What is Quantum Computing?

- Particles on a very small scale (e.g. electrons), do not follow classical mechanics. Instead, they show a different set of laws, which we call quantum mechanics.

- Quantum compuing tries to represent data using quantum properties, and those laws, and perform (quantum) operations on this data.

- For e.g., superposition and entanglement are two quantum properties exhibited at the small scale only.

Basic Terms

Classical Bit.

- Take a regular bit b.
- b can only have a value of 0 or 1. i.e., it is present in exactly 1 of 2 states.
- Hence b = 0 **OR** b = 1.

Qubit

- However, a qubit q is like a vector on the unit circle.

$$q = \alpha i + \beta j$$

- It is present in both states 0 and 1.
- Probability of being present in state 0 = α^2.
- Probability of being present in state 1 = β^2.

- In other words, a qubit is a combination of the states 0 and 1. It can have any values for α and β such that $\alpha^2 + \beta^2 = 1$.

Basic Terms

Classical 2-bit system.

- Take a variable X, which can be formed from 2-bits.
- Hence, $X = a_1 a_2$.
- At any point of time, X only has 1 of 2^2 possible values.
- e.g. X = 01. (it could also be 00, 10 or 11).

2-Qubit System.

- In a 2-qubit system, the value of the qubit, is a combination of **ALL** 4 states.
- X = a |00> + b |01 + c |10> + d |11>.
- Hence, think of it like, some part of X is present in 00, some part in 01 and so on.
- Here, the probability of X being in state 00 is a^2, of being in state 01 is b^2 and so on.
- $a^2 + b^2 + c^2 + d^2 = 1$.
- This is similar to X being a 4D unit vector.

Superposition

- This concept of a qubit being present in all 4 states, with a certain probability of being in each state, is called superposition.

Representation of a Qubit

- Since a qubit exists in multiple states, it is represented as follows.

$$|\psi\rangle = a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

- Here, $|0\rangle$ and $|1\rangle$ are the orthonormal basis. They are defined in matrix form as:

$$\left\{ |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

- Similarly, for a n-qubit system, there are 2^n orthonormal basis. For e.g., for a 2-qubit system, these are

$$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, |01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, |10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, |11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Quantum Gates vs Classical Gates

- **Classical Not gate**

NOT gate	
A	$\overline{A}$
0	1
1	0

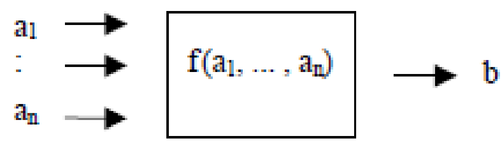
- **Quantum Gates**
- One gate is called the Pauli-X gate. It is the equivalent of a NOT gate.

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- For e.g., say a qubit $|\psi\rangle = a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$
- Then, P * $|\psi\rangle$ = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$ $\begin{bmatrix} b \\ a \end{bmatrix}$ =

More Quantum Gates (universal)

- The three basic classical gates, AND, OR and NOT can be used to solve any classical function of the form

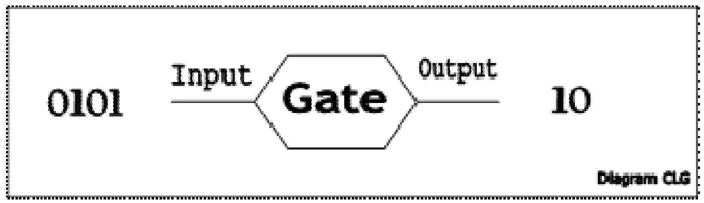


- Hence, together they can be combined to do universal classical computation.
- A NAND gate can be used to simulate all the three gates, and hence it can also be used to do universal computation.

- Similar to this, there is a gate, called the Toffoli gate, which is reversible. Quantum computers can only simulate reversible gates. Since a quantum computer can execute the toffoli gate, it can also be used to do universal classical computation.

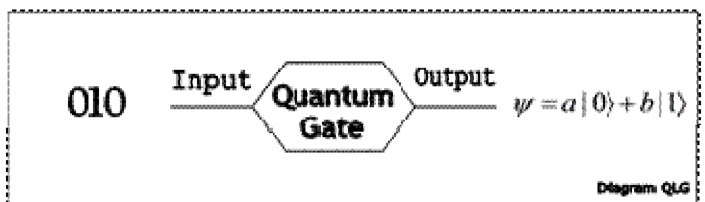
How to evaluate a function **f**

Classical Computation



- A classical gate takes input as 0101, and gives output as 10.

Quantum Computation



- When a quantum gate takes input (say 010), it gives the output as a superposition of multiple states, and not a single state.

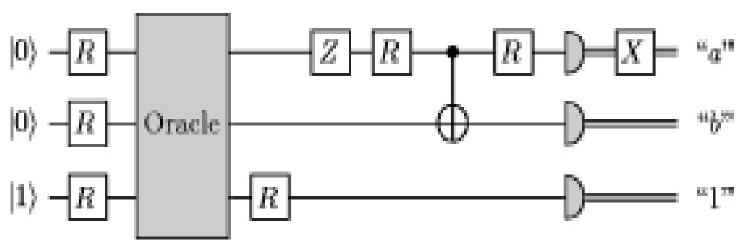
Why is this helpful?

- Since there is a gate, (Toffoli Gate) which is universal, a quantum computer can do every computation that a classical computer can do.

- However, when using a quantum gate, we get the output as a superposition of multiple states, and not a single state. This is exploited to achieve faster results, than what we can achieve by using a classical gate.

