is underway or has occurred and report it
- Real-time monitoring
  - ► or, as close as possible
  - monitor attacks to provide data about their nature, severity, and results
- Intrusion verification and notification
  - intrusion detection systems (IDS)
  - typical detection systems monitor various aspects of the system, looking for actions or information indicating an attack
    - example: denial of access to a system when user repeatedly enters incorrect password

## **Prevention is not enough!**

Prevention systems are never perfect.

No bank ever says: "Our safe is so good, we don't need an alarm system."

No museum ever says: "Our door and window locks are so good, we don't need night watchmen."

Detection and response are how we get security in the real world, and they're the only way we can possibly get security in the cyberspace world.

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Bruce Schneier, Counterpane Internet Security, Inc<sub>11.34</sub>

### **Outline of implementation**

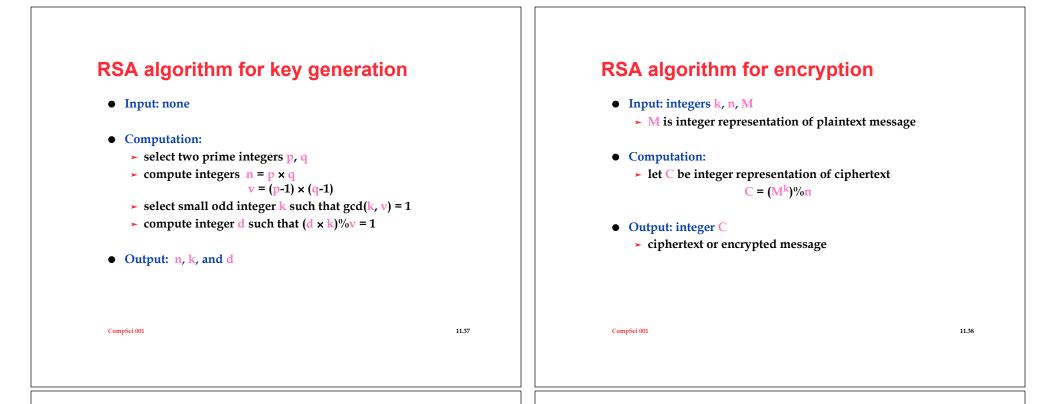
- RSA algorithm for key generation
  - select two prime numbers p, q
  - $\succ$  compute  $n = p \times q$ 
    - $\mathbf{v} = (\mathbf{p}\textbf{-}\mathbf{1}) \times (\mathbf{q}\textbf{-}\mathbf{1})$
  - select small odd integer k such that gcd(k, v) = 1
    - $gcu(\mathbf{K}, \mathbf{V})$
  - compute d such that $(d \times k)%v = 1$

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- RSA algorithm for encryption/decryption
  - ► encryption: compute E(M) = (M<sup>k</sup>)%n
  - ► decryption: compute  $D(M) = (E(M)^d)^{\%}n$

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## **RSA algorithm for decryption**

- Input: integers d, n, C
  - **~** C is integer representation of ciphertext message
- Computation:
  - let D be integer representation of decrypted ciphertext D = (C<sup>d</sup>)%n
- Output: integer D
  - decrypted message

This seems hard ...

- How to find big primes?
- How to find mod inverse?
- How to compute greatest common divisor?
- How to translate text input to numeric values?
- Most importantly: RSA manipulates big numbers
  - ► Java integers are of limited size
  - how can we handle this?
- Two key items make the implementation easier
  - ► understanding the math
  - ► Java's BigInteger class

