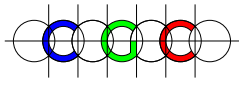
Surface Details

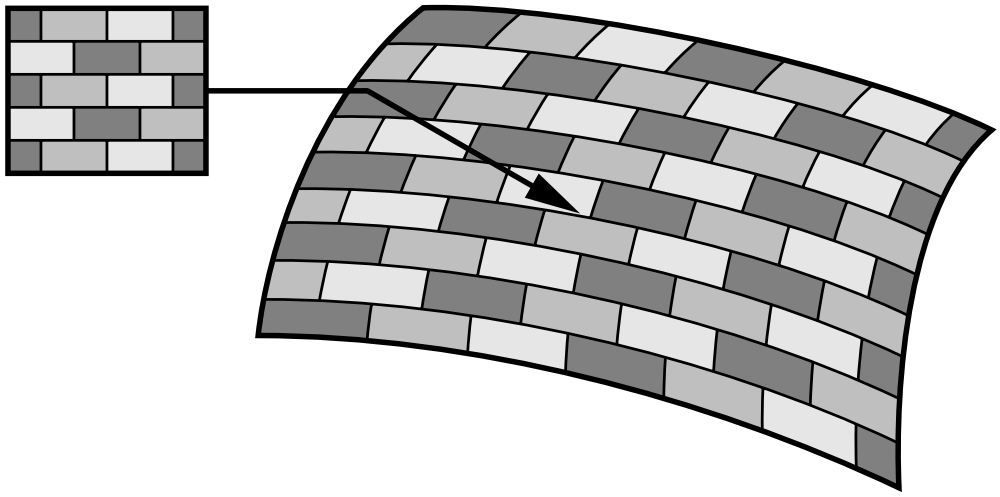
- ☆ Incorporate fine details in the scene.
- ☆ Modeling with polygons is impractical.
- ☆ Map an image (texture/pattern) on the surface (Catmull, 1974); (Blin & Newell, 1976).

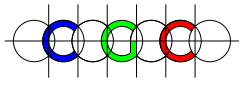
- ☆ *Texture map*
  - Models patterns, rough surfaces, 3D effects.
- ☆ *Solid textures*
  - (3D textures) to model wood grain, stains, marble, etc.
- ☆ *Bump mapping*
  - Displace normals to create shading effects.
- ☆ *Environment mapping*
  - Reflections of environment on shiny surfaces.
- ☆ *Displacement mapping*
  - Perturb the position of some pixels.



Texture Maps

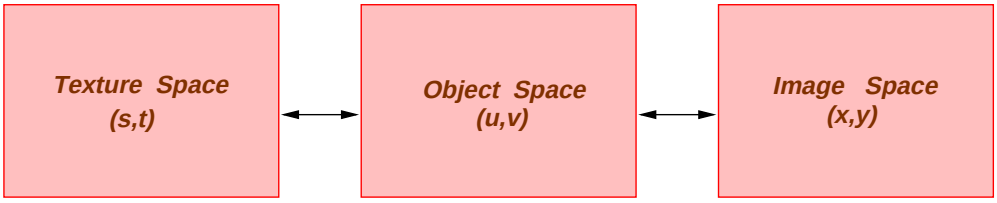
- ☆ Maps an image on a surface.
- ☆ Each element is called *texel*.
- ☆ Textures are fixed patterns, procedurally generated, or digitized images.

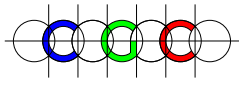




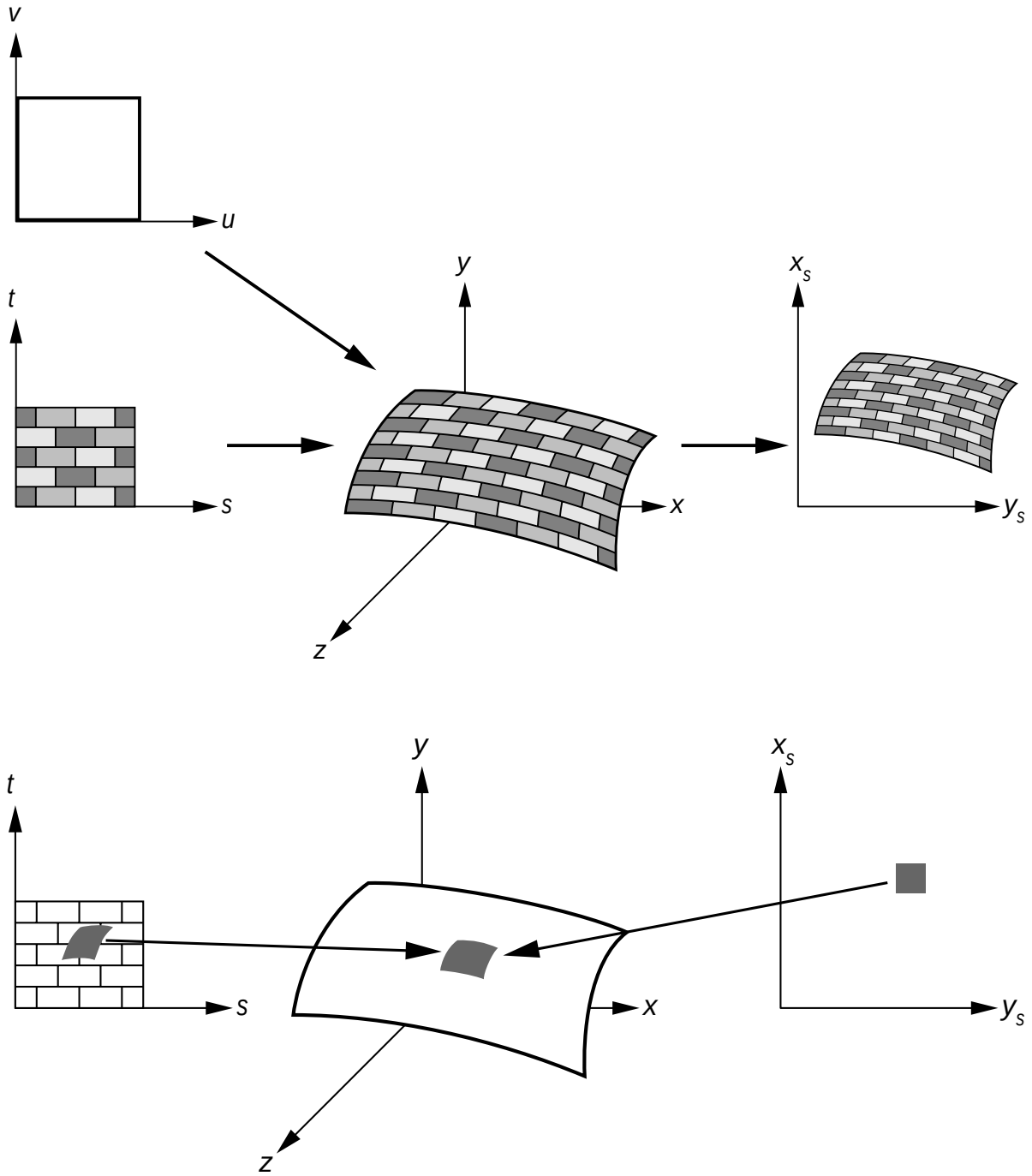
Texture Maps

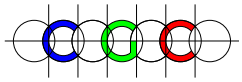
- ☆ Texture map has its own coordinate system; st-coordinate system.
- ☆ Surface has its own coordinate system; uv-coordinates.
- ☆ Pixels are referenced in the window coordinate system (Cartesian coordinates).



