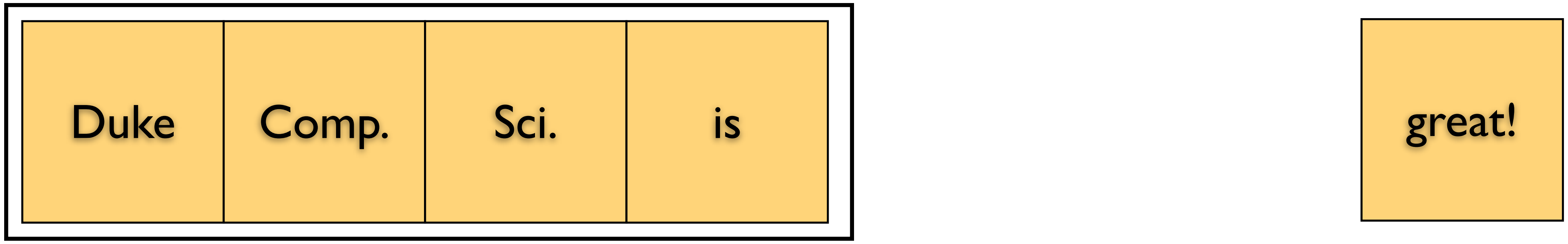
Amortized Analysis

(or: How ArrayLists work)

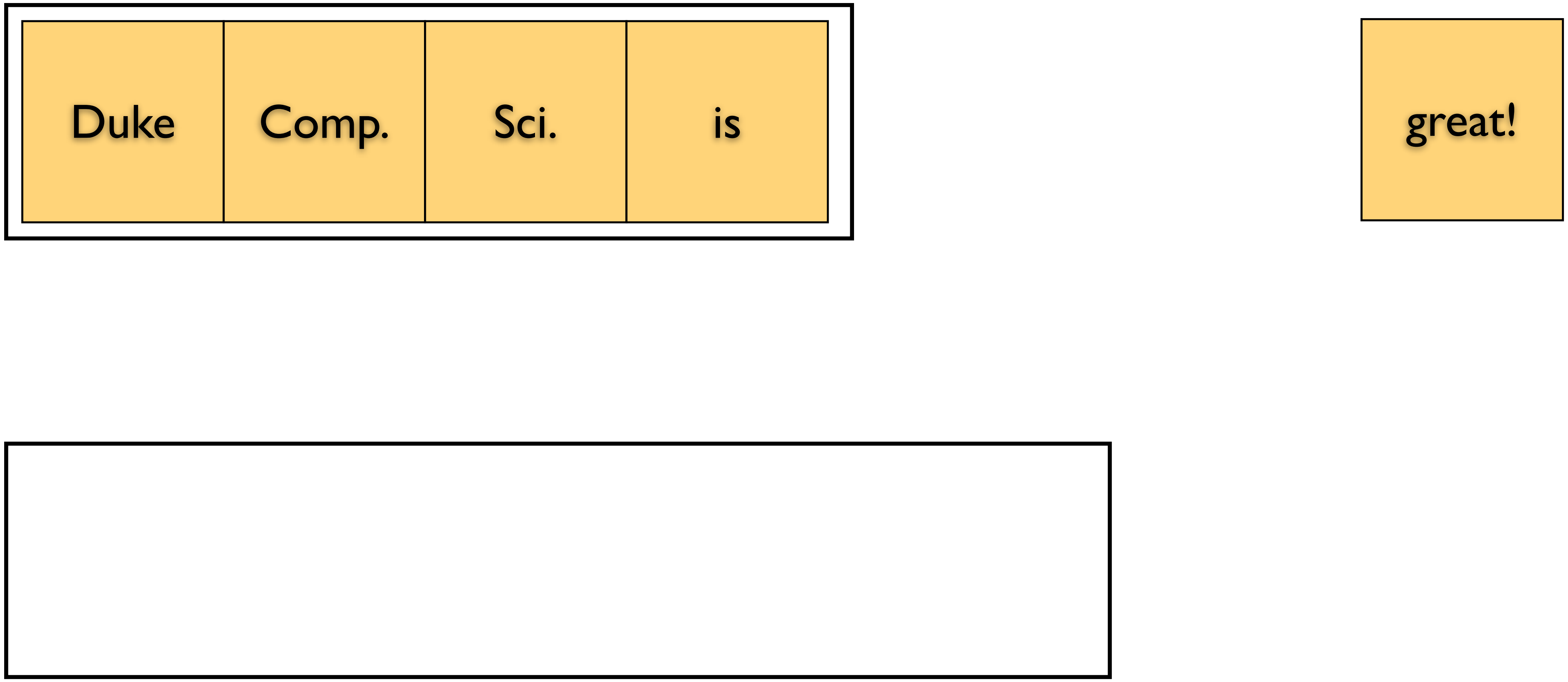
Recall ExpandingArray



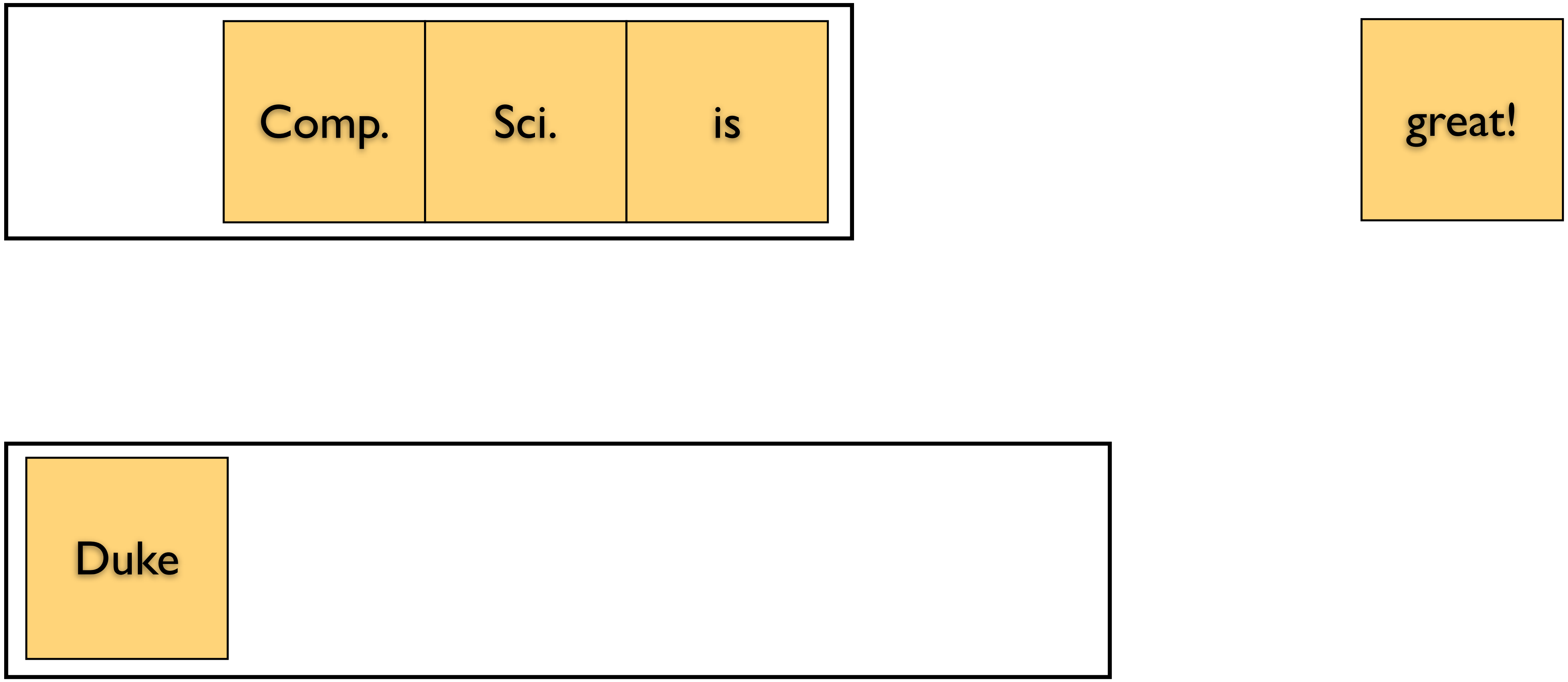
Duke | Comp. | Sci. | is

great!

