CPS 296.1 - Computational Microeconomics: Game Theory, Social Choice, and Mechanism Design

Homework 3 (due 11/18)

Note the rules for assignments on the course web page. Show all your work, but circle your final answer. Contact Vince (conitzer@cs.duke.edu) with any questions.

1. (Stochastic games.)

Consider the following two-state two-player zero-sum stochastic game with discount factor $\delta = .9$ (i.e., you care about the next period only 90% as much as you care about the current period). (Each of the matrices corresponds to a state.)

30, -30 .4	0, 0 .4
0, 0 .4	20, -20 .4

-1, 1 .8	-1, 1 0
-2, 2 .9	0, 0 0

Each entry of each matrix gives the utilities for the players, as well as the probability of transitioning to the *other* state given that this entry was played. For example, if in state 1, (U, L) is played, then the next period the game will be in state 2 with probability .4 (and in state 1 with probability .6).

(40 points.) Find an equilibrium of this game in stationary strategies. You should give each player's (possibly mixed) strategy for each state, as well as the value of each state to player 1. (Recall that the value of a state includes not only the immediate rewards, but also the whole discounted sequence of future rewards. That is, given equilibrium strategies σ_1, σ_2 for the players, for any state $s \in S$,

$$v_1(s) = R_1^{\sigma_1,\sigma_2}(s) + \delta \sum_{s' \in S} P^{\sigma_1,\sigma_2}(\text{next state is } s' | \text{current state is } s) v_1(s')$$

where $R_1^{\sigma_1,\sigma_2}(s)$ is the expected immediate reward for player 1 in state s given that the players are playing σ_1, σ_2, and the conditional probabilities $P^{\sigma_1,\sigma_2}(\cdot|\cdot)$ are also taken according to the equilibrium (which is why σ_1, σ_2 is in the superscript). You are allowed to solve this game by computer (e.g., by coding up Shapley's algorithm), but it should be possible to solve it by hand, if you think about the game carefully.

2. (Coalitional games.)

Consider again the archaic Babylonian Talmud game from class. Again, the agents ("wives") have claims of 100, 200, and 300, respectively. Now suppose that the estate that needs to be divided is 250.

2a. (15 points.) Write down the characteristic function of the game. That is, for every nonempty subset of agents, write down the value of that subset. Remember that each coalition can choose to get 0, or to pay off everyone outside the coalition in full and divide the remainder of the estate. (Be careful – the rest of the question depends on this!)

2b. (15 points.) How much does the Shapley value give to each agent?

2c. (15 points.) Is the Shapley value outcome in the core? Why (not)?

2d. (15 points.) How much does the nucleolus give to each agent? Write down the list of excesses for the nucleolus in decreasing order, and argue why it is impossible to better. (This question will probably require you to play around with the payoffs a little to find the nucleolus.)