Worst-Case Optimal Redistribution of VCG Payments in Multi-Unit Auctions

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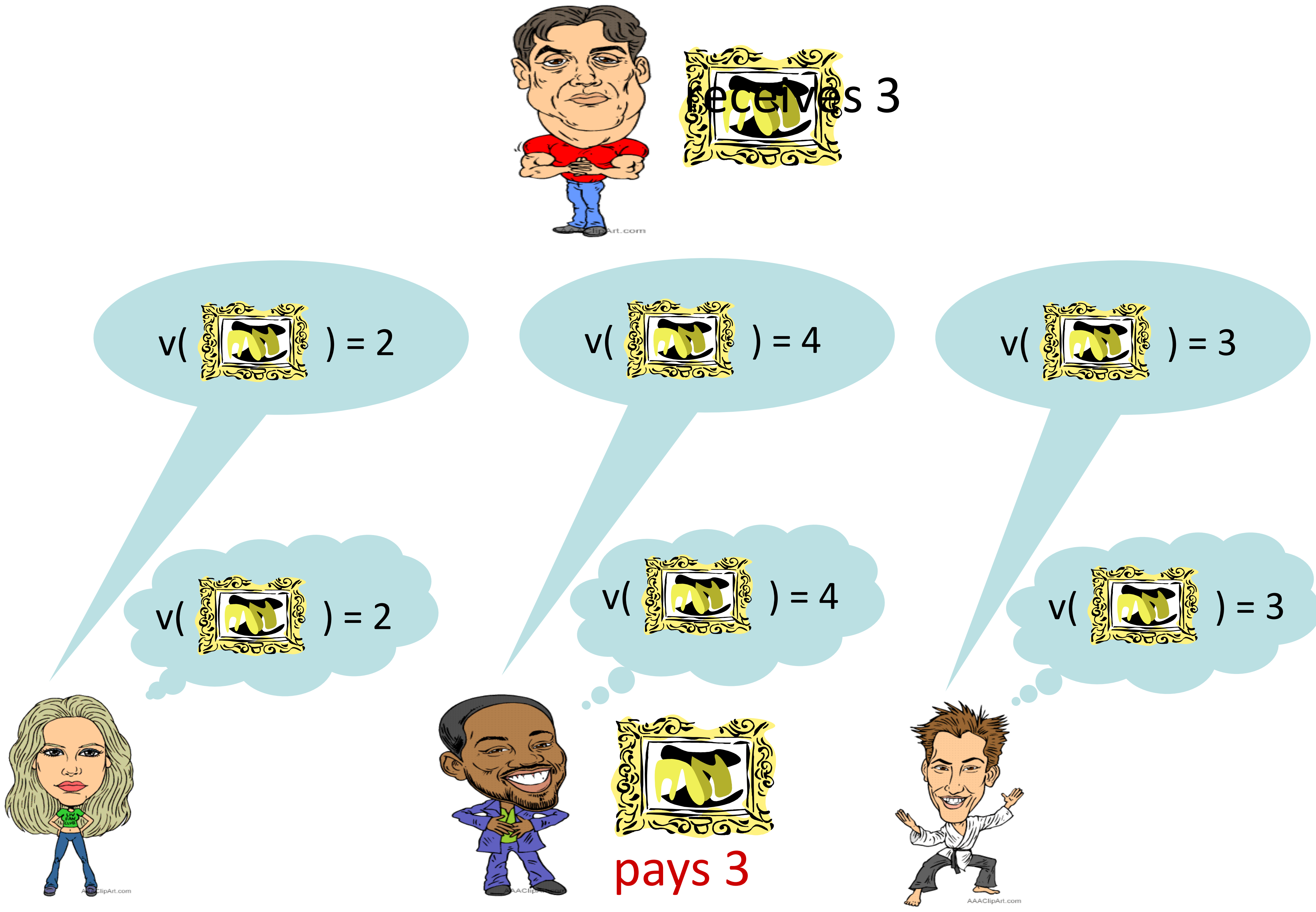
Guest Lecture CPS 196.1/296.1 Fall 09