Recall ExpandingArray



Recall ExpandingArray



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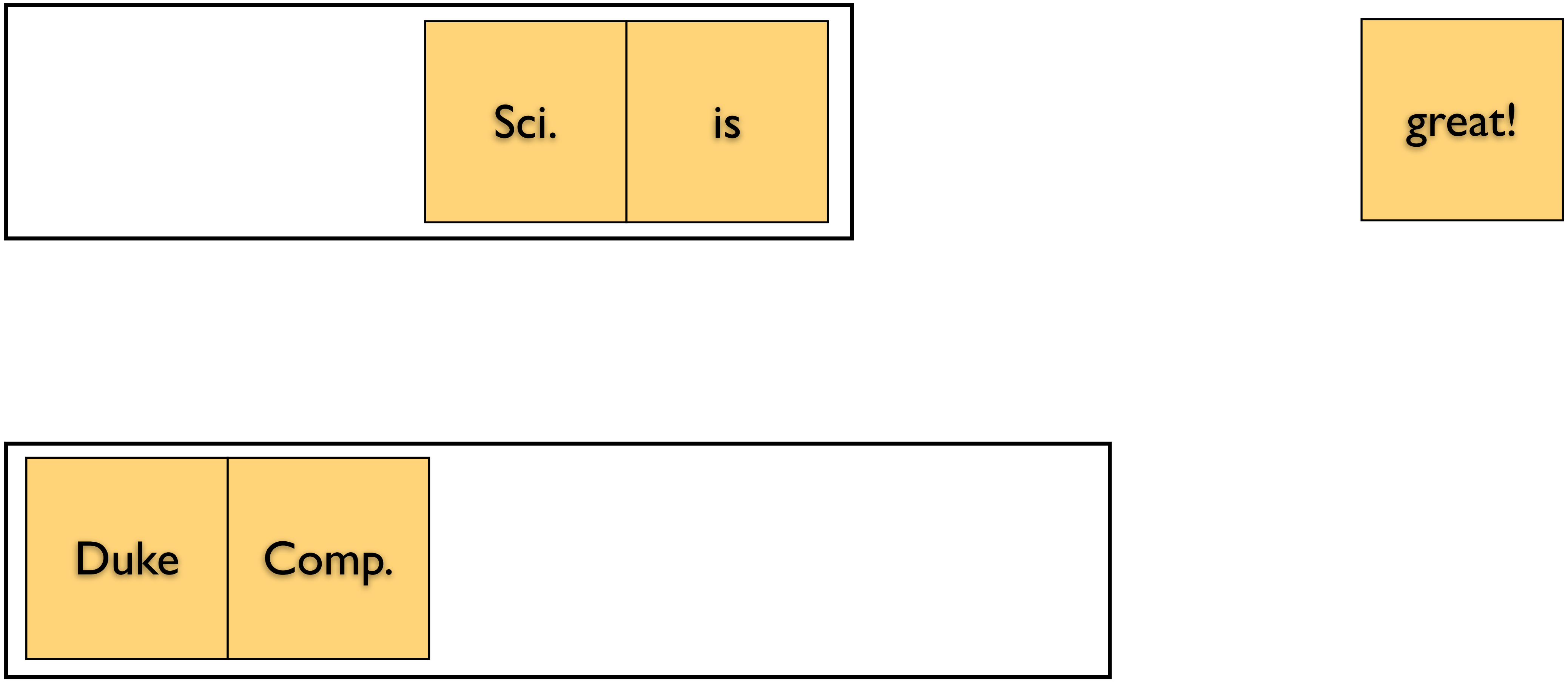
Recall ExpandingArray



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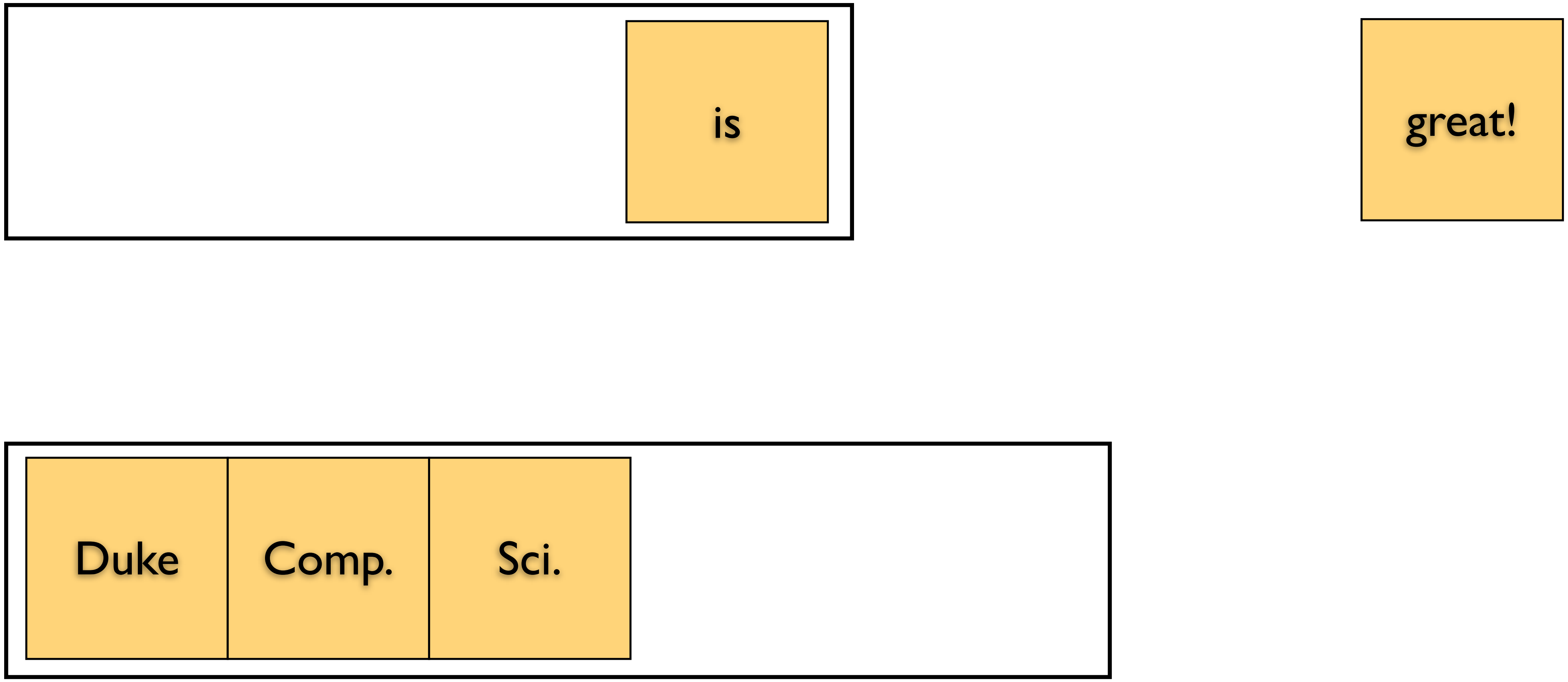