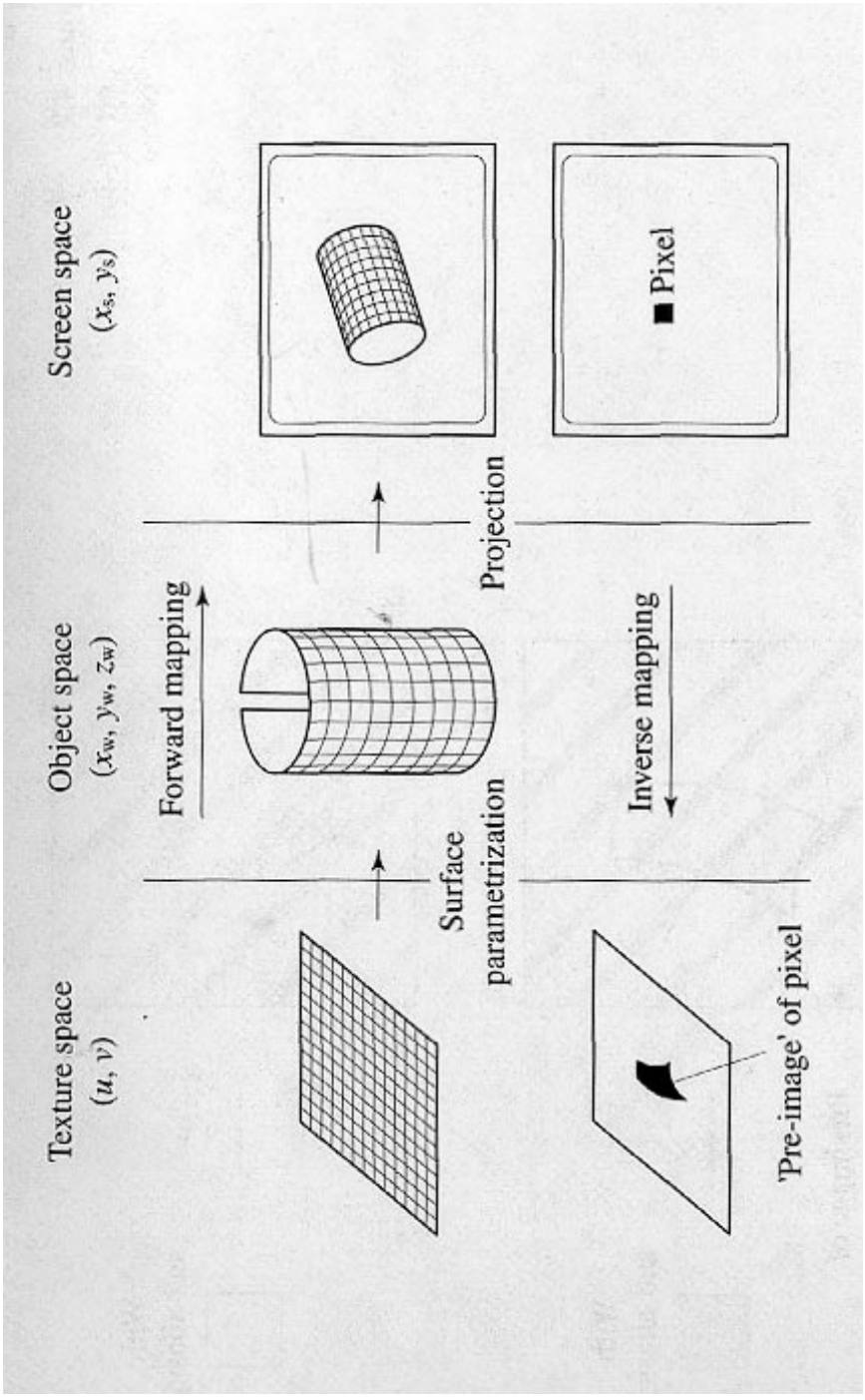


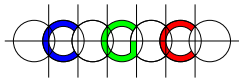
Texture Maps





Texture Mapping





Texture Mapping

Forward mapping: (Texture scanning)

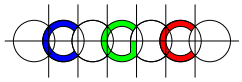
- ☆ Map texture pattern to the object space.

$$\begin{aligned} u &= f_u(s,t) = a_u s + b_u t + c, \\ v &= f_v(s,t) = a_v s + b_v t + c. \end{aligned}$$

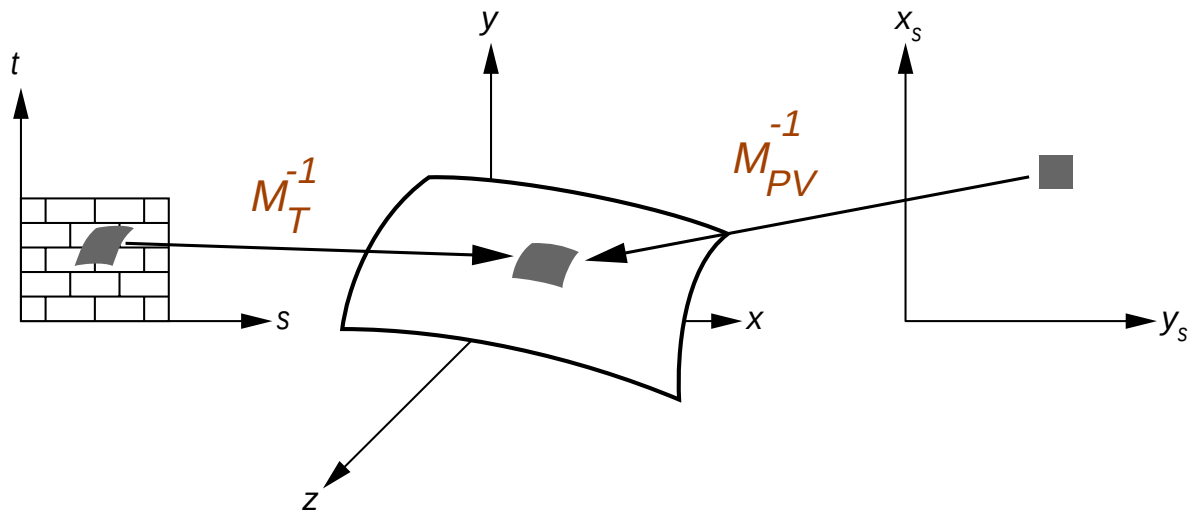
- ☆ Map object space to window coordinate system.
  Use modelview/projection transformations.

Drawback: Selected texture patch usual does not match with pixel boundaries.

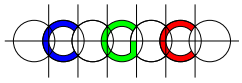
- ☆ Requires fractional pixel calculations.



Inverse Mapping



- ☆ Map screen coordinate system to object space.
- ☆ Map object coordinate system to texture space.
- ☆ Avoids fractional pixel calculations.
- ☆ Allows anti-aliasing.
- ☆ Requires calculating inverse transformations; M_{PV}^{-1}, M_T^{-1}.
  - M_{PV}^{-1} can be computed from projection and modelview matrices (`gluUnproject`)
  - Computing M_T^{-1} is not easy.