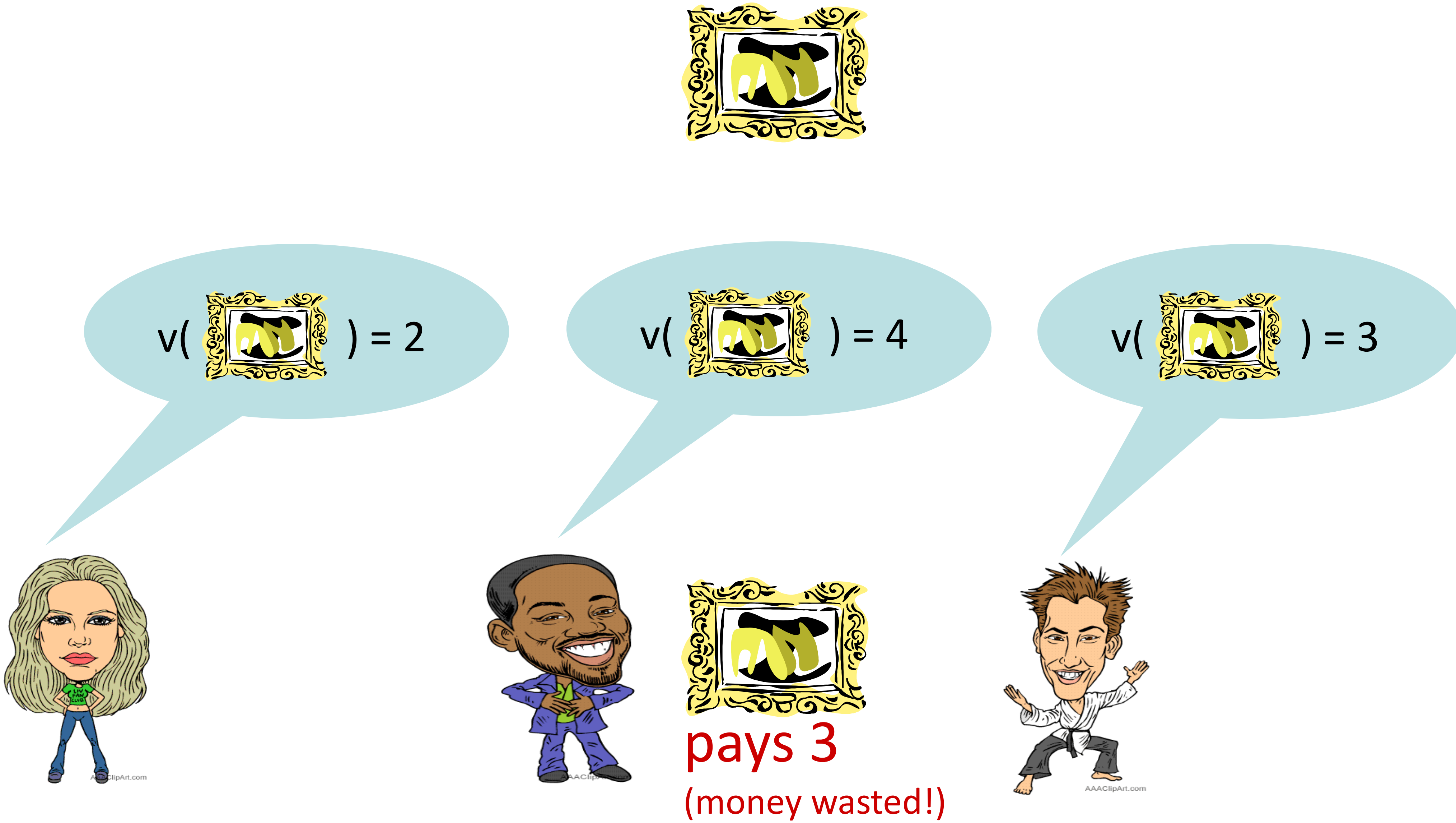
This talk covers material from:

Guo and Conitzer, “Worst-Case Optimal Redistribution of VCG Payments in Multi-Unit Auctions” GEB 2009