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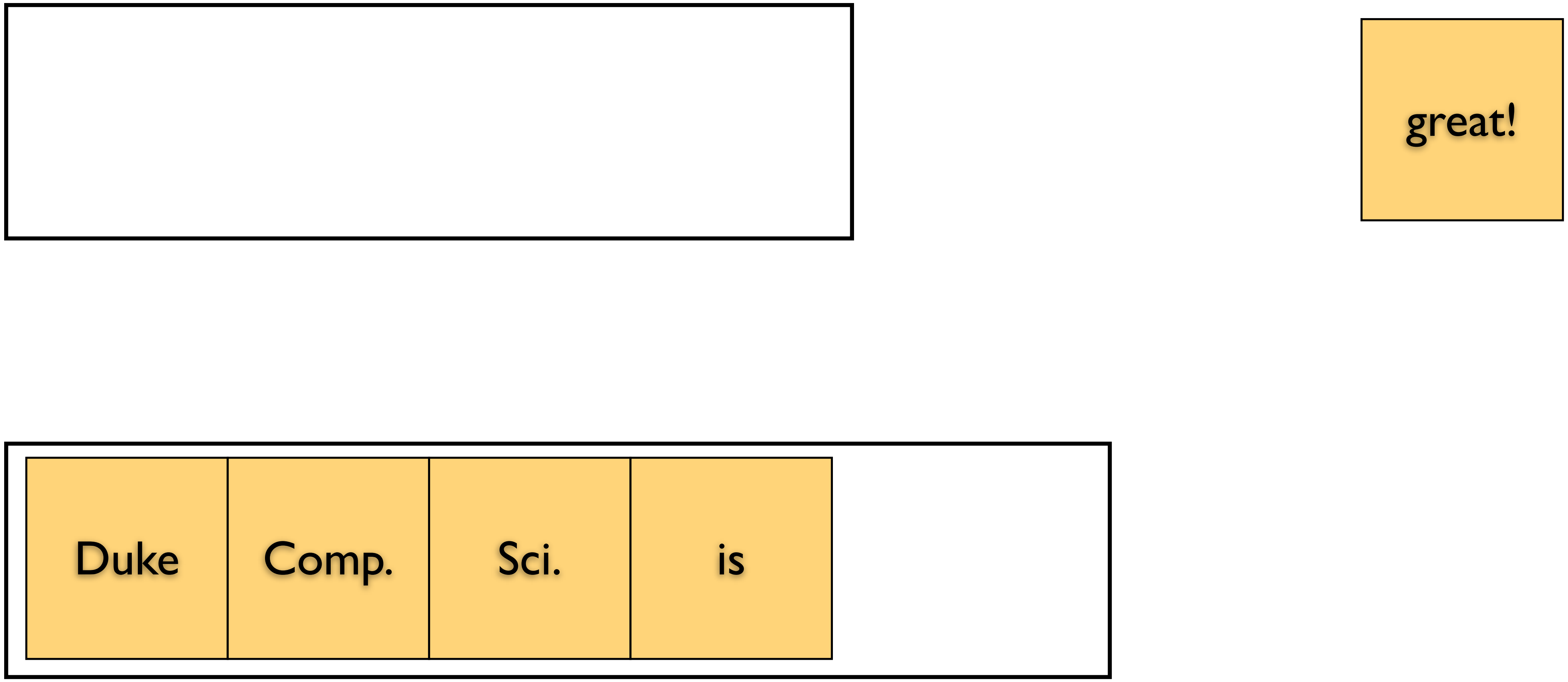
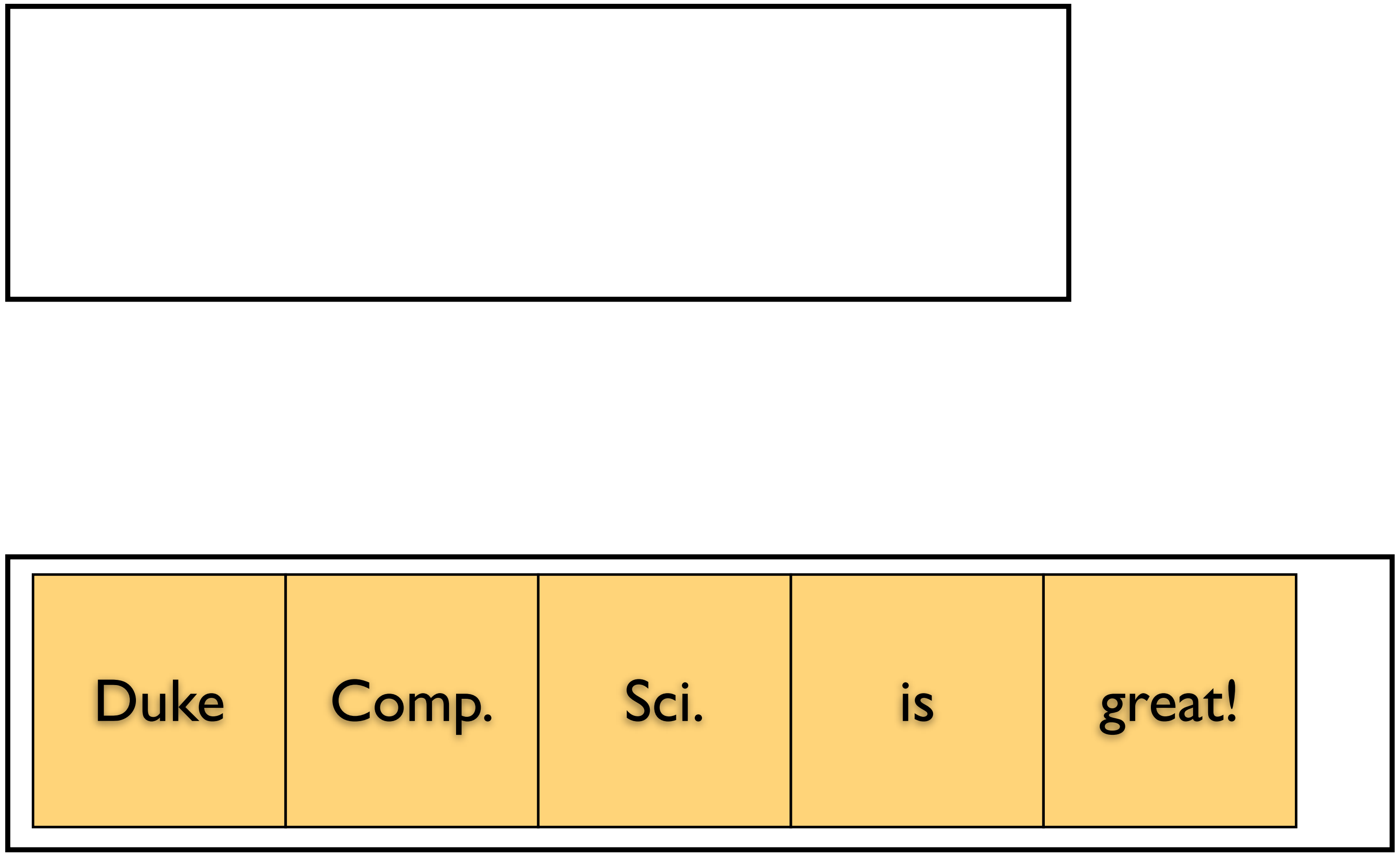


Recall ExpandingArray



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Adding one each time leads to $$\frac{n(n+1)}{2} \in O(n^2)$$

Why not Linked Lists?

Why not Linked Lists?

`ExpandingArray`

.get(): $O(1)$ *You couldn't hope for better!*

.add(): $O(n)$ *Which means $O(n^2)$ for n operations...*

Linked List

Re: DNA: Good at splicing, too!

.get(): $O(n)$ *Which means $O(n^2)$ for n operations...*

.add(): $O(1)$ *Best it can be!*

What we want:

.get(): $O(1)$ *Best it can be!*

.add(): $O(1)$ *Best it can be!*

It can be done!

ArrayList & StringBuilder, for example

Also: StringBuffer, C++'s vector, and Python's list. *Not* Matlab's array.

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ArrayList & StringBuilder, for example

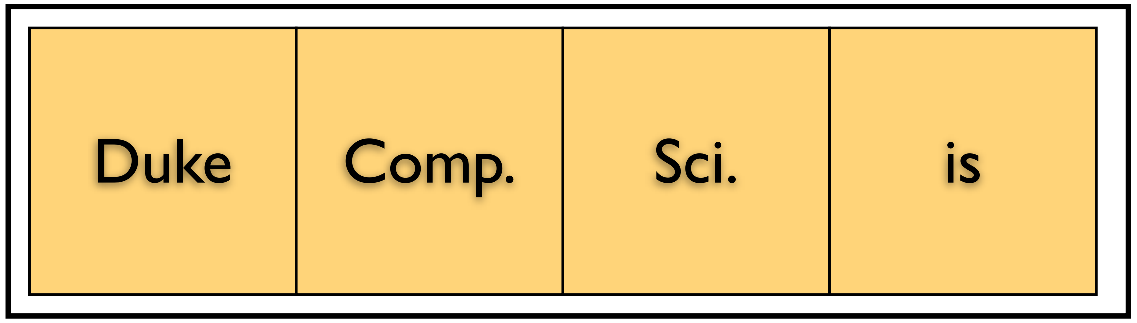
What we want:

.get(): $O(1)$ *Best it can be!*

.add(): $O(1)$ *Best it can be!*

Also: StringBuffer, C++'s vector, and Python's list. *Not* Matlab's array.