Quantum Oracle

- In our program, we will also use what is called the quantum oracle.
- An oracle is the portion of an algorithm which can be regarded as a “black box” whose behavior can be relied upon



  - Theoretically, its implementation does not need to be specified
  - However, in practice, the implementation must be considered

Quantum Oracle…

- Why do we use oracles?
  - Conceptually simplifies algorithms
  - An oracle hides the details of the implementation, and allows us to focus on the algorithm.

- An oracle can be made up of quantum gates, or it can be made up of classical gates. An oracle, given any input X, gives us the output f(X).

Measuring a quantum state.

- When we try to measure the value of a quantum state, it collapses to a single basis, just like a regular classical bit.
- A quantum algorithm with classical inputs has to find a way to evolve them into near-classical outputs again for efficient read-out, even though the intermediate state of the system will be decidedly unclassical.

DEALING INTEGER PROGRAMS WITH ADIABATIC QUANTUM COMPUTING

Outcome of this Research Project

- We develop model of adiabatic quantum computing. We can simulate small adiabatic quantum computer on MATLAB.
- Our model of adiabatic quantum computing produces results that are in accordance with research papers describing state of the art research in Adiabatic quantum computing.
- We are able to describe general mechanism of solving integer programs using adiabatic quantum computer.
- We are able to show that adiabatic quantum computer can solve optimization problem in constant time adiabatic evolution.

Schrödinger's Equation

$$ih\frac{d}{d(t)}\left|\psi(t)\right\rangle = H\left|\psi(t)\right\rangle$$

- h stands for Plank's constant

- Operator H stands for Hamiltonian.

- What does Hamiltonian do?

- Hamiltonian describes energy contents of the system. How?

Schrödinger's Equation

- How to solve Schrödinger's equation?
- The solution of Schrödinger's equation is

$$\left|\psi(t)\right\rangle = \exp\left(-\frac{iHt}{h}\right)\left|\psi(0)\right\rangle$$

- Evolution of one quantum state to another is Unitary.
- Possible when Hamiltonian is a Hermitian Matrix.

Hamiltonian

- Hamiltonian is an operator which we represent by a Hermitian matrix.
- Eigen values of Hamiltonian represent spectrum of possible energy levels (states) of a quantum system.
- The eigenvector (eigenstate) associated with is the lowest eigen-value energy is the ground state of the Hamiltonian.
- What is the lowest eigenstate (ground state) of this Hamiltonian?

$$H = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$X = [1 \quad 0 \quad 0]'$$

Classical Optimization in terms of Quantum states

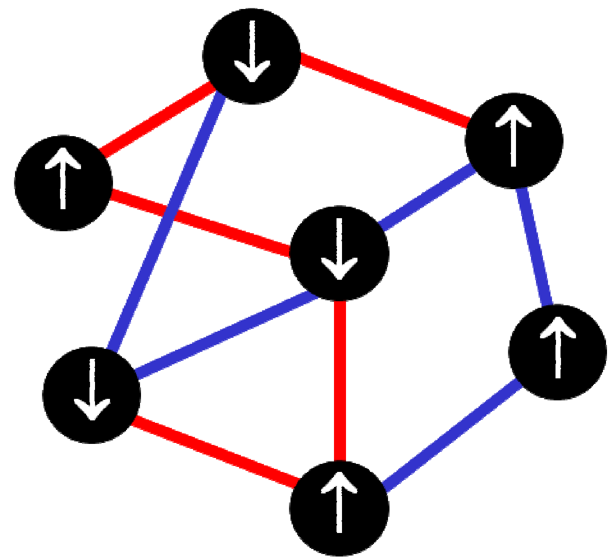
Given: f: $\{0,1\}^n \rightarrow N$, f(x) for $=x_1,\ldots\ldots x_n$,
Objective: find x_{min} which minimizes f

$$H = \begin{bmatrix} f(x_{000}) & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & f(x_{111}) \end{bmatrix}$$

|x> are the eigenvectors
f(x) are the eigenvalues
The answer = state with minimal eigenvalue

Ground state solution

- Which ↑↓ spin distribution maximizes the number of red edges?
- Analogous to a combinatorial optimization problem.
- Lowest energy question for magnetic materials.



Courtesy Dorit Aharonov
UC Berkeley

The **ground state** of the magnet is solution to our optimization problem!

Adiabatic Evolution

$$i\frac{d|\psi(t)\rangle}{dt} = H(t)\,|\,\psi(t)\rangle$$

Adiabatic theorem: [BornFock '28, Kato '51]

$$H(0) \longrightarrow H(T)$$

$|\psi(0)\rangle$ Ground state of H(0) $\longrightarrow$ $|\psi(T)\rangle$ Ground state of H(T)

$$T >> \frac{1}{\min_s \{\gamma(t)\}^2} \qquad \gamma(t) = E_1(t) - E_0(t)$$

Let us start our project

- Imagine there are *m* bidders:

$$B_1, B_2, B_3, ..., B_m$$

- Suppose there are *n* items

$$I_1, I_2, I_3, ..., I_n$$

- The auctioneer will accept bids that maximizes his payoff.

Quantum Auctions

- An auctioneer gives p qubits to each bidder.
- The initial state of all qubits is $|000..0\rangle$
- We can parse our quantum register $|x\rangle$ as follows

$$|x\rangle = |\text{Item\#,bidder}_1\text{\_bid ; Item\#,bidder}_2\text{\_bid;.....;Item\#,bidder}_m\text{\_bid}\rangle$$

Quantum Auctions

- Let us mention some rules
  - Bidders will prepare their respective qubits and hand them over to the auctioneer.
  - Auctioneer cannot assign same item to multiple bidders.
  - Such an assignment sill be infeasible.
  - Auctioneer will select payoffs from available feasible quantum states that we describe next.