Second-price (Vickrey) auction

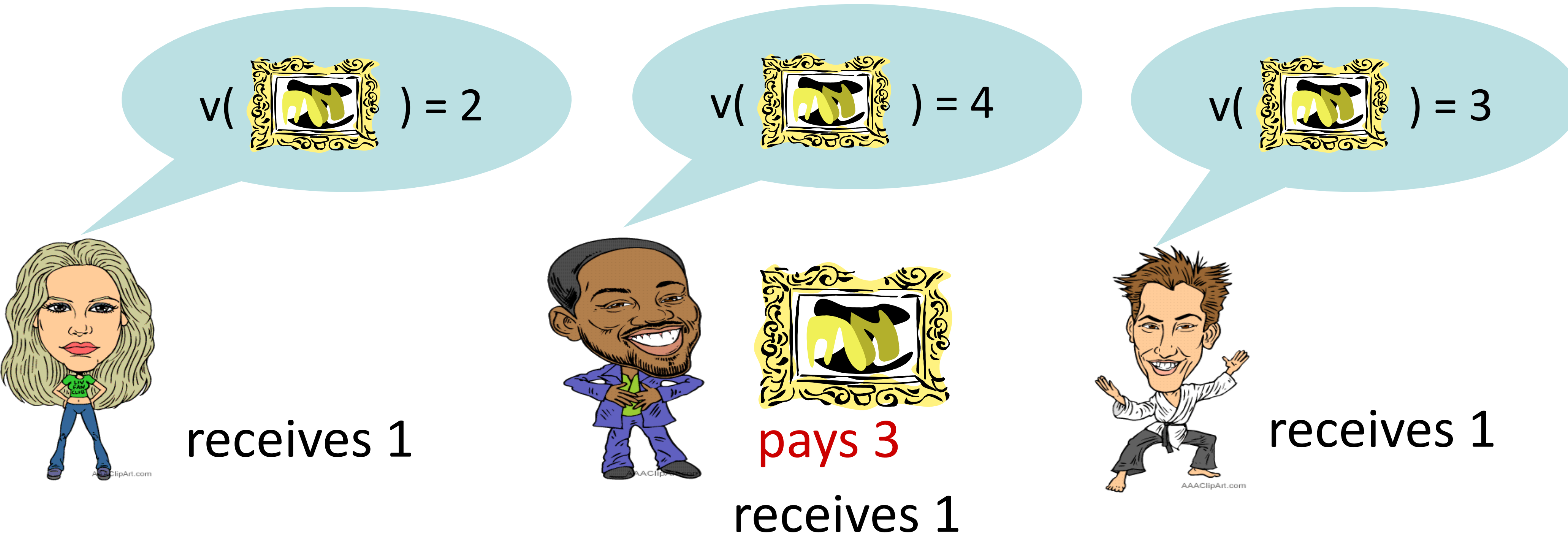


Vickrey auction without a seller



Can we redistribute the payment?

Idea: give everyone 1/n of the payment



not incentive compatible

Bidding higher can increase your redistribution payment

Incentive compatible redistribution

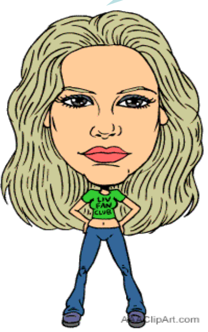
[Bailey 97, Porter et al. 04, Cavallo 06]

Idea: give everyone 1/n of second-highest **other** bid

v() = 2

v() = 4

v() = 3



receives 1

pays 3

receives 2/3

receives 2/3

2/3 wasted (22%)

incentive compatible

Your redistribution does not depend on your bid;

incentives are the same as in Vickrey

Bailey-Cavallo mechanism…

- Bids: $V_1 \geq V_2 \geq V_3 \geq \ldots \geq V_n \geq 0$
- First run Vickrey auction
- Payment is V_2
- First two bidders receive V_3/n
- Remaining bidders receive V_2/n
- Total redistributed: $2V_3/n+(n-2)V_2/n$

$R_1 = V_3/n$

$R_2 = V_3/n$

$R_3 = V_2/n$

$R_4 = V_2/n$

…

$R_{n-1} = V_2/n$

$R_n = V_2/n$

Can we do better?

Desirable properties

- Incentive compatibility
- Individual rationality: bidder's utility always nonnegative
- Efficiency: bidder with highest valuation gets item
- Non-deficit: sum of payments is nonnegative
  - i.e. total VCG payment ≥ total redistribution
- (Strong) budget balance: sum of payments is zero
  - i.e. total VCG payment = total redistribution
- **Impossible to get all**
- We sacrifice budget balance
  - Try to get approximate budget balance
- Other work sacrifices: incentive compatibility [Parkes 01], efficiency [Faltings 04], non-deficit [Bailey 97], budget balance [Cavallo 06]

Another redistribution mechanism

- Bids: $V_1 \geq V_2 \geq V_3 \geq V_4 \geq \ldots \geq V_n \geq 0$
- First run Vickrey
- Redistribution:

  Receive 1/(n-2) * second-highest **other** bid, - 2/[(n-2)(n-3)] third-highest **other** bid
- Total redistributed:

  $V_2 - 6V_4/[(n-2)(n-3)]$
- Efficient & incentive compatible
- Individually rational & non-deficit (for large enough n)

$R_1 = V_3/(n-2) - 2/[(n-2)(n-3)]V_4$

$R_2 = V_3/(n-2) - 2/[(n-2)(n-3)]V_4$

$R_3 = V_2/(n-2) - 2/[(n-2)(n-3)]V_4$

$R_4 = V_2/(n-2) - 2/[(n-2)(n-3)]V_3$

...