Parametric Representation

Curves: Coordinates are represented as functions of one parameter.

$$\gamma(t) = (x(t), y(t)), \quad t \in [a, b]$$

Line: $\ell : (a_1 + b_1 t, a_2 + b_2 t),\ t \in \mathbb{R}$

Surfaces: Coordinates of each point is represented as a function of u and v.

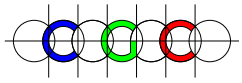
$$S(u, v) = (x(u, v), y(u, v), z(u, v)).$$

★ *Sphere:*

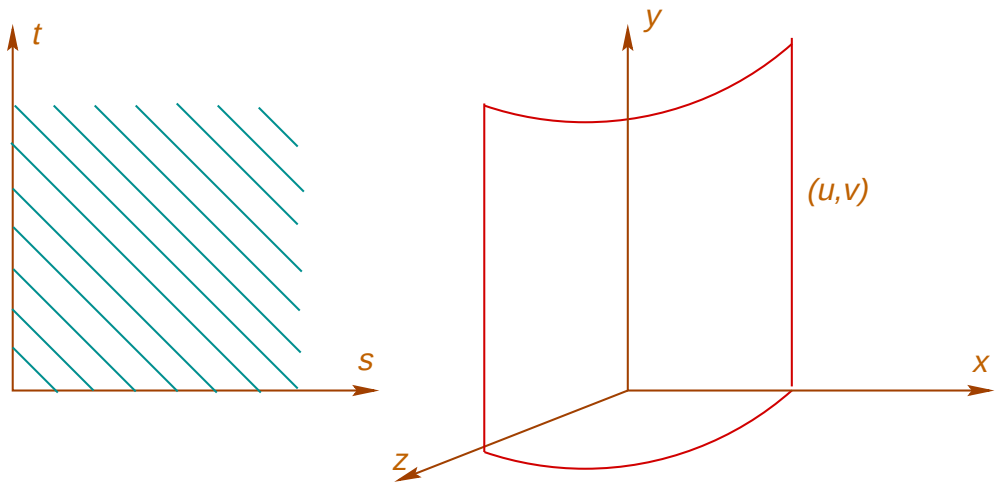
$$S(u, v) = (r \cos u \cos v, r \cos u \sin v, r \sin u)$$
$$-\pi/2 \leq u \leq \pi/2, 0 \leq v \leq 2\pi.$$

★ *Cylinder:*

$$S(u, v) = (r \cos u, r \sin u, v)$$
$$0 \leq u \leq 2\pi, 0 \leq v \leq h.$$



Inverse Mapping: An Example



$$u = \theta, v = y \qquad 0 \le \theta \le \pi/2, 0 \le y \le 1$$

$x = \sin\theta, z = \cos\theta, y = v.$

$M_{PV}^{-1} : u = \sin^{-1} x, \quad v = y.$

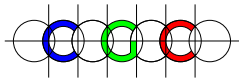
Map texture origin to left bottom corner of the surface

$$u = s\pi/2, \quad v = t.$$

Projected pixels are mapped to texture with M_T^{-1}:

$$M_T^{-1} : s = \frac{2u}{\pi} \qquad t = v.$$

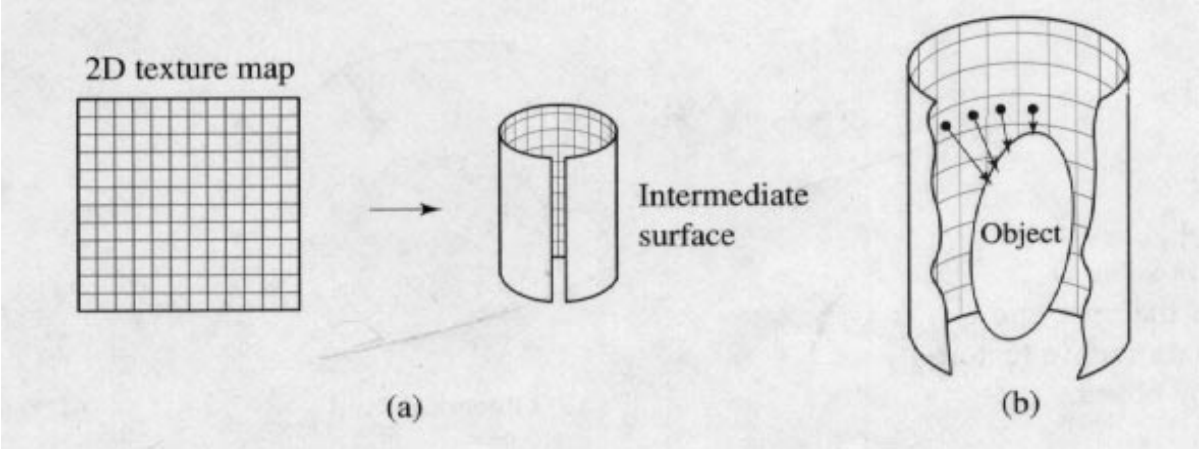
$$s = \frac{2}{\pi}\sin^{-1} x \qquad t = z.$$



Object to Texture Mapping

How does one define a reasonable M_T^{-1}?

(Bier & Sloan, 1986): Two-step process:

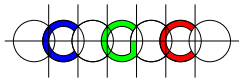


***S-mapping*:** Mapping from a 2D texture space to a simple 3D surface, e.g., cylinder.

$$T(u, v) \rightarrow T'(x_i, y_i, z_i).$$

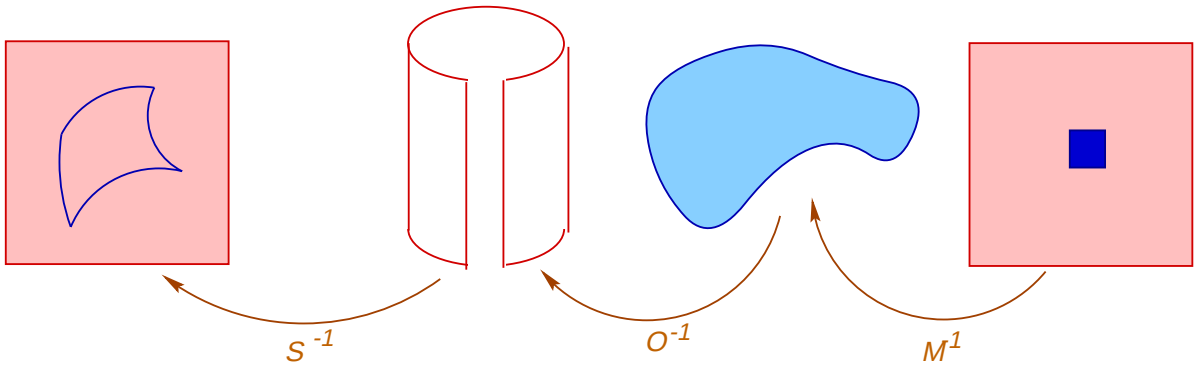
***O-mapping*:** Mapping from the 3D texture pattern onto the object surface.

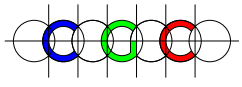
$$T'(x_i, y_i, z_i) \rightarrow O(x_w, y_w, z_w).$$



Two-Step Inverse Mapping

Inverse mapping: Apply them in the reverse order!