Example of Superposition of Bids

- Suppose we have two bidder B1 and B2 and 1 item
- Bidder1 puts \$2 on the item while Bidder2 puts \$3 on the item
- The resulting state of |x> will be

$$U_1|00\rangle \rightarrow \frac{|00\rangle + |10\rangle}{\sqrt{2}} \text{ for Bidder1}$$

$$U_2|00\rangle \rightarrow \frac{|00\rangle + |11\rangle}{\sqrt{2}} \text{ for Bidder2}$$

Superposition

- The resulting state of |x> will be the superposition of qubits prepared by Bidder1 and that by Bidder2. That is,

$$\left|x\right\rangle = \frac{\left|0000\right\rangle + \left|0011\right\rangle + \left|1000\right\rangle + \left|1011\right\rangle}{2}$$

Infeasible

- In vector form

$$\left|x\right\rangle = \frac{1}{2}[1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0]^T$$

Preparing Hamiltonians

- Initial Hamiltonian W will contain entries corresponding to the number of ones in each quantum state.

$$W = diag\{0,1,1,2,1,2,2,3,1,2,2,3,2,3,3,4\}$$

- Final Hamiltonian H_f will contain diagonal entries corresponding to –ve of payoffs for each quantum bid state.

$$H_f = -diag\{0,1,2,3,1,0,0,0,2,0,0,0,3,0,0,0\}$$

Preparing Hamiltonian

- We want |x> to be the ground state of W.
- Is this really the case?
- No!
- So we modify our initial Hamiltonian as

$$H_i = UWU^{\dagger}$$

$$U = U_1 \otimes U_2$$

- So that now |x> is the lowest eigenstate of H_i

Quantum Auction Protocol

- Start with |x> the ground state of Hi.
- Change Hi slowly so that it becomes Hf
- When Hamiltonian is changing |x> is also changing.
- The final state |x> will be the ground state of Hf which encodes our solution.
- Auctioneer will measure the final state and announce the winner.

QUANTUM AUCTIONS PROTOCOL

$$H(s) = (1-s)H_i + sH_f$$

$$H = H_i \longrightarrow H = H_f$$

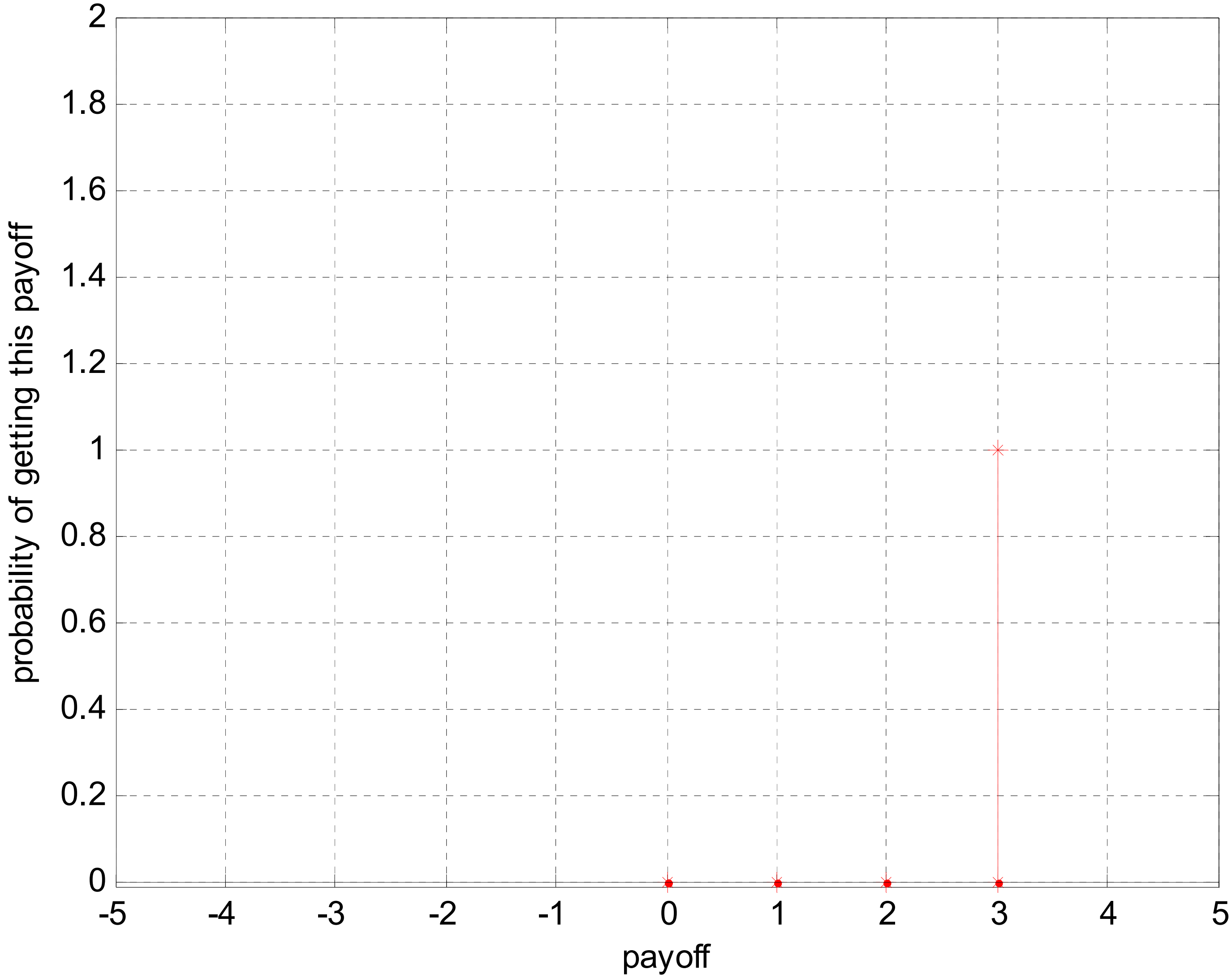
$|x\rangle$ Ground state of H(0) $\longrightarrow$ $|x\rangle$ Ground state of H(f)