$R_{n-1} = V_2/(n-2) - 2/[(n-2)(n-3)]V_3$

$R_n = V_2/(n-2) - 2/[(n-2)(n-3)]V_3$

Comparing redistributions

- Bailey-Cavallo: $\sum R_i = 2V_3/n+(n-2)V_2/n$
- Second mechanism: $\sum R_i = V_2-6V_4/[(n-2)(n-3)]$
- Sometimes the first mechanism redistributes more
- Sometimes the second redistributes more
- Both redistribute 100% in some cases
- What about the **worst** case?
- Bailey-Cavallo worst case: $V3=0$
  - percentage redistributed: $1-2/n$
- Second mechanism worst case: $V2=V4$
  - percentage redistributed: $1-6/[(n-2)(n-3)]$
- For large enough n, $1-6/[(n-2)(n-3)] \geq 1-2/n$, so second is better (in the worst case)

Generalization: linear redistribution mechanisms

- Run Vickrey
- Amount redistributed to bidder:

$$C_0 + C_1 S_1 + C_2 S_2 + ... + C_{n-1} S_{n-1}$$

  where S_j is the j-th highest **other** bid
- Bailey-Cavallo: $C_2 = 1/n$
- Second mechanism: $C_2 = 1/(n-2)$, $C_3 = - 2/[(n-2)(n-3)]$

- Bidder's redistribution does not depend on own bid, so incentive compatible
- Efficient
- Other properties?

Redistribution to each bidder

Recall: $R = C_0 + C_1 S_1 + C_2 S_2 + \ldots + C_{n-1} S_{n-1}$

$R_1 = C_0 + C_1 V_2 + C_2 V_3 + C_3 V_4 + \ldots + C_i V_{i+1} + \ldots + C_{n-1} V_n$

$R_2 = C_0 + C_1 V_1 + C_2 V_3 + C_3 V_4 + \ldots + C_i V_{i+1} + \ldots + C_{n-1} V_n$

$R_3 = C_0 + C_1 V_1 + C_2 V_2 + C_3 V_4 + \ldots + C_i V_{i+1} + \ldots + C_{n-1} V_n$

$R_4 = C_0 + C_1 V_1 + C_2 V_2 + C_3 V_3 + \ldots + C_i V_{i+1} + \ldots + C_{n-1} V_n$

...

$R_{n-1} = C_0 + C_1 V_1 + C_2 V_2 + C_3 V_3 + \ldots + C_i V_i \quad + \ldots + C_{n-1} V_n$

$R_n = C_0 + C_1 V_1 + C_2 V_2 + C_3 V_3 + \ldots + C_i V_i \quad + \ldots + C_{n-1} V_{n-1}$

Individual rationality & non-deficit

- Individual rationality:

equivalent to

$$R_n=C_0+C_1V_1+C_2V_2+C_3V_3+\ldots+C_iV_i+\ldots+C_{n-1}V_{n-1} \geq 0$$

$$\text{for all } V_1\geq V_2\geq V_3\geq\ldots \geq V_{n-1}\geq 0$$

- Non-deficit:

$\sum R_i\leq V_2$ for all $V_1\geq V_2\geq V_3\geq\ldots \geq V_{n-1}\geq V_n\geq 0$

Worst-case optimal (linear) redistribution

Try to maximize worst-case redistribution %

Variables: C_i ,K

Maximize K

subject to:

$R_n \geq 0$ for all $V_1 \geq V_2 \geq V_3 \geq \ldots \geq V_{n-1} \geq 0$

$\sum R_i \leq V_2$ for all $V_1 \geq V_2 \geq V_3 \geq \ldots \geq V_n \geq 0$

$\sum R_i \geq K V_2$ for all $V_1 \geq V_2 \geq V_3 \geq \ldots \geq V_n \geq 0$