S-Mapping

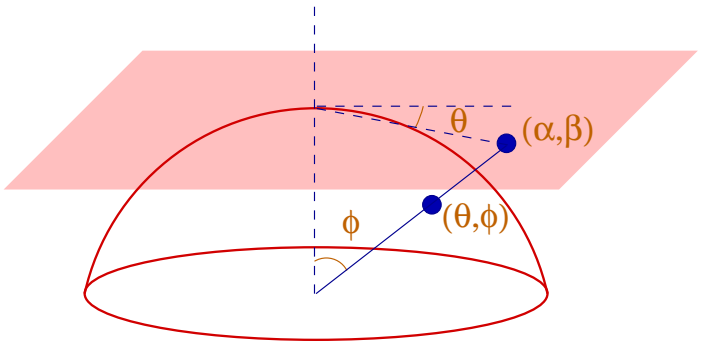
Four possible surfaces: Plane, cylinder, sphere, box.

Cylinder: (θ, z)

$$S : (\theta, z) \rightarrow \left[\tfrac{r}{c}(\theta - \theta_0), \tfrac{1}{d}(z - z_0)\right]$$

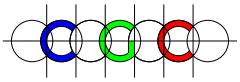
- ☆ c, d: Scaling factors.
- ☆ Texture origin is mapped to the point (θ_0, z_0) on the cylinder.

Sphere: (θ, φ), stereographic projection.

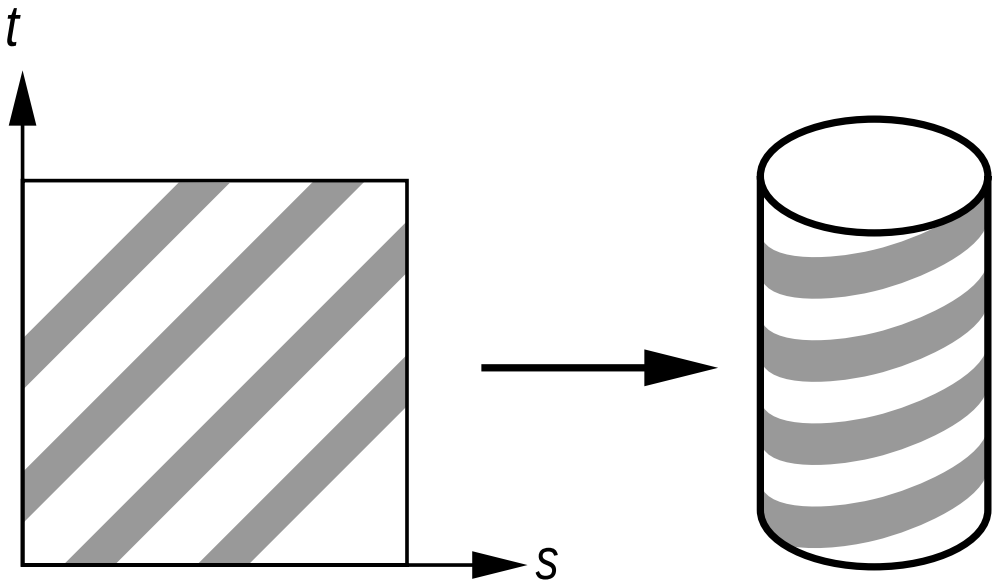


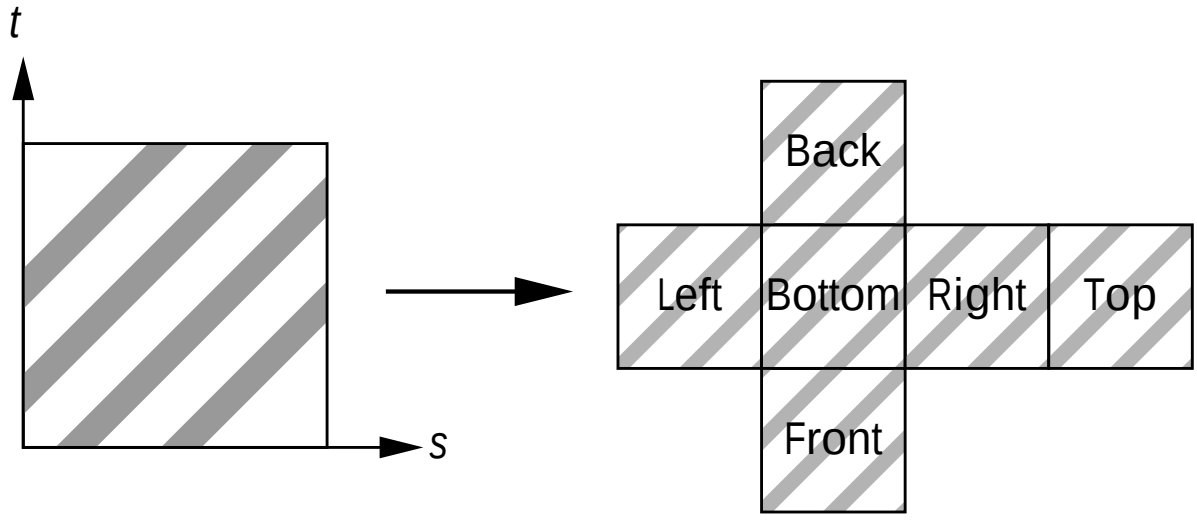
$$S : (\theta, \varphi) \rightarrow \left[\frac{2\alpha}{1 + (1 + \alpha^2 + \beta^2)^{1/2}}, \frac{2\beta}{1 + (1 + \alpha^2 + \beta^2)^{1/2}}\right]$$

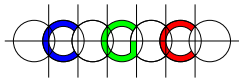
$$\alpha = \tan\varphi \cos\theta,\ \beta = \tan\varphi \sin\theta.$$