Solution

$$T >> \frac{1}{\min_s \{\gamma(t)\}^2}$$

$$\gamma(t) = E_1(t) - E_0(t)$$

DEMO OUTPUT



2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
payoff
probability of getting this payoff

DEMO

LET US SEE MATLAB
DEMONSTRATION OF QUANTUM
AUCTION PROTOCOL

Quantum Computer and Integer Programming

- Previous example gives insight into quantum computer solving small integer program namely: Auction winner determination problem.

- Can we extend this approach to solve integer programs in general???

Quantum Computer and Integer Programming

- Good News
  - It turns out that we can use quantum computer solve integer programs using Adiabatic quantum computing. How?
  - Can we generalize the procedure we followed in the last example.

Quantum Computer and Integer Programming

- Prepare Hf the final Hamiltonian which encodes our solution. Can we do this efficiently?
  - For 2^d possible values of input variables, we can calculate corresponding objective values using single black box query!
  - If this is done using NMR Quanutum information processor, the corresponding 2^d objective values appear as frequency peaks on NMR spectrometer.
  - These frequencies correspond to eigen values since Energy ~ frequency

Quantum Computer and Integer Programming

- Prepare Hc the constraint Hamiltonian containing 1, -1 entries. The ith entry of this Hamiltonian is 1 if corresponding combination of variable values which is infeasible. How to do this
  - Out of 2^d possible values of input variables, we can determine what combinations of variable values are infeasible using single query to quantum black box!
  - If this is done using NMR Quantum information processor, the mixture of feasible and infeasible states appear as frequency peaks with corresponding phase shifted (0 or 180) on NMR spectrometer.

Quantum Computer and Integer Programming

- Prepare |x> using superposition of all input variable values. |x> is the initial state of our system.

- Evolve Hc to Hm. This evolution is simple. The intermediate Hamiltonian Hm has eigen values same as those of Hc but only incremented by 1.

- Evolve Hm to Hf and measure the final state which encodes our solution.

Quantum Computer and Integer Programming

- Adiabatic Quantum Computer solving Integer Program

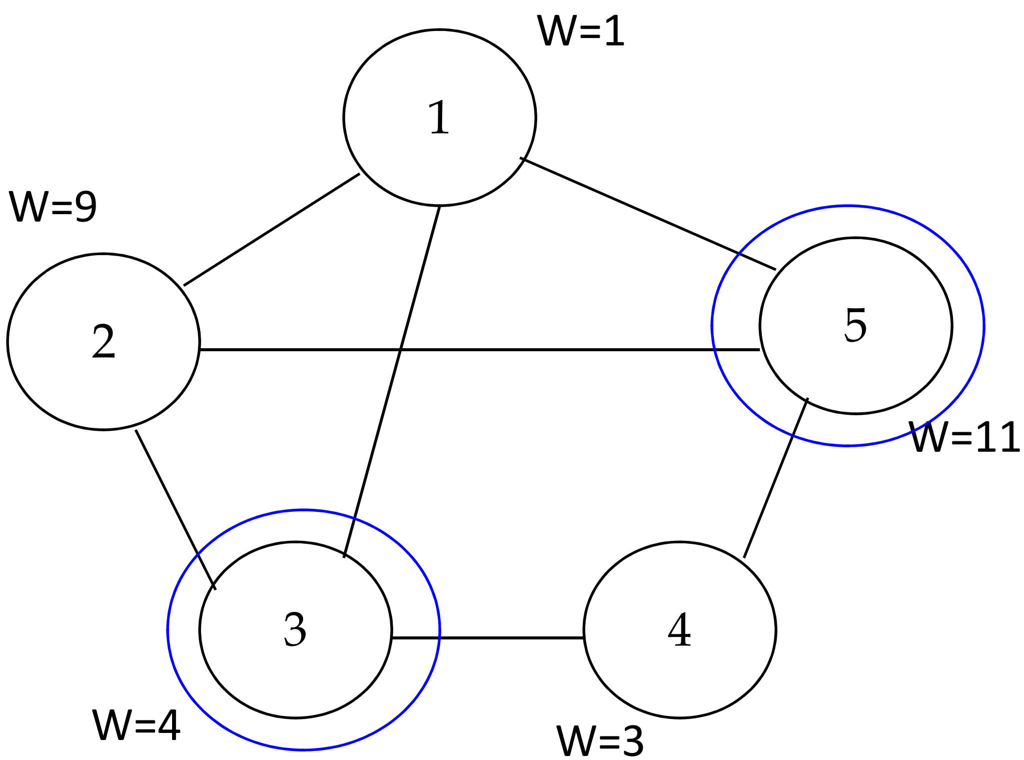
$$H = H_c \xrightarrow{H = H_c - sI_{n\times n}} H = H_m \xrightarrow{H = (1-s)H_m + H_f} H = H_f$$

Phase 1: Removal of Infeasible States from superposition

Phase 2: Selecting Best feasible state from filtered superposition

$$|x\rangle_{INITIAL} \longrightarrow |x\rangle_{FEASIBLE} \longrightarrow |x\rangle_{OPTIMAL}$$