R_i as defined in previous slides

Transformation into linear program

- **Claim**: $C_0=0$
- **Lemma**: $Q_1X_1+Q_2X_2+Q_3X_3+...+Q_kX_k\geq 0$ for all $X_1\geq X_2\geq ...\geq X_k\geq 0$

  is equivalent to

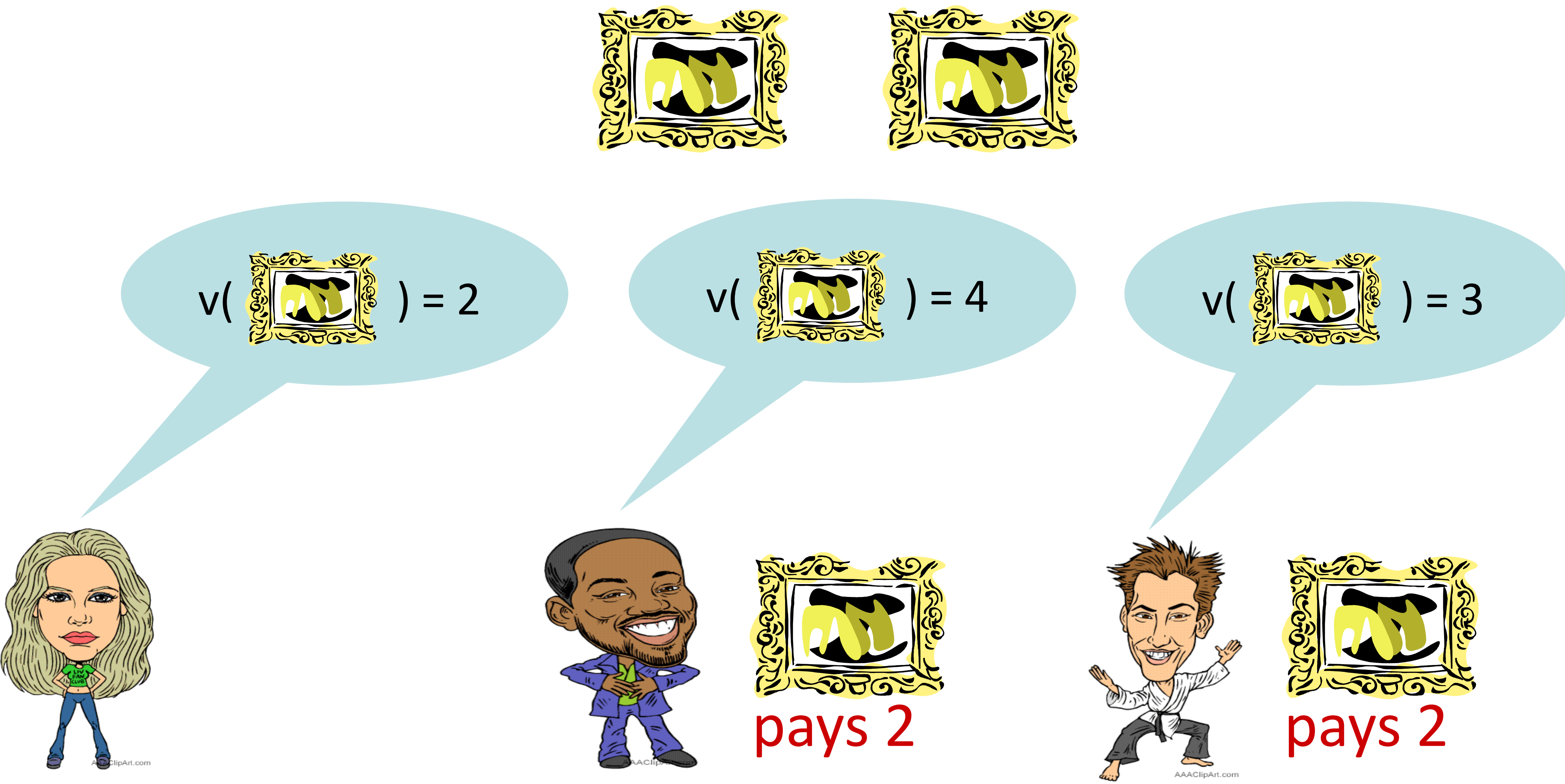
  $Q_1+Q_2+...+Q_i\geq 0$ for $i=1$ to k
- Using this lemma, can write all constraints as linear inequalities over the C_i

