CPS 196.2 - Intro to Computational Economics

Written Assignment 1: kidney exchange, voting (due 10/18 before class)

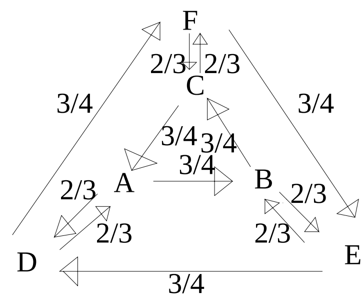
Please read the rules for assignments on the course web page. Contact Mingyu (mingyu@cs.duke.edu) or Vince (conitzer@cs.duke.edu) with any questions.

1. Kidney exchange with probabilities.

In kidney exchange, so far, we have assumed that there is either an edge from one vertex to another, or not. This is not completely realistic; in reality, we do not always know with certainty if a donor is compatible with a patient. We will model this to some limited extent here.

In particular, we associate a probability with every edge, which is the probability of compatibility. We choose cycles, taking these probabilities into account. Right before any planned surgery (after we choose the cycles), we perform some advanced tests to determine compatibility. Let us assume that these tests are flawless: they predict exactly whether a donor and a patient are compatible. If they are incompatible, we do not perform the surgery. Moreover, in this case, we must cancel *every* surgery in the cycle (because we cannot use the donor associated with the patient whose operation was canceled, which leaves another patient without a kidney, so we cannot use her donor, etc.). So, if the tests reveal one or more incompatibilities in a cycle, the whole cycle is canceled. For simplicity, let us assume that we cannot re-match these canceled patients and donors. We will assume that compatibilities are independent: if donor A and patient B turn out to be compatible, this gives us no information about whether donor C and patient D are compatible. Our objective is to maximize the *expected* number of surgeries that are not canceled.

Consider the following example:



a. If we choose the cycle DFE, what is the probability that it is not canceled (i.e. that all the edges turn out to be compatible)? What about the cycle AD? What about the cycle ABED? What about ABEDFC?

b. What is the optimal solution? Explain your reasoning.

c. Suppose we change all occurrences of 3/4 to some other value. At what value does another solution become equally good as the one in **b**? Explain your reasoning.

d. Consider the cycle-based integer program from class for the kidney exchange problem (the one with a variable for each cycle). How can you modify this program to take the probabilities into account? (Remember that it is OK to multiply *parameters* with each other—but multiplying *variables* with each other is not allowed if you want an (integer) linear program.)

2. Strategic voting with probabilities.

We will consider an election with 4 candidates, A, B, C, D. We will consider the predicament of a voter who has a utility of 3 for A winning, a utility of 2 for B winning, a utility of 1 for C winning, and a utility of 0 for D winning. Hence, the voter's truthful vote would be $A \succ B \succ C \succ D$. But the voter wants to maximize his expected utility, and is willing to misreport his preferences if this is to his benefit. That is, he is voting strategically.

In the below, if candidates are tied, the tie will be broken uniformly at random. For example, if A and C end up tied for the win, then A is chosen with probability .5, and C is chosen with probability .5, resulting in an expected utility of 2 for our strategic voter.

a. Suppose that the strategic voter (somehow) knows that the (two) other votes will be $B \succ C \succ D \succ A$ and $C \succ A \succ B \succ D$. For each of Borda and Copeland, say whether voting truthfully is optimal for our strategic voter (and give the expected utility for the strategic voter); if not, give another vote that gives a higher expected utility (and give the expected utility for this vote and for the truthful vote).

b. Now suppose that the strategic voter (somehow) knows that the (two) other votes will be $B \succ D \succ C \succ A$ and $D \succ C \succ A \succ B$. Again, for each of Borda and Copeland, say whether voting truthfully is optimal for our strategic voter (and give the expected utility for the strategic voter); if not, give another vote that gives a higher expected utility (and give the expected utility for this vote and for the truthful vote).

c. Now suppose the strategic voter believes that with probability .5, the other votes will be as in **a** ($B \succ C \succ D \succ A$ and $C \succ A \succ B \succ D$), and with probability .5, the other votes will be as in **b** ($B \succ D \succ C \succ A$ and $D \succ C \succ A \succ B$). Again, for each of Borda and Copeland, say whether voting truthfully is optimal for our strategic voter (and give the expected utility for the strategic voter); if not, give another vote that gives a higher expected utility (and give the expected utility for this vote and for the truthful vote).

(Comment: it is not clear why the strategic voter would have these beliefs over the other votes. Those voters may also be strategic, in which case this should be taken into account. The game theory portion of the course will provide techniques for doing this. Nevertheless, even when using game theory, a strategic voter will have beliefs over what the other voters will do.)

Instructions for turning in your homework: Turn in your writeup at the **beginning** of class. *Do not come to class late because you are still working on the homework!* If you know you will be late for class, turn in your homework earlier (e.g. under Vince's door, LSRC D207). You are welcome to go to Mingyu and Vince's office hours or to send them e-mail with questions.