Let us Solve Maximum Weight Independent Set Problem using QC

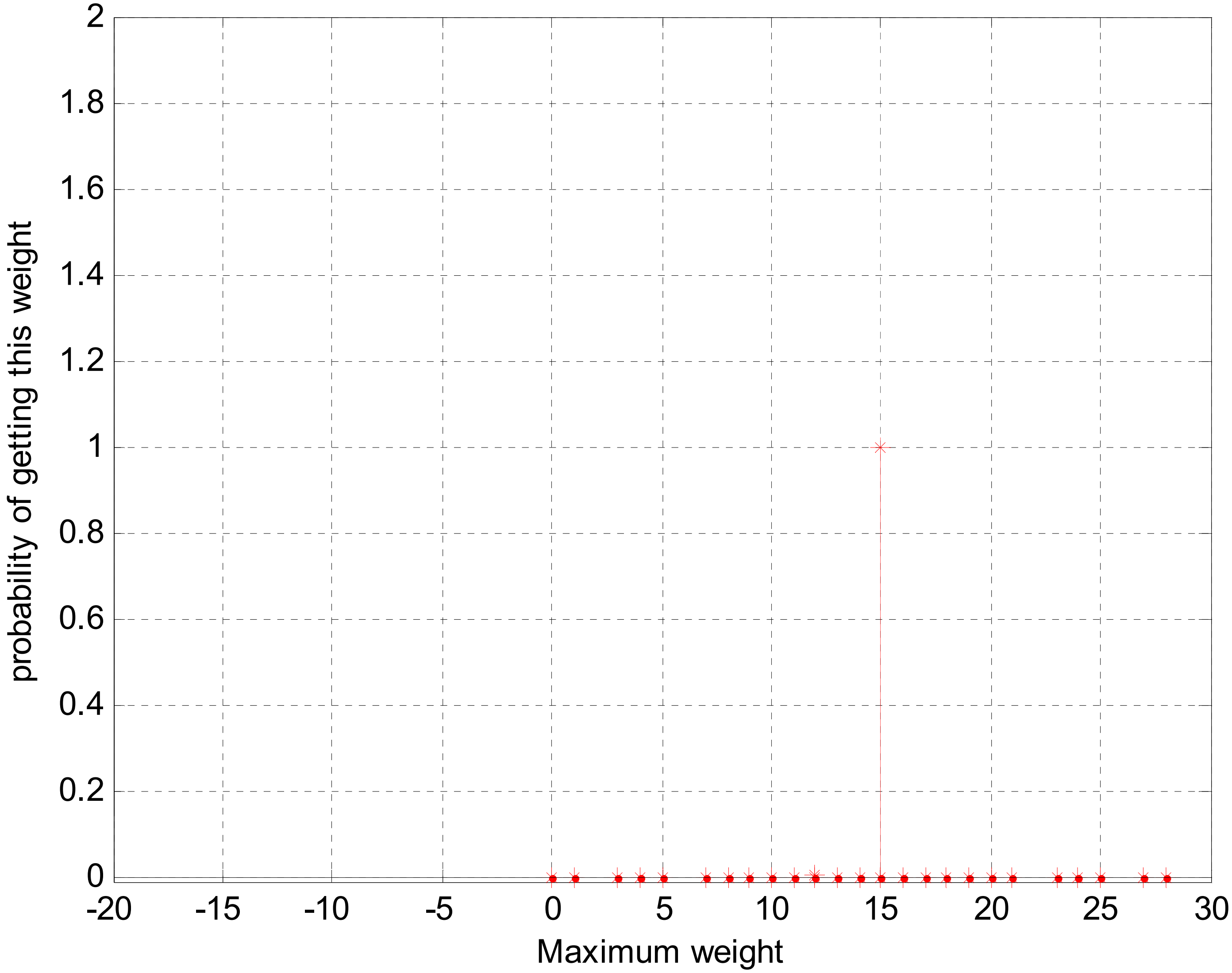


Maximum Weight of Independent vertices is : 15

DEMO

- Let us see MATLAB demonstration of Adiabatic quantum computer solving Maximum weight independent set of the graph.

DEMO OUTPUT



2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
probability of getting this weight
-20
-15
-10
-5
0
5
10
15
20
25
30
Maximum weight

How efficient is Adiabatic Quantum Computation

- The good news is that time spent in the evolution of Hamiltonian doesn't depend upon the size of Hamiltonian!!!

- Time depends on the difference between two lowest eigenvalues also known as spectral gap.