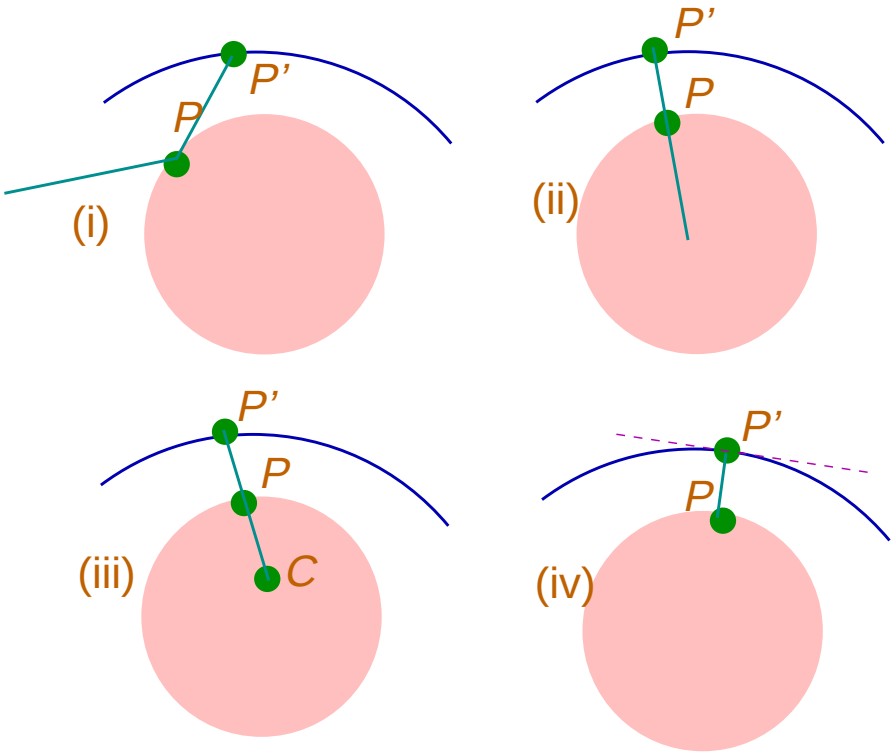
S-Mapping



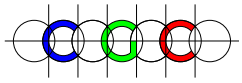




O-Mapping



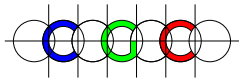
- ☆ , : Intermediate surface
- ☆ P': Intersection of , with a ray ρ emanating from P.
- ☆ Direction of ρ:
  - (i) Reflection direction
  - (ii) Normal of O at P
  - (iii) $\overline{CP}$; C: centroid of O
  - (iv) $-\rho$: Normal of , at P'
- ☆ (ii), (iii) are bad if , is cylinder.



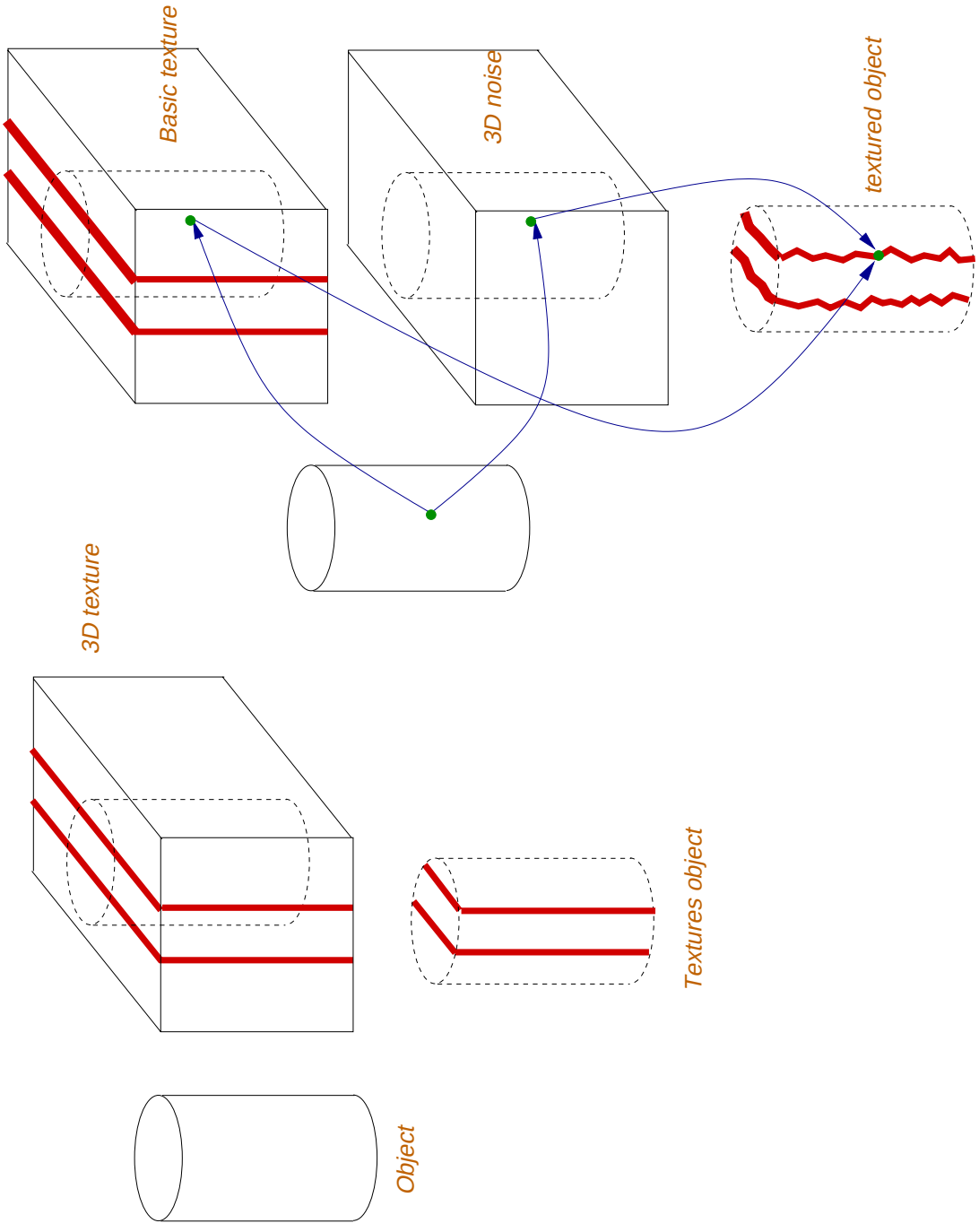
3D Textures

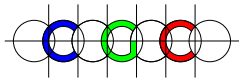
Introduced by Peachey and Perlin in 1985.

- ☆ Define texture to be a 3D image.
  - Carving out an object from a 3D solid material.
- ☆ Ignoring scaling, M_T is identity.
- ☆ Distortion is minimized.
- ☆ Three dimensional vector fields can be mapped coherently.
- ☆ Texture is generated by a procedure.
- ☆ *Example:* Wood grain can be mapped as a set of cylinders with respect to a prespecified axis.