Backed by an array!



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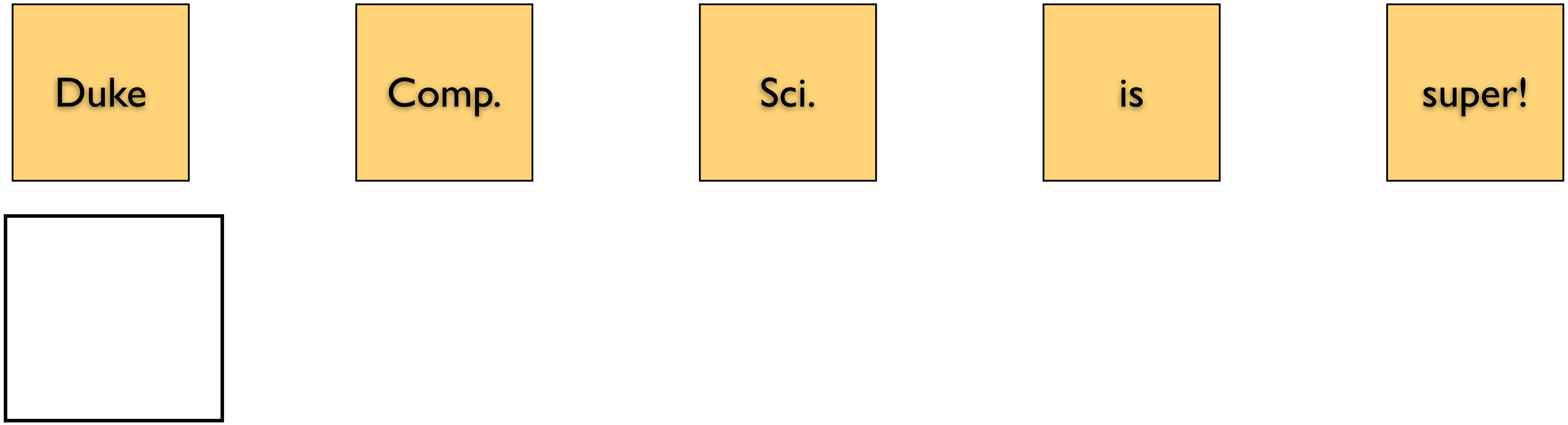
What we want:

.get(): *O(1)* *Best it can be!*

.add(): *O(...)* *...?*

Add more than one?

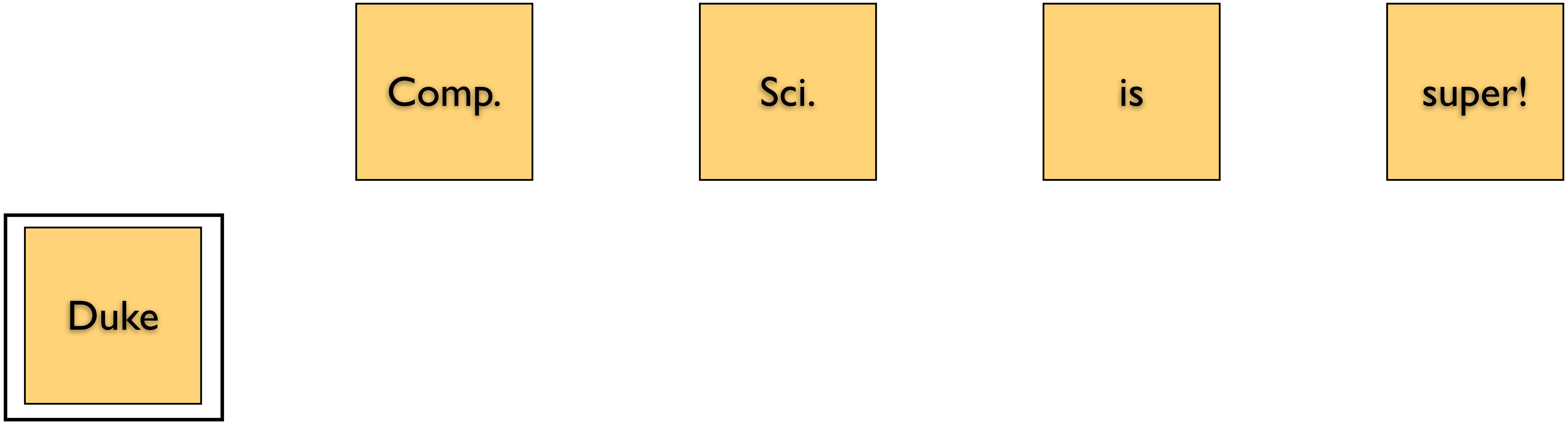


Duke Comp. Sci. is super!

Operations: 0

Adds: 0

Add more than one?



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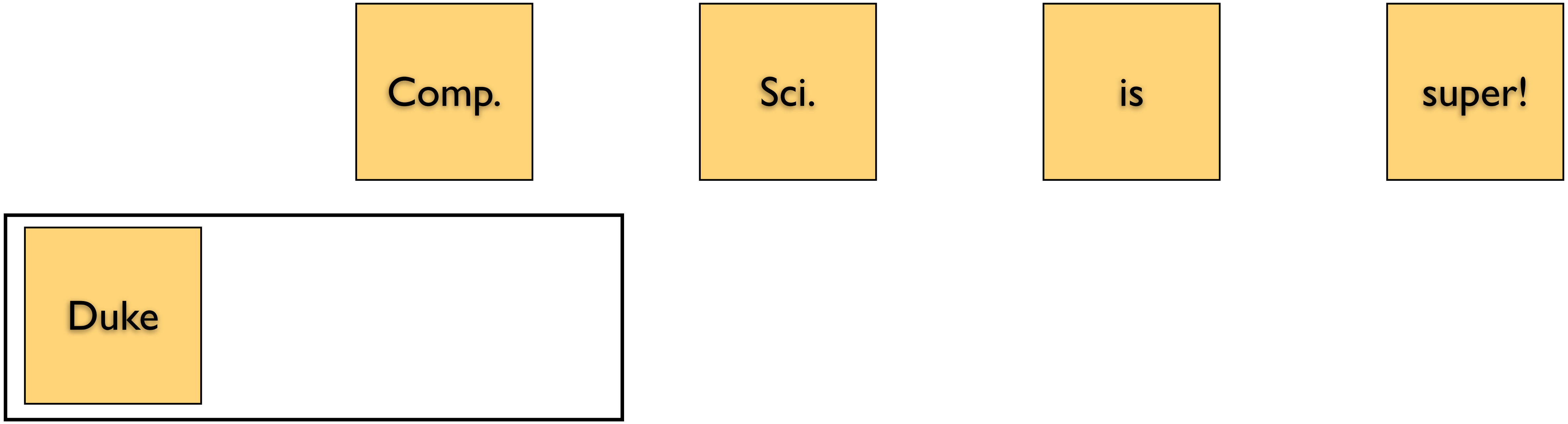
Operations: 1

Adds: 1

Add more than one?