Worst-case optimal **remaining** %

- n=5: 27% (40%)
- n=6: 16% (33%)
- n=7: 9.5% (29%)
- n=8: 5.5% (25%)
- n=9: 3.1% (22%)
- n=10: 1.8% (20%)
- n=15: 0.085% (13%)
- n=20: 3.6 e-5 (10%)
- n=30: 5.4 e-8 (7%)

- the data in the parenthesis are for Bailey-Cavallo mechanism

m-unit auction with unit demand: VCG (m+1th price) mechanism



Incentive compatible

Our techniques can be generalized to this setting

m+1th price mechanism

Variables: C_i ,K

Maximize K

subject to:

$R_n \geq 0$ for all $V_1 \geq V_2 \geq V_3 \geq \ldots \geq V_{n-1} \geq 0$

$\sum R_i \leq V_2$ for all $V_1 \geq V_2 \geq V_3 \geq \ldots \geq V_n \geq 0$

$\sum R_i \geq K\, V_2$ for all $V_1 \geq V_2 \geq V_3 \geq \ldots \geq V_n \geq 0$

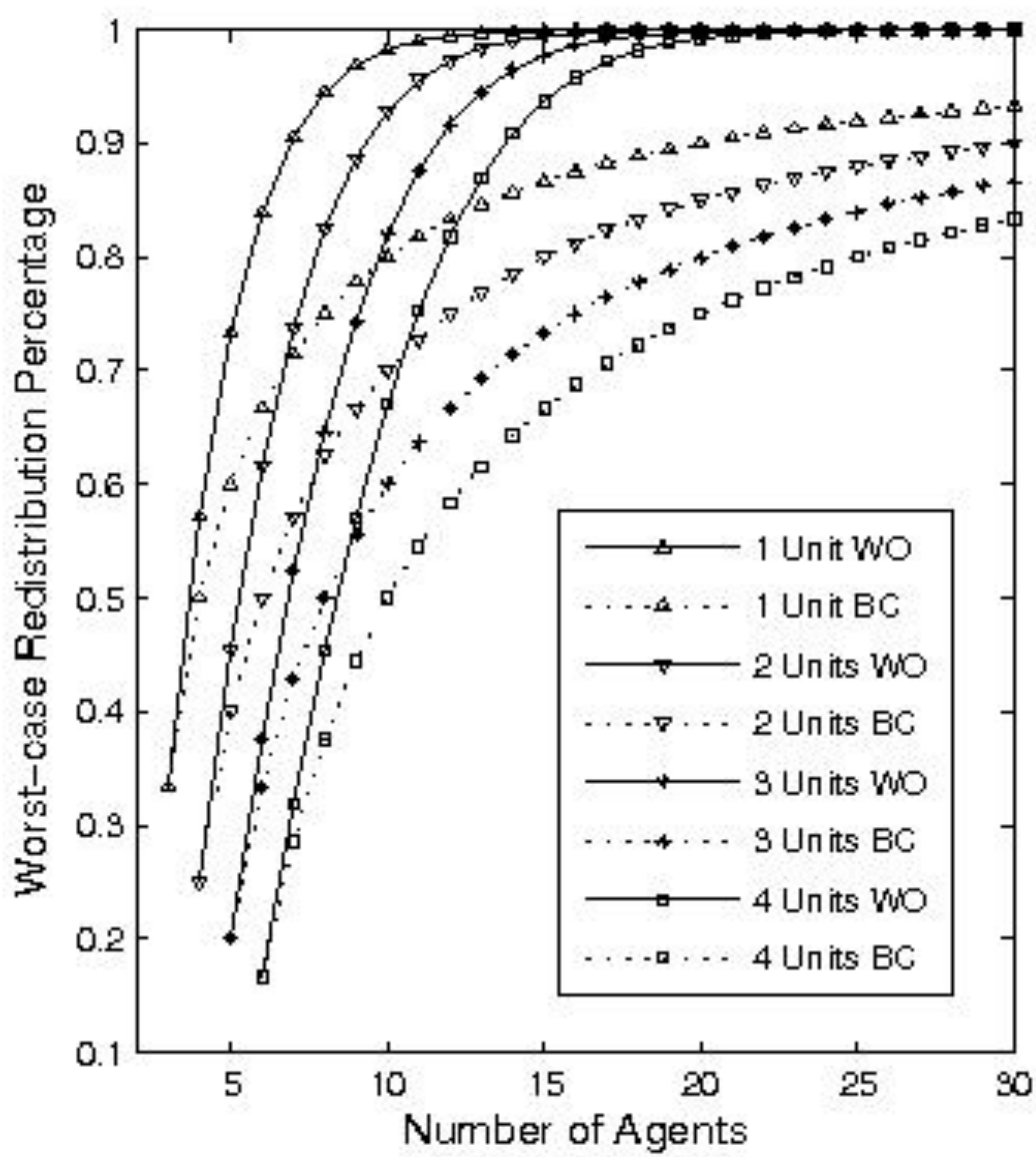
R_i as defined in previous slides

Only need to change V_2 into mV_{m+1}

Results for m+1th price auction

BC = Bailey-Cavallo

WO = Worst-case Optimal



Analytical characterization of WO mechanism

$$k^* = 1 - \frac{\binom{n-1}{m}}{\Sigma_{j=m}^{n-1}\binom{n-1}{j}}$$

$$c_i^* = \frac{(-1)^{i+m-1}(n-m)\binom{n-1}{m-1}}{i\Sigma_{j=m}^{n-1}\binom{n-1}{j}} \frac{1}{\binom{n-1}{i}} \Sigma_{j=i}^{n-1}\binom{n-1}{j}$$

$$for\ i = m+1, \ldots, n-1$$

- Unique optimum
- Can show: for fixed m, as n goes to infinity, worst-case redistribution percentage approaches 100% linearly
- Rate of convergence 1/2

Worst-case optimality outside the linear family

- **Theorem:** The worst-case optimal **linear** redistribution mechanism is also worst-case optimal among **all** VCG redistribution mechanisms that are
  - deterministic,
  - anonymous,
  - incentive compatible,
  - efficient,
  - non-deficit

- Individual rationality is not mentioned
  - Sacrificing individual rationality does not help
- Not **uniquely** worst-case optimal

Remarks

- Moulin's paper “Almost budget-balanced VCG mechanisms to assign multiple objects”

  pursues different worst-case objective (minimize waste/efficiency)
  - Results in same mechanism in the unit-demand setting (!)
  - Different mechanism results after removing individual rationality
  - Also mentions the idea of removing non-deficit property, without solving for the actual mechanism

More general settings:

multi-unit auction with nonincreasing marginal values

- A bid consists of m elements: b1,b2,...,bm