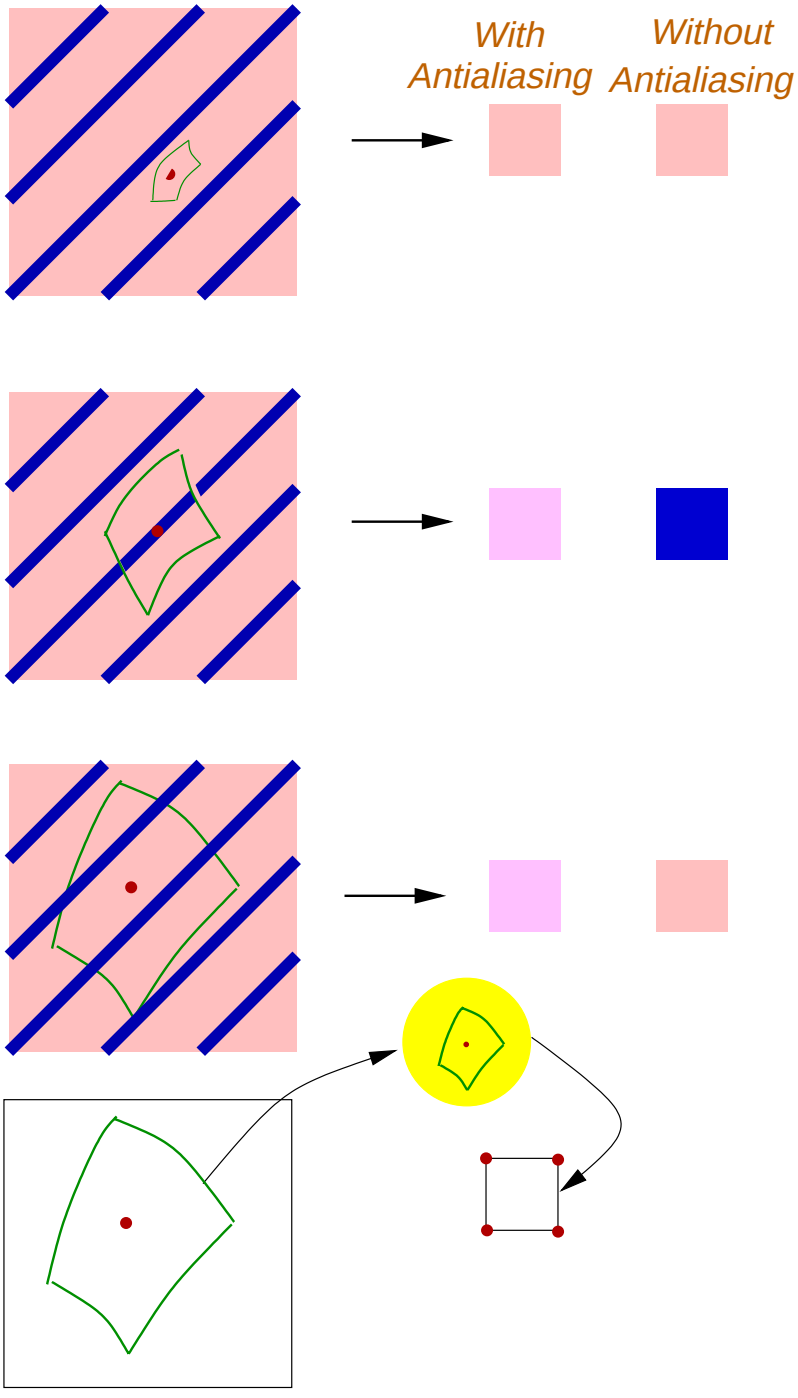
3D Textures

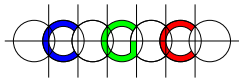

Basic texture
3D noise
textured object
3D texture
Textures object
Object



Anti-aliasing & Texture Mapping

Aliasing is particularly visible in periodic and coherent textures.



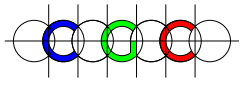


Anti-aliasing & Texture Mapping

- ☆ A pixel is mapped to a curvilinear quadrilateral.
- ☆ A single pixel may cover many texels.
- ☆ Compute a weighted sum of texel values covered by the pixel.
- ☆ Summation is called *filtering*.

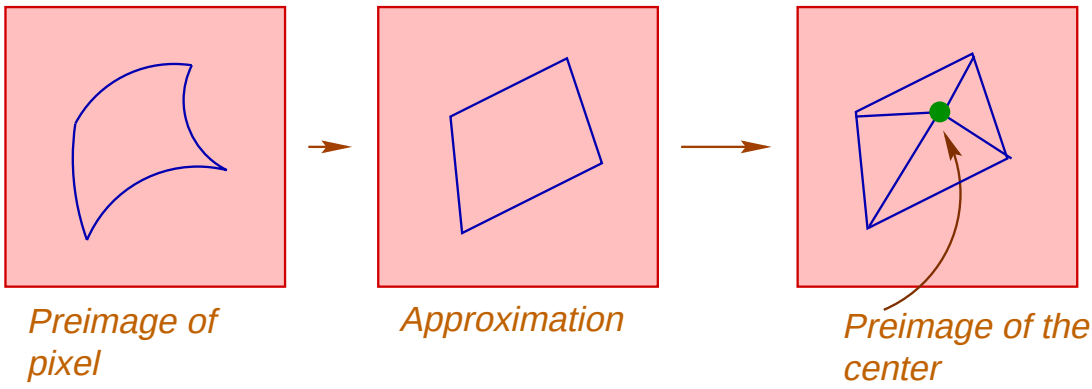
Two-step process:

- ☆ Define and approximate the texture over which filtering is performed.
- ☆ Integrate by weighing and summing the texel values within the filtering area.



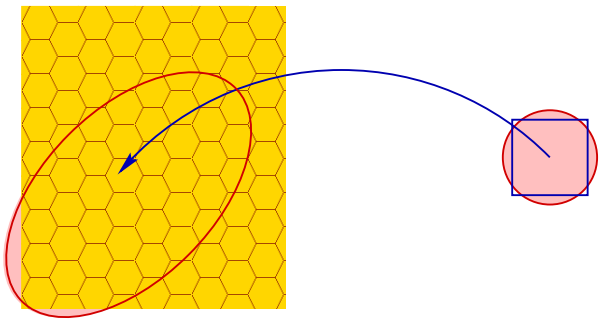
Anti-aliasing: Weight Functions

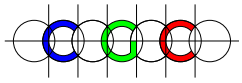
Blinn & Newell, (1976): Pyramid weight function



Greene & Heckbert, (1986): Elliptical weighted average

- ☆ Approximate a pixel by circle.
- ☆ Circle always maps to an ellipse in the texture space.
- ☆ Find the texels that lie inside the ellipse.
- ☆ Use a look-up table to determine the weighted value of each texel.