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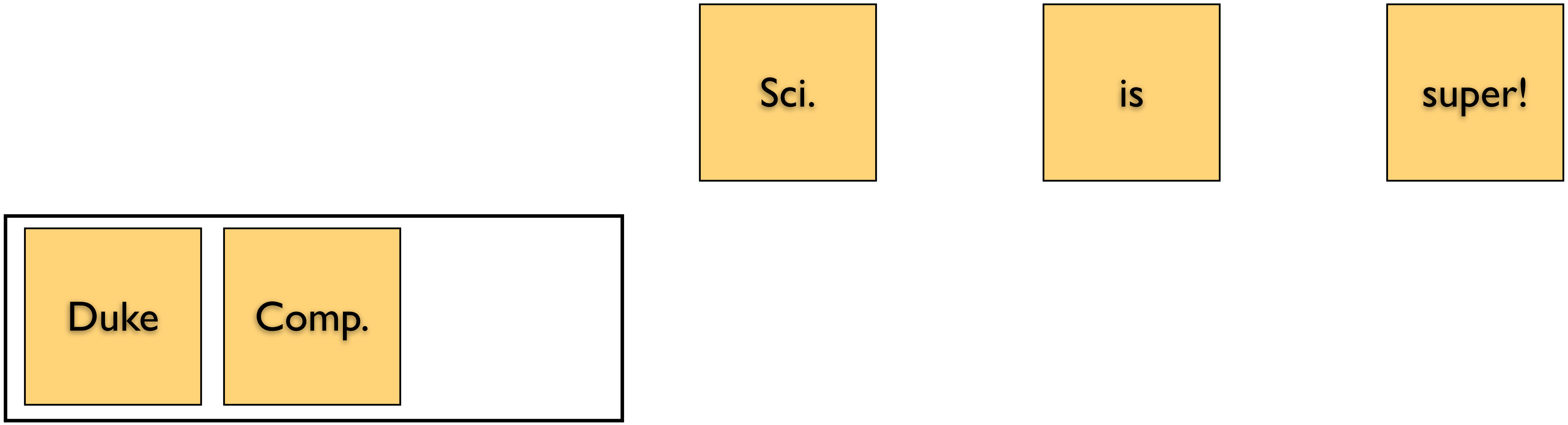
Operations: 2

Adds: 1

Add more than one?



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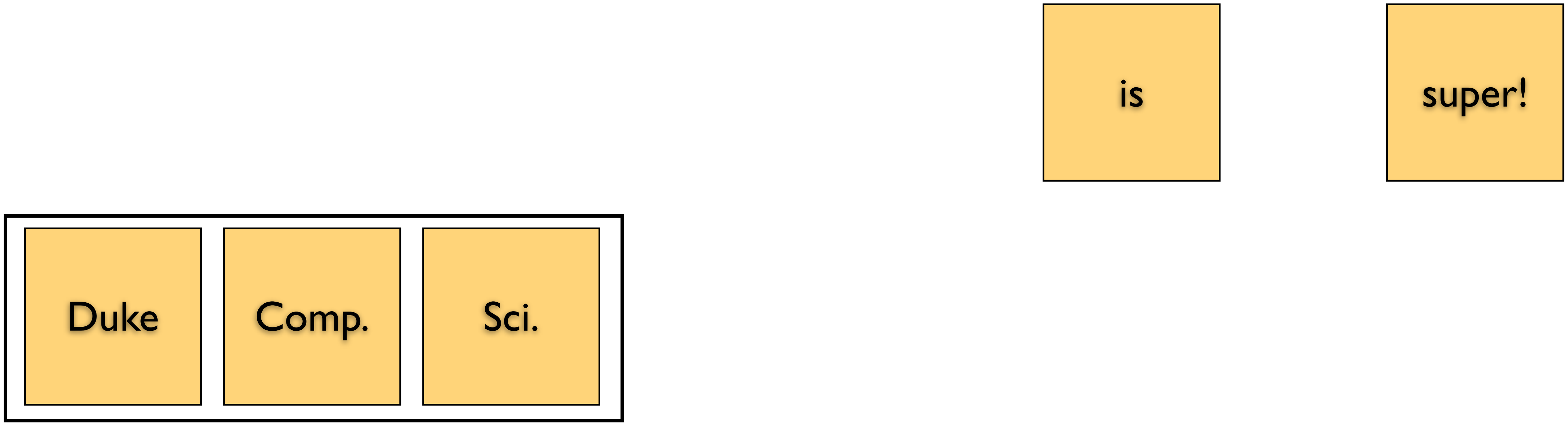
Operations: 3

Adds: 2

Add more than one?



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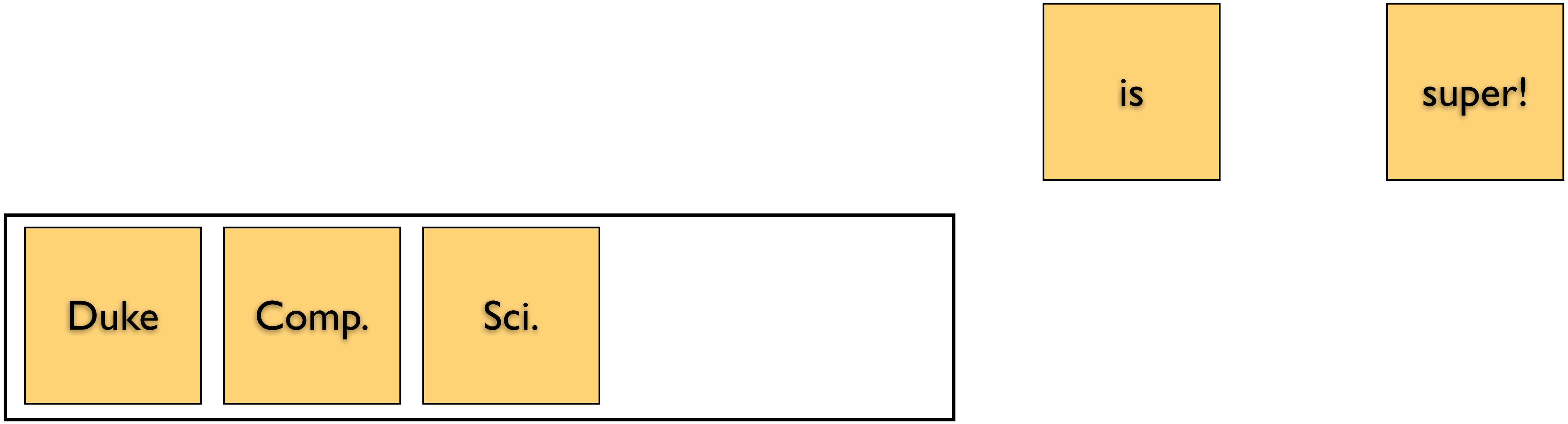
Operations: 4

Adds: 3

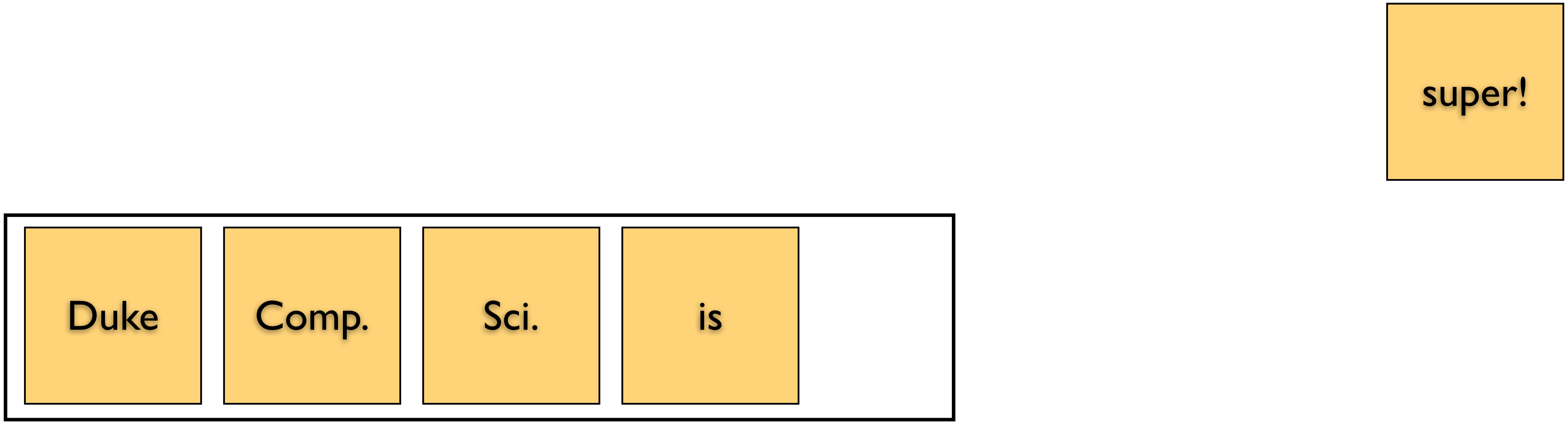
Add more than one?