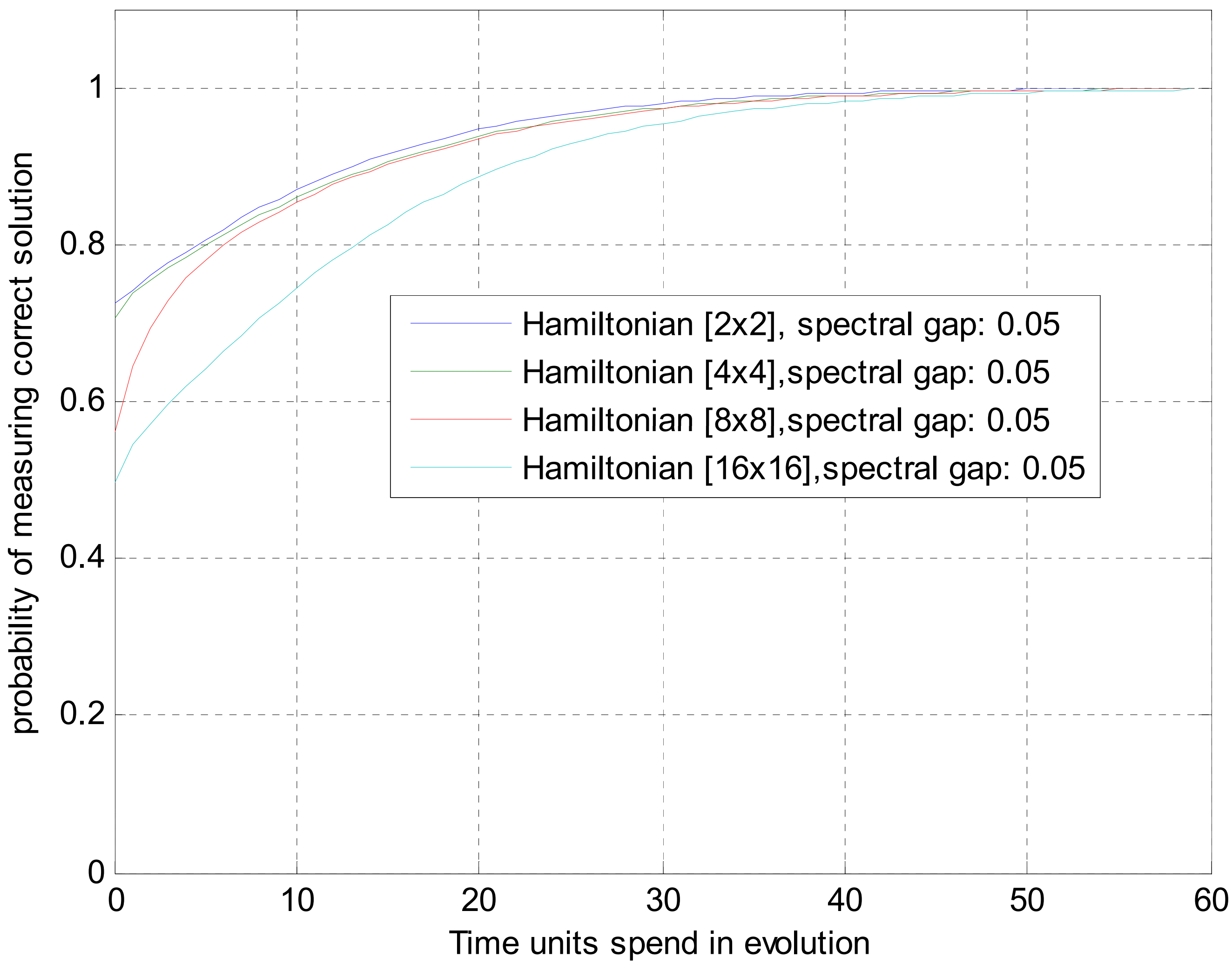
$$T >> \frac{1}{\min_s \{\gamma(t)\}^2}$$

$$\gamma(t) = E_1(t) - E_0(t)$$

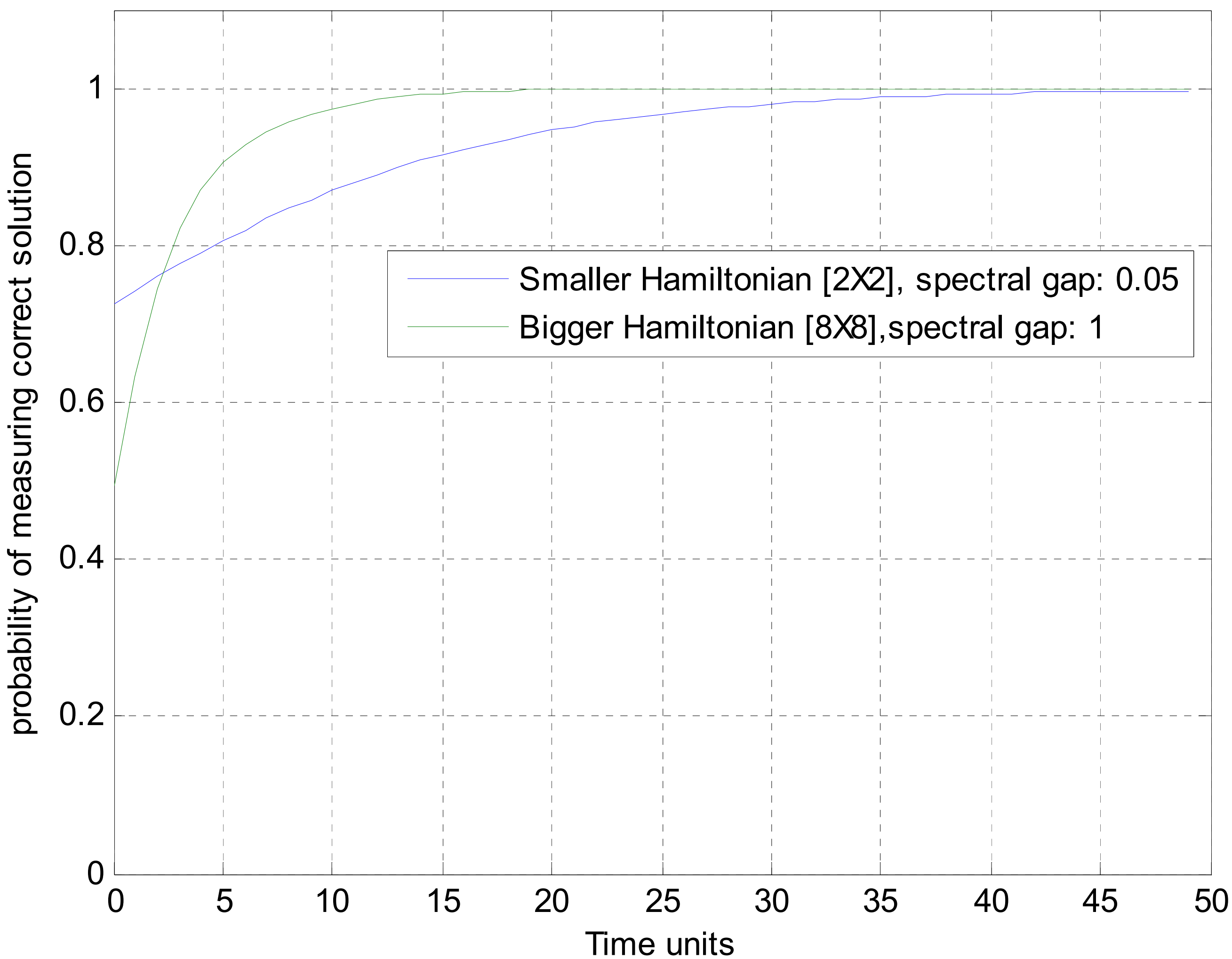
How efficient is Adiabatic Quantum Computation