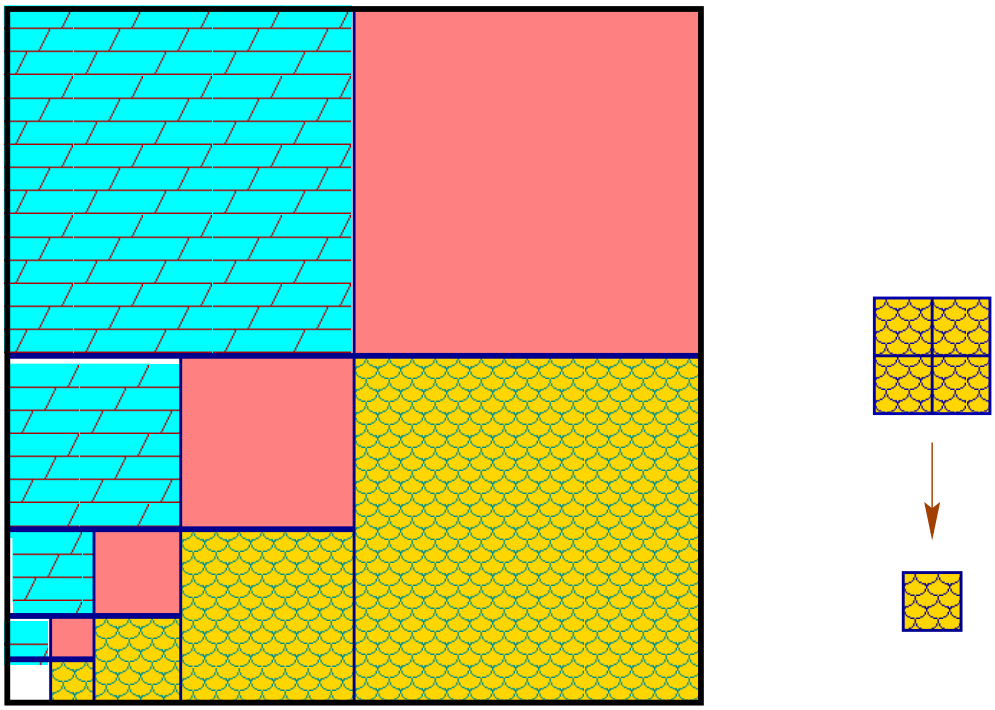




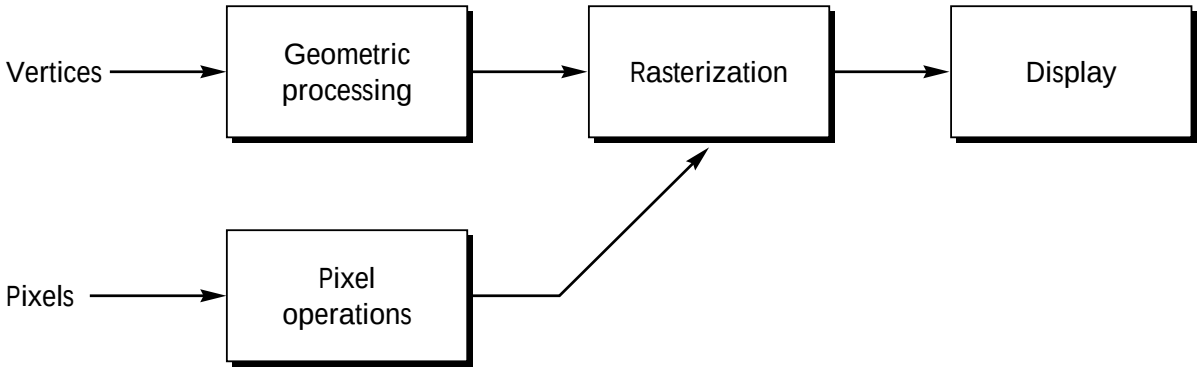
Anti-aliasing & Texture Mapping

Mip-mapping: *multum in parvo*
(Williams, 1983)

- ☆ Store many texture images.
- ☆ i-th image is obtained by scaling down the previous image by half along each axis.
- ☆ Effectively a 3D database.
- ☆ Given a pixel, search in the image with an appropriate resolution.



Texture Mapping in OpenGL

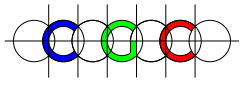


- ☆ Relies on the pipeline architecture
- ☆ Texture mapping is done at the rasterization stage.

```
glTexImage2D (GL_TEXTURE_2D, level, comp, w, h,
              border, format, type, tarray)

GLubyte my_texel[512][512];
glTexImage2D (GL_TEXTURE_2D, 0, 3, 512, 512, 0
              GL_RGB, GLuint, tarray);
glEnable(GL_TEXTURE_2D);
```

- ☆ *level*: Multiple levels of texture maps; 0 for one level.
- ☆ *comp*: integer between 1 and 4; specifies how many of R, G, B, and A components specified.
- ☆ *format*: Format of the texture map.



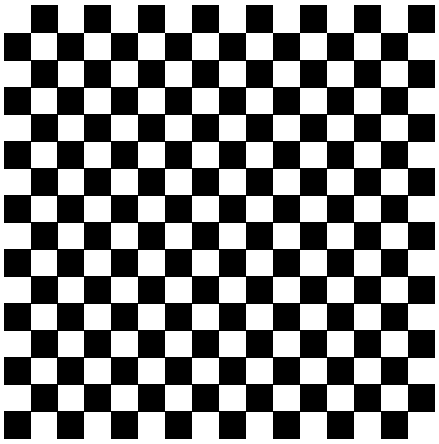
Texture Mapping in OpenGL

`glTexCoord2f` (s, t)

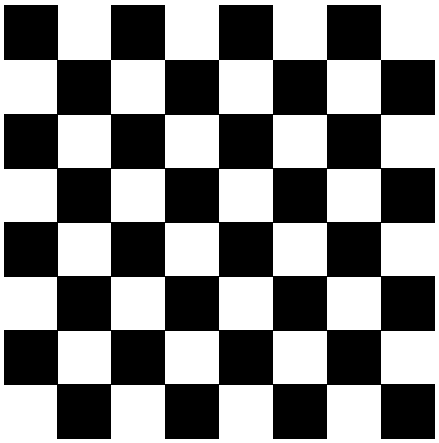
Assigns the two dimensional texture coordinates to a vertex.

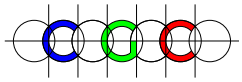
```
glBegin(GL_QUAD);
  glTexCoord2f (0.0, 0.0);
  glVertex3f (x1, y1, z1);
  glTexCoord2f (1.0, 0.0);
  glVertex3f (x2, y2, z2);
  glTexCoord2f (1.0, 1.0);
  glVertex3f (x3, y3, z3);
  glTexCoord2f (0.0, 1.0);
  glVertex3f (x4, y4, z4);
glEnd
```

(a)



(b)





Texture Mapping in OpenGL

- ☆ Texture objects

  `glGenTextures`(n, *names); `glBindTexture`(target, name);

- ☆ Repeating the texture pattern

  `glTexParameteri`(GL_TEXTURE_WRAP_S,GL_REPEAT)

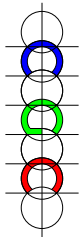
  `glTexParameteri` (GL_TEXTURE_WRAP_S,GL_CLAMP)

  Use GL_TEXTURE_WRAP_T for t-coordinates

- ☆ Assign a pixel color to the nearest texel

  `glTexParameterf` (GL_TEXTURE2D, GL_TEXTURE_MAG_FILTER, GL_NEAREST)

  `glTexParameterf` (GL_TEXTURE2D, GL_TEXTURE_MIN_FILTER, GL_NEAREST)



- ☆ GL_LINEAR: Interpolates the color using a 2×2 average.

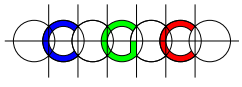
Bump Mapping

Perturb normals in the illumination model calculations.