Operations: 7

Adds: 3

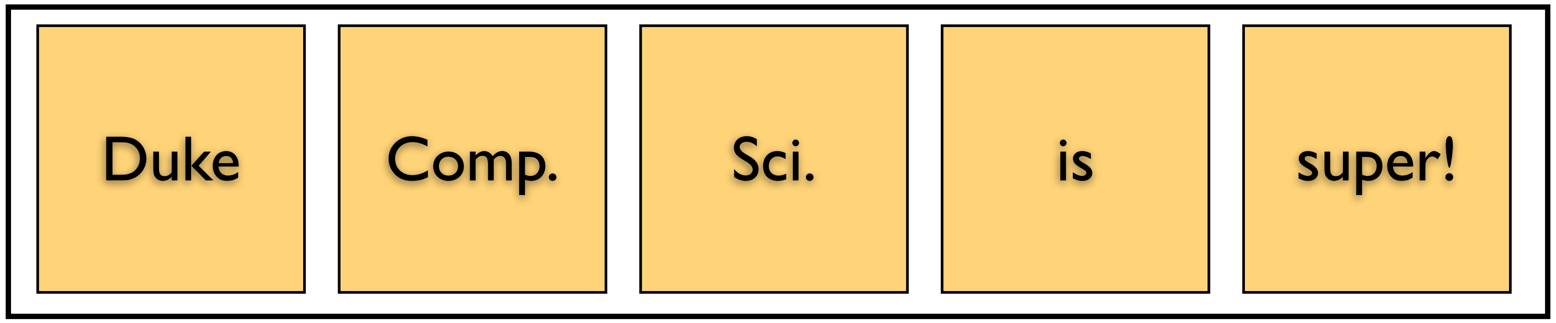
Add more than one?



Operations: 8

Adds: 4

Add more than one?



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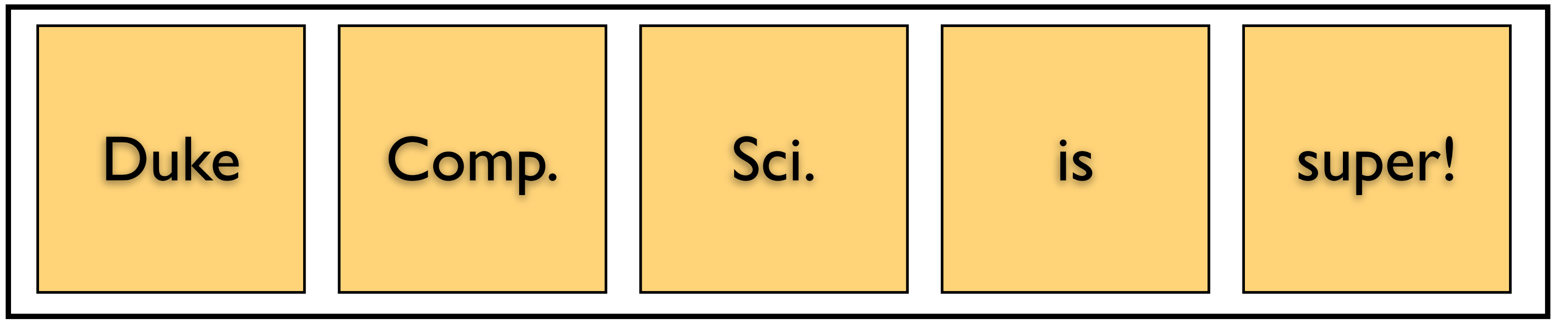
Operations: 9

Adds: 5

Add more than one?



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Operations: 9

Adds: 5

http://goo.gl/eLp8l

Adding *n* elements, expanding by *k*

O(1) most of the time

O(n) sometimes

$$\sum_{i=1}^{n} \left[1 + \frac{i}{k} \right]$$

Adding *n* elements, expanding by *k*

O(1) most of the time

O(n) sometimes

$$\sum_{i=1}^{n}\left[1+\frac{i}{k}\right]=n+\sum_{i=1}^{n}\frac{i}{k}=n+\frac{1}{k}\sum_{i=1}^{n}i$$

Adding *n* elements, expanding by *k*

O(1) most of the time

O(n) sometimes

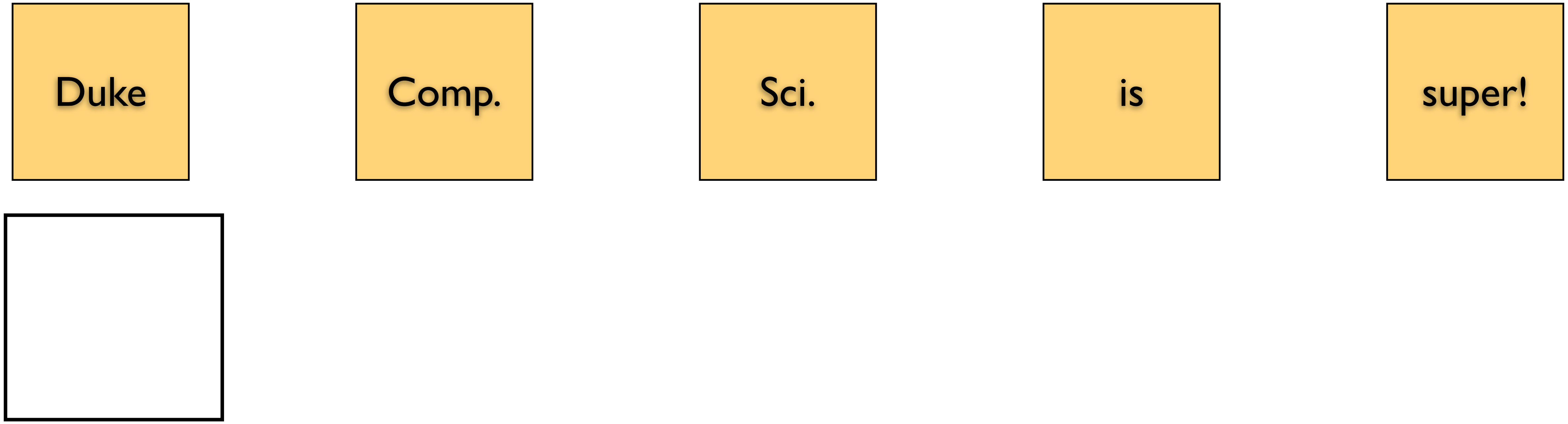
Note the worst-case behavior still dominates!

$$\sum_{i=1}^{n}\left[1+\frac{i}{k}\right]=n+\sum_{i=1}^{n}\frac{i}{k}=n+\frac{1}{k}\sum_{i=1}^{n}i$$

$$\in O(n^2)$$

Uh-oh...