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### What is a BigInteger? Using BigInteger • If we understand what mathematical computations are • Java class to represent and perform operations on integers of involved in the RSA algorithm, we can use Java's arbitrary precision BigInteger methods to perform them • Provides analogues to Java's primitive integer operations, e.g. addition and subtraction • To declare a BigInteger named B multiplication and division BigInteger B; • Along with operations for modular arithmetic • Predefined constants ► gcd calculation BigInteger.ZERO generation of primes BigInteger.ONE http://java.sun.com/j2se/1.5.0/docs/api/ CompSci 001 11 41 CompSci 001 11 42 Randomly generated primes probablePrime • Example: randomly generate two BigInteger primes named BigInteger probablePrime(int b, Random rng) p and q of bit length 32: • Returns random positive BigInteger of bit length b that is "probably" prime /\* create a random number generator \*/ probability that BigInteger is not prime < 2<sup>-100</sup> Random rng = new Random(); • Random is Java's class for random number generation /\* declare p and q as type BigInteger \*/ • The following statement Random rng = new Random(); BigInteger p, q; creates a new random number generator named rng • What about *randomized algorithms* in general? /\* assign values to p and q as required \*/ p = BigInteger.probablePrime(32, rng); q = BigInteger.probablePrime(32, rng CompSci 001 11.43 11.44

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Integer operations
                                                                                          Greatest common divisor
                                                                                           • The greatest common divisor of two numbers x and y is the
    • Suppose have declared and assigned values for p and q and
                                                                                              largest number that divides both x and y
       now want to perform integer operations on them
                                                                                               this is usually written as gcd(x,y)
        use methods add, subtract, multiply, divide
                                                                                           • Example: gcd(20,30) = 10
        result of BigInteger operations is a BigInteger
                                                                                               ► 20 is divided by 1,2,4,5,10,20
                                                                                               ► 30 is divided by 1,2,3,5,6,10,15,30
    • Examples:
                                                                                           • Example: gcd(13,15) = 1
            BigInteger w = p.add(q);
                                                                                               ► 13 is divided by 1,13
            BigInteger x = p.subtract(q);
                                                                                               ► 15 is divided by 1,3,5,15
           BigInteger y = p.multiply(q);
                                                                                           • When the gcd of two numbers is one, these numbers are said
            BigInteger z = p.divide(q);
                                                                                              to be relatively prime
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  Euler's Phi Function
                                                                                         Relative primes
                                                                                           • Suppose we have an integer x and want to find an odd integer
• For a positive integer n, \phi(n) is the number of positive integers
                                                                                              z such that
  less than n and relatively prime to n
                                                                                               > 1 < z < x, and
• Examples:
                                                                                               z is relatively prime to x
   (4) = 2
                      1,2
   (4) = 2
                      1,2,3 (but 2 is not relatively prime to 4)
   (5) = 4
                      1,2,3,4
                                                                                           • We know that x and z are relatively prime if their greatest
                                                                                              common divisor is one
• For any prime number p,
                 \phi(p) = p-1
                                                                                               randomly generate prime values for z until gcd(x,z)=1
• For any integer n that is the product of two distinct primes p and
                                                                                               ► if x is a product of distinct primes, there is a value of z
  q,
                                                                                                  satisfying this equality
                \phi(n) = \phi(p)\phi(q)
                      = (p-1)(q-1)
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                                                                11.47
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### **Relative** BigInteger primes **Multiplicative identities and inverses** • Suppose we have declared a BigInteger x and assigned it a • The multiplicative identity is the element e such that value e \* x = x \* e = x• **Declare a** BigInteger z for all elements $x \in X$ • Assign a prime value to z using the probablePrime method specifying an input bit length smaller than that of x gives • The multiplicative inverse of x is the element x<sup>-1</sup> such that a value z<x $x * x^{-1} = x^{-1} * x = 1$ • The expression (x.gcd(z)).equals(BigInteger.ONE) • The multiplicative inverse of x mod n is the element x<sup>-1</sup> such that returns true if gcd(x,z)=1 and false otherwise $(x * x^{-1}) \mod n = (x^{-1} * x) \mod n = 1$ • While the above expression evaluates to false, assign a new ► x and x<sup>-1</sup> are inverses only in multiplication mod n random to z CompSci 001 11.49 CompSci 001 11 50 modInverse • Suppose we have declared BigInteger variables x, y and assigned values to them • We want to find a BigInteger z such that (x\*z) %y = (z\*x) %y = 1that is, we want to find the inverse of $\times$ mod $\vee$ and assign its value to z • This is accomplished by the following statement: BigInteger z = x.modInverse(y); CompSci 001 11.51