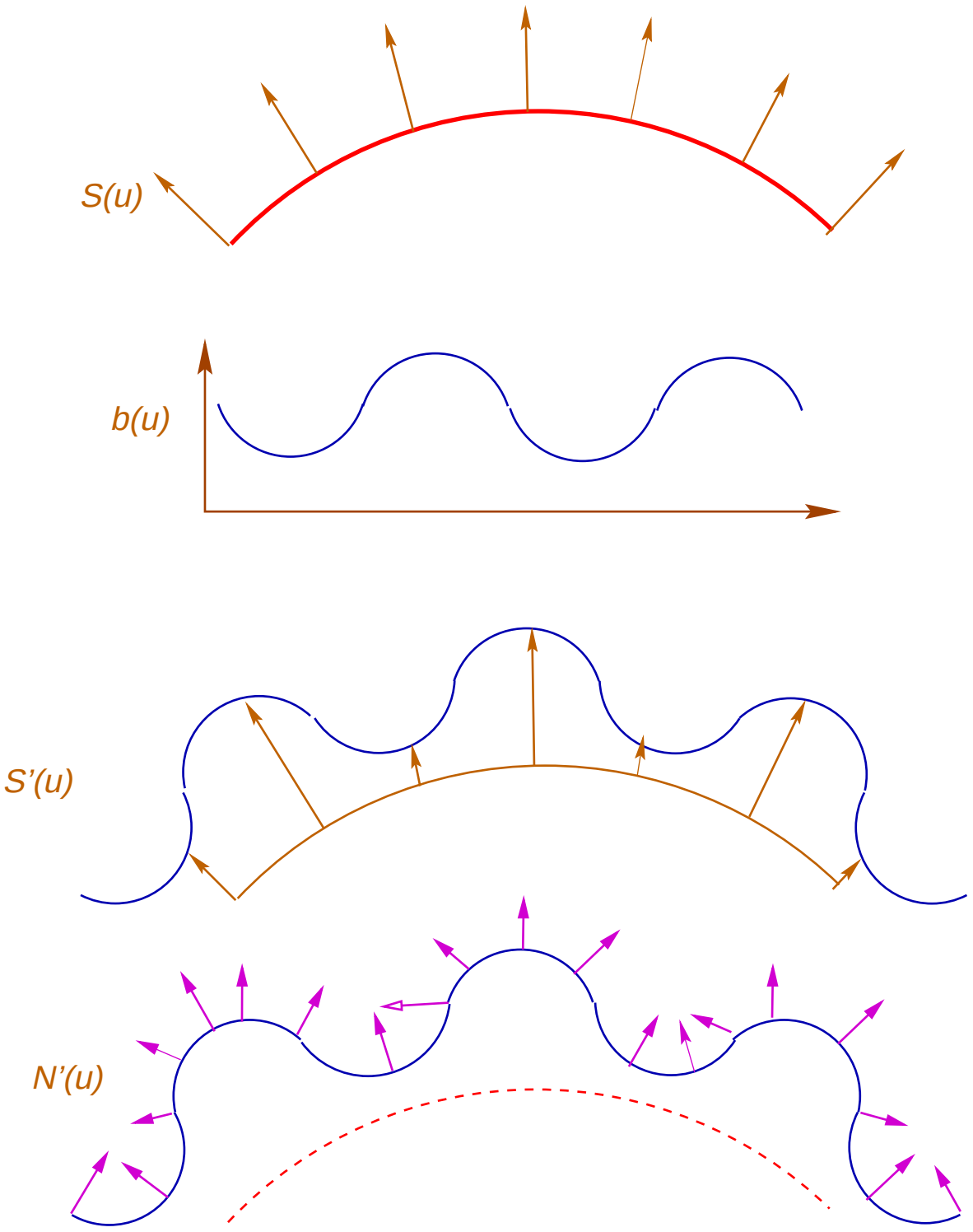
- ☆ $S(u,v)$: Parameterized surface
- ☆ $S_u = \partial S(u,v)/\partial u$, $S_v = \partial S(u,v)/\partial v$.
- ☆ $N(u,v) = S_u \times S_v$; $n = N/|N|$
- ☆ $b(u,v)$: Bump function

$S'(u,v) = S(u,v) + b(u,v) \cdot n \qquad N'(u,v) = S'_u \times S_v$

$$
\begin{aligned}
S'_u &= \frac{\partial}{\partial u}(S(u,v) + b(u,v) \cdot n) \\
&= S_u + b_u \cdot n + b n_u \\
&\approx S_u + b_u \cdot n \\
S'_v &\approx S_v + b_v \cdot n \\
N' &\approx S_u \times S_v + b_u(S_u \times n) + b_v(n \times S_v) + \\
&\quad b_u b_v (n \times n) \\
&= S_u \times S_v + b_u(S_u \times n) + b_v(n \times S_v)
\end{aligned}
$$



Bump Mapping

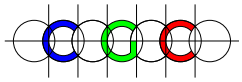


Bump Mapping

- ☆ Define bump functions analytically.
- ☆ Use look-up tables for bump functions.
- ☆ Approximate b_u, b_v with finite differences.
- ☆ Random pattern vs regular patterns.

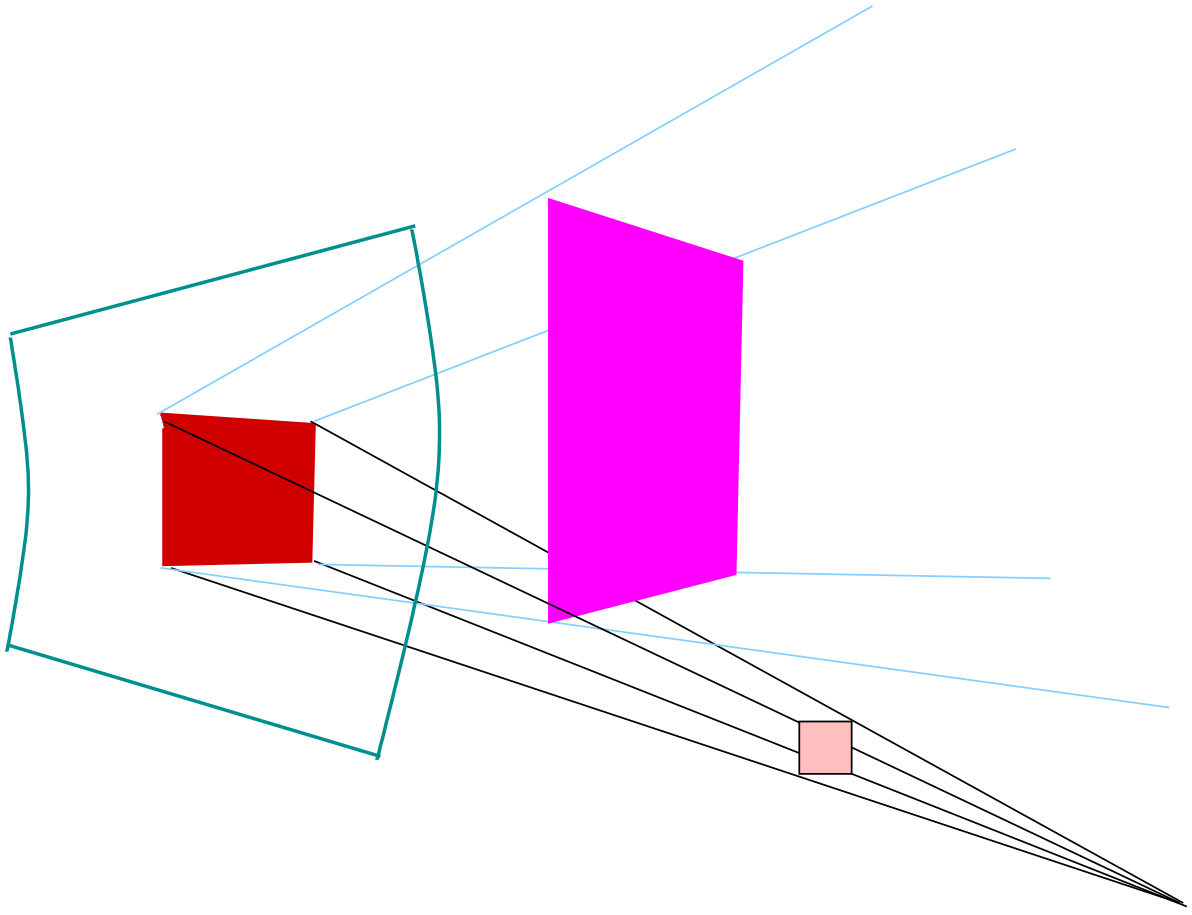
Displacement Mapping:

- ☆ Perturb normals as well as local coordinate system
- ☆ Used to render anisotropic objects.



Environment Mapping

- ☆ Reflects the surrounding environment on the surface of shiny objects.
- ☆ Similar to texture mapping.
- ☆ Pattern depends on the viewpoint.



- ☆ Store environment maps as 2D images.