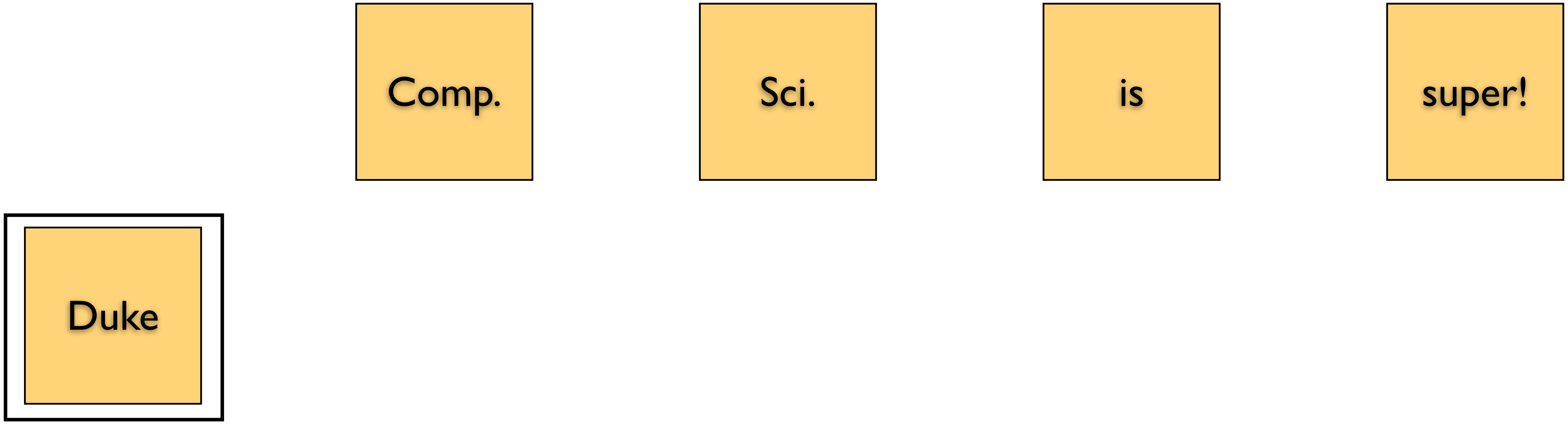
Adding didn’t work...



Operations: 0

Adds: 0

Adding didn't work...



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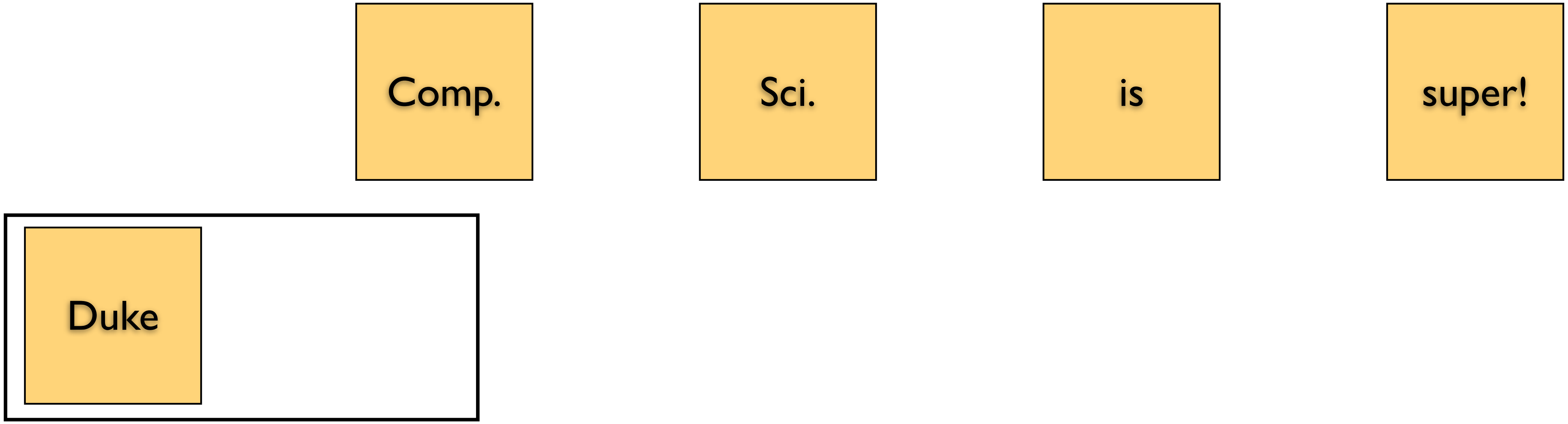
Operations: 1

Adds: 1

Adding didn't work...



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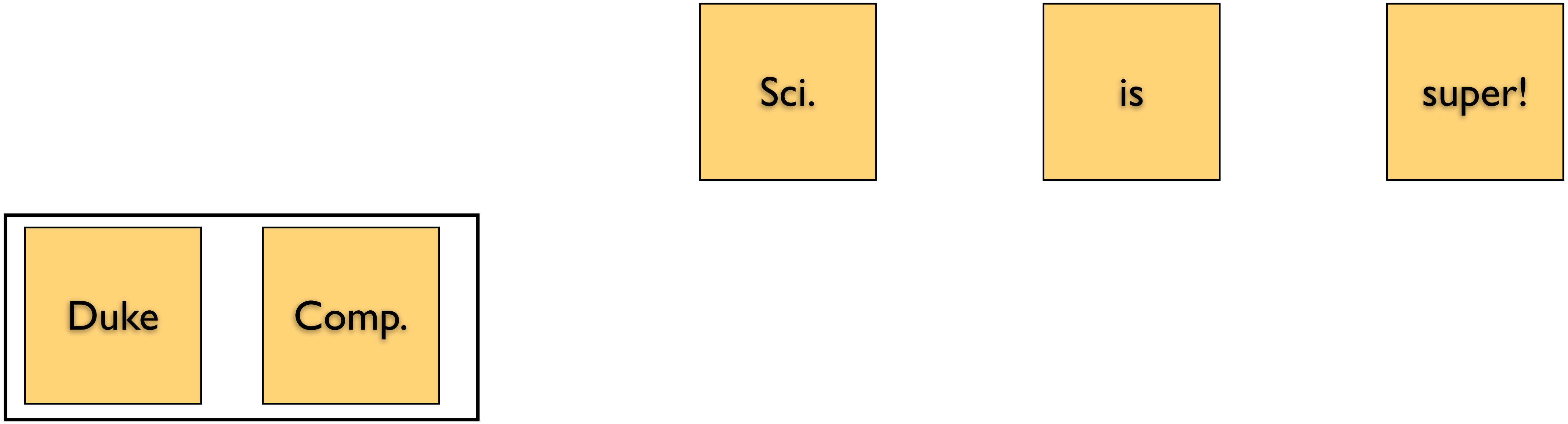
Operations: 2

Adds: 1

Adding didn’t work...



Sci.

is

super!

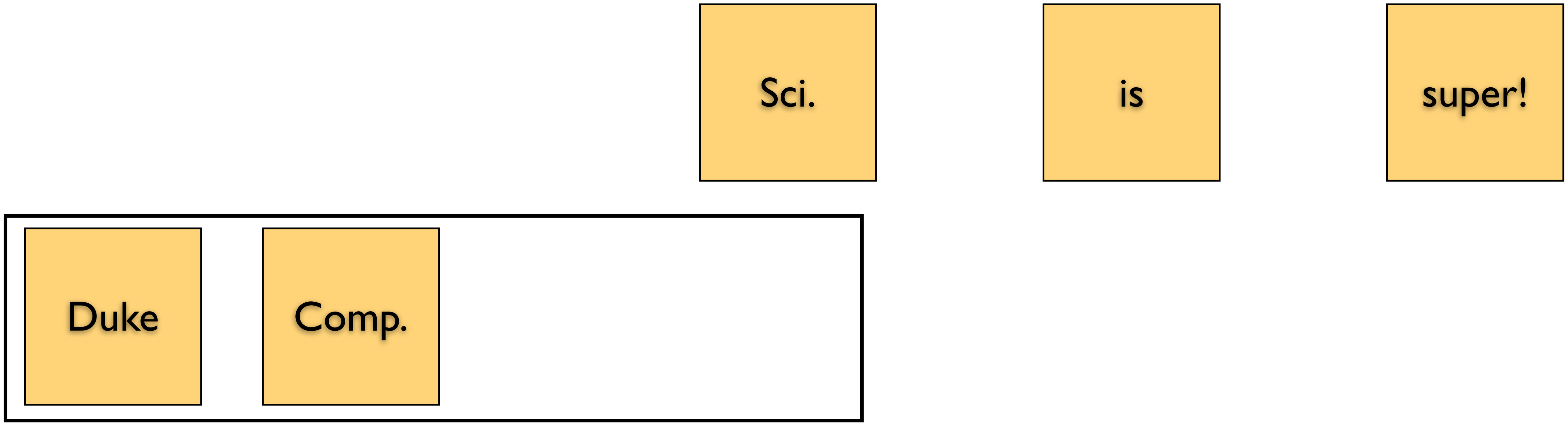
Duke

Comp.