- Time spent in evolution depends ONLY on difference between two smallest normalized Eigenvalues of evolving Hamiltonian.
- Time spent in evolution DOESNOT depend on the number of qubits i.e. size of the problem as such.
- Can we show this?

How efficient is Adiabatic Quantum Computation



Efficiency of Hamiltonian



Time Complexity of our Adiabatic Quantum Computing Algorithm

- Bad News:
  - As we increase size of the problem the normalized spectral gap can decrease exponentially in terms of number of qubits!

- Good News:
  - We can nullify the exponential decrease in the spectral gap by exponentially increasing the eigenvalues!

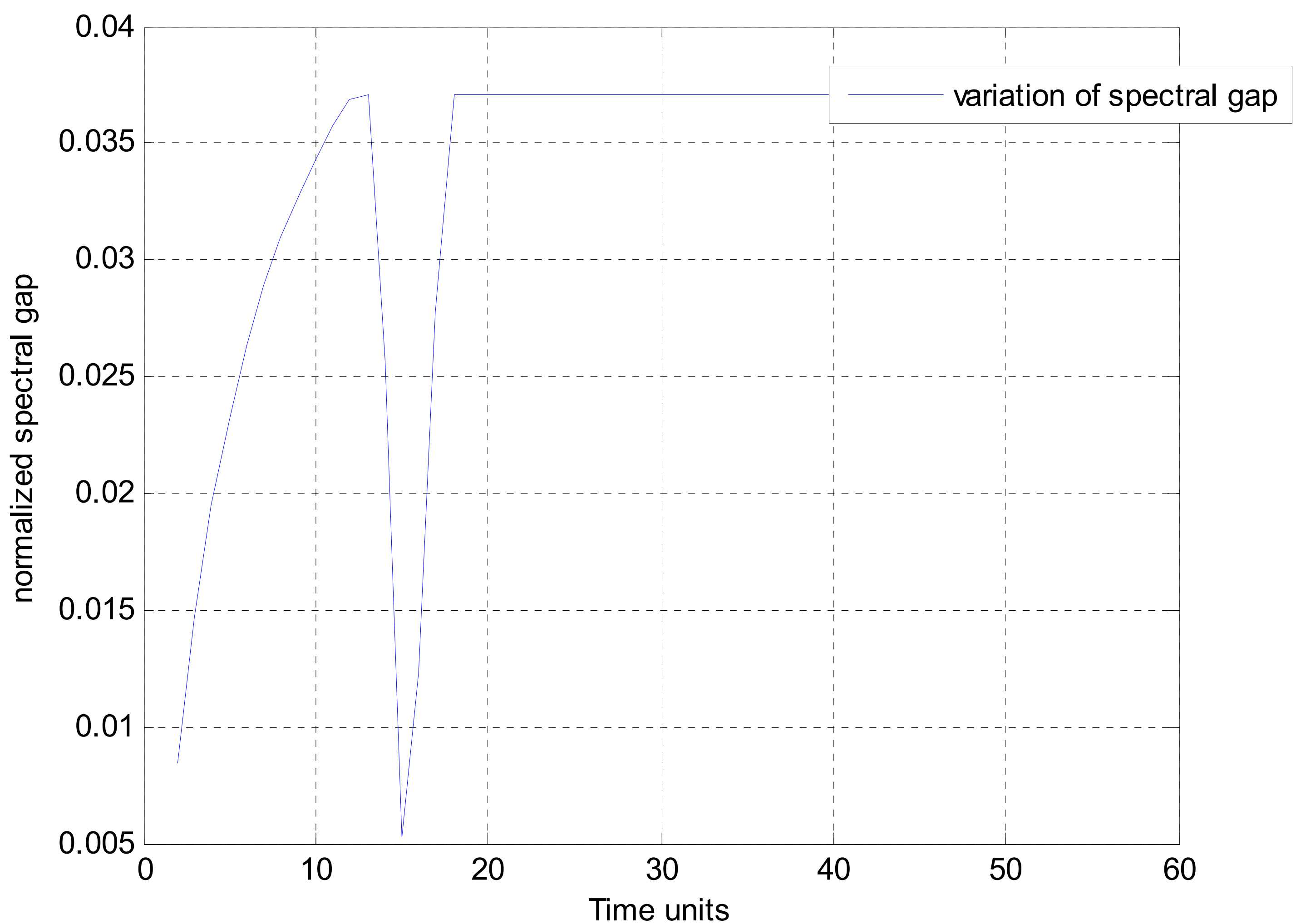
Time Complexity of our Adiabatic Quantum Computing Algorithm

- In our Maximum weight problem, the eigenvalues of final Hamiltonian may not be simply the sum of feasible combinations of the weights of vertices.