  bi = utility(i units) – utility(i-1 units)

  b1≥b2≥...≥bm≥0

Approach

- We construct a mechanism that has the same worst-case performance as the earlier WCO mechanism.
- Multi-unit auction with unit demand is a special case of multi-unit auction with nonincreasing marginal value.

- The new mechanism is optimal in the worst case.

Construction details omitted

Even more general setting?

- If marginal values are not required to be nonincreasing, the worst-case redistribution percentage is 0

  Proof by example

  The original VCG mechanism is already worst-case optimal

- Same for general combinatorial auction

Undominated redistribution mechanisms [AAMAS 08]

- Sometimes redistribution mechanisms are dominated
  - another redistribution mechanism always redistributes at least as much to each agent and sometimes more
  - WCO mechanism is dominated
- We characterized mechanisms that are undominated
- We proposed two techniques for transforming any dominated redistribution mechanisms into one that dominates it
- Experimentally, the techniques significantly improve known redistribution mechanisms

- Related paper (with Apt and Markakis) [WINE 08]: variant where other mechanism redistributes at least as much and sometimes more *in total*

Optimal-in-expectation redistribution mechanism [AAMAS 08]

- Goal: find optimal-in-expectation (strategy-proof) redistribution mechanism
  - Analytical solution for optimal linear mechanism (OEL)
  - Discretization methodology for getting (guaranteed) almost-optimal mechanisms
- For small cases can solve for very finely discretized mechanism
- For large cases OEL is almost optimal

Better redistribution with inefficient allocation in multi-unit auctions with unit demand [EC 08]

- Inefficient mechanisms can lead to higher welfare
  - The agents' total efficiency is smaller when the allocation is inefficient
  - But, the total payment can also be smaller (or more can be redistributed)
  - The net effect could be an increase in the total utility

    (total utility = total efficiency - total payment)
- Goal: design competitive mechanism, against the omnipotent allocation
- By allocating inefficiently (e.g., burning units, excluding agents, partitioning), we obtain more competitive mechanisms

Thank you for your attention!