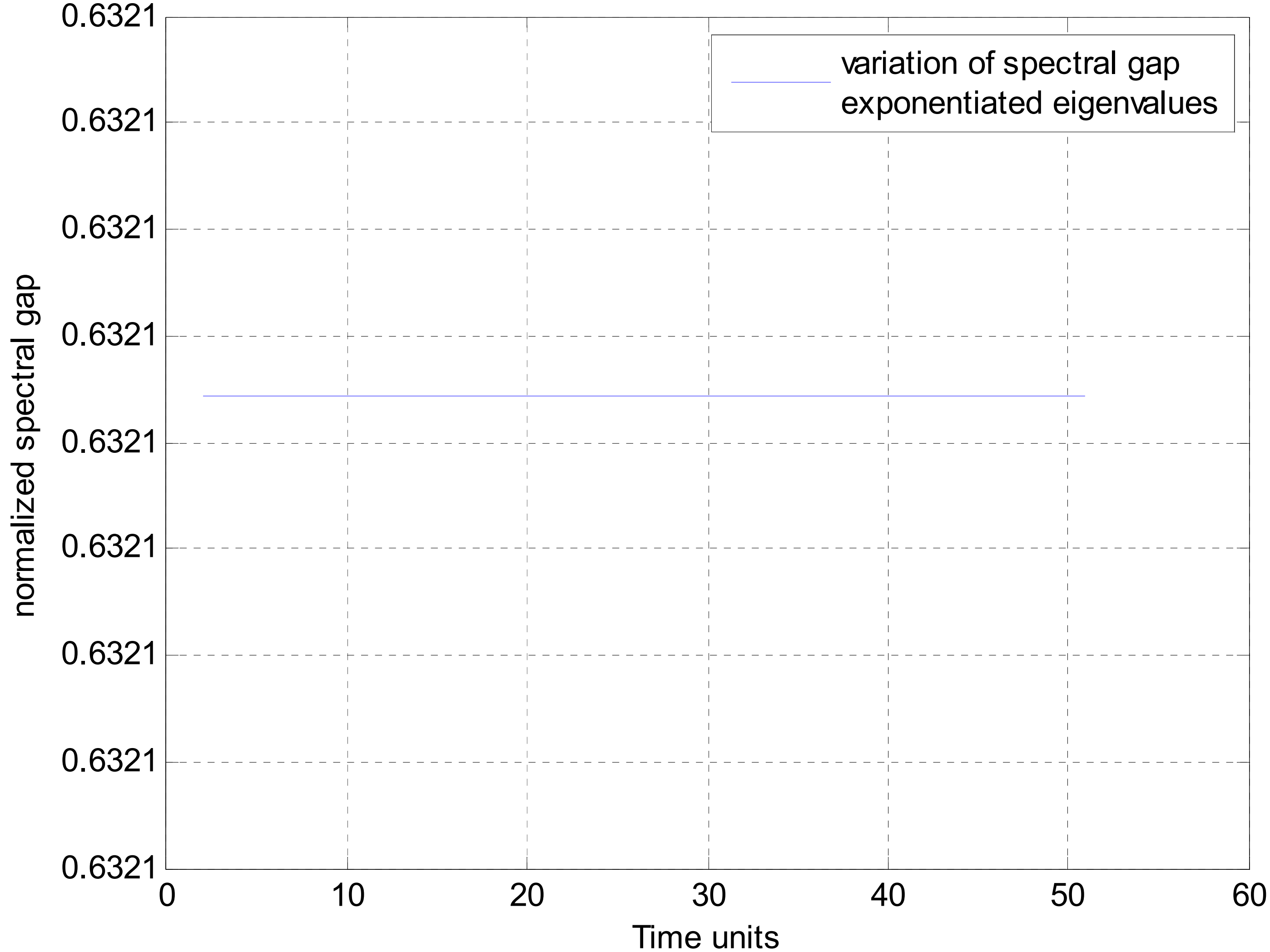
- We can set eigenvalues to be the exponent of sum of feasible weights.

- The resulting quantum algorithm is implemented in LIP_Final2_mod.m

Variation in spectral Gap



Variation in spectral gap with exponentiated eigenvlaues



How to test our simulation

- Please remember you will be simulating a quantum computer on classical computer.
- How to measure time spent in evolution?
- So we have fixed time step of 1/50. (You can decrease that if you want to)
  - Add more vertices by adding more weights to the weight vector.
  - The output should contain only 1 long stem regardless of how many vertices you add.
  - If this is not the case then adiabatic evolution wasn't completed in constant time. In such a situation please drop me an e-mail at *ahsan@cs.duke.edu*

Time Complexity of our Adiabatic Quantum Computing Algorithm

- Adiabatic Quantum computing CAN solve problem hard optimization problems such that the evolution of Hamiltonians takes CONSTANT TIME.

- This idea is not new since Andercut and Ali 2004 showed similar result for searching in item in unstructured database.

Important questions and Future Work

- How efficiently can we prepare Hamiltonians.
- Study Non linear evolution of Hamiltonians.
- Attempt to tackle exponential number of constraints in a linear program using quantum computer.
- Build general purpose Quantum computer.

THANK YOU