Operations: 3

Adds: 2

Adding didn't work...



Sci.

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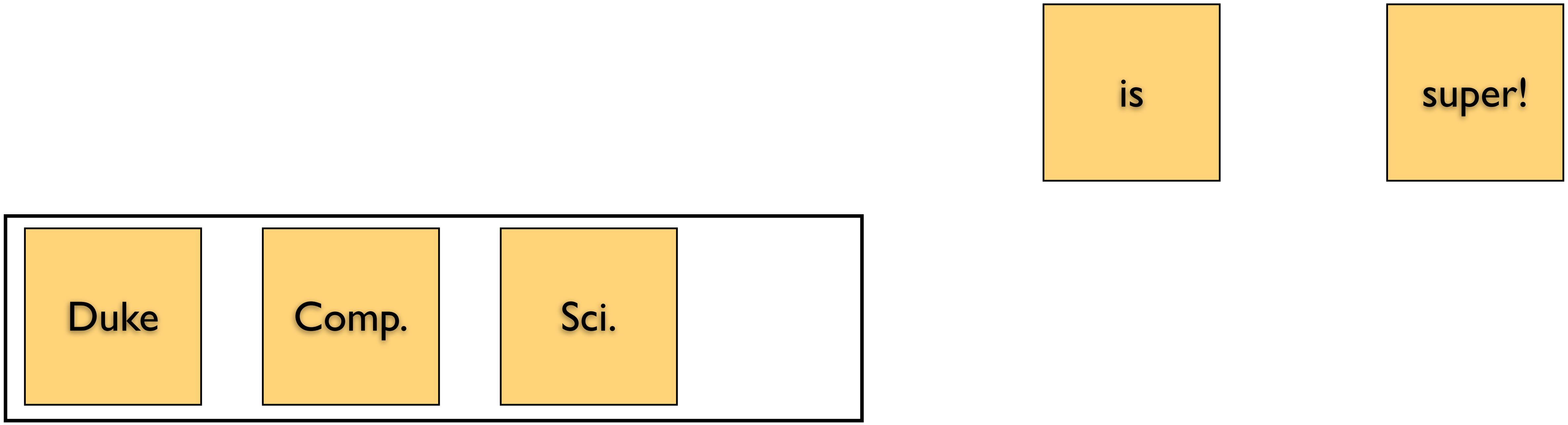
Comp.

Operations: 5

Adds: 2

Adding didn’t work...



is

super!

Duke

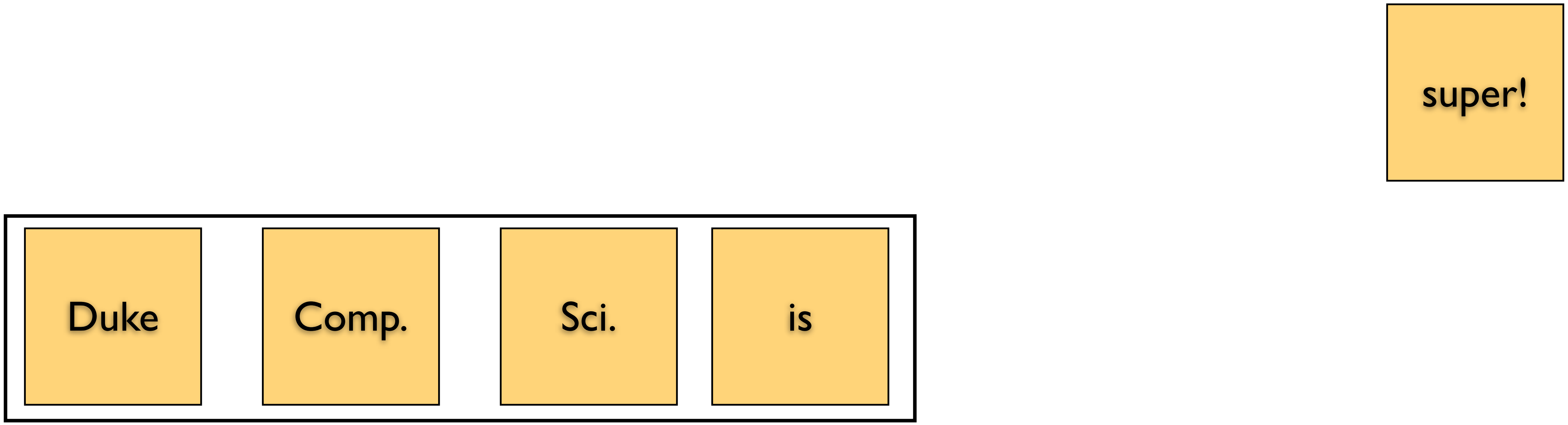
Comp.

Sci.

Operations: 6

Adds: 3

Adding didn’t work...



super!

Duke

Comp.

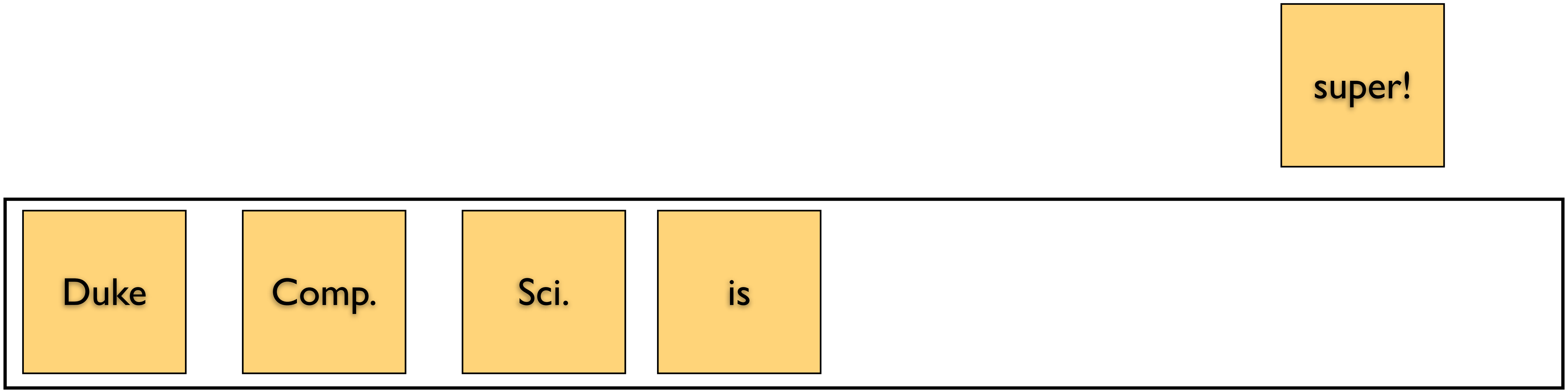
Sci.

is

Operations: 7

Adds: 4

Adding didn’t work...



super!

Duke

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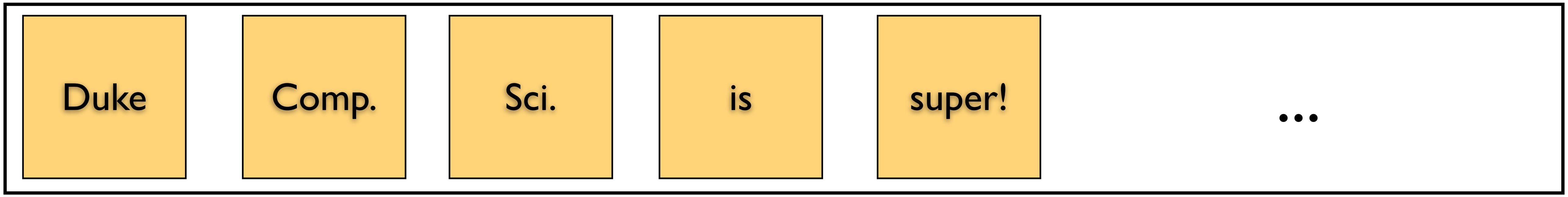
Operations: 11

Adds: 4

Adding didn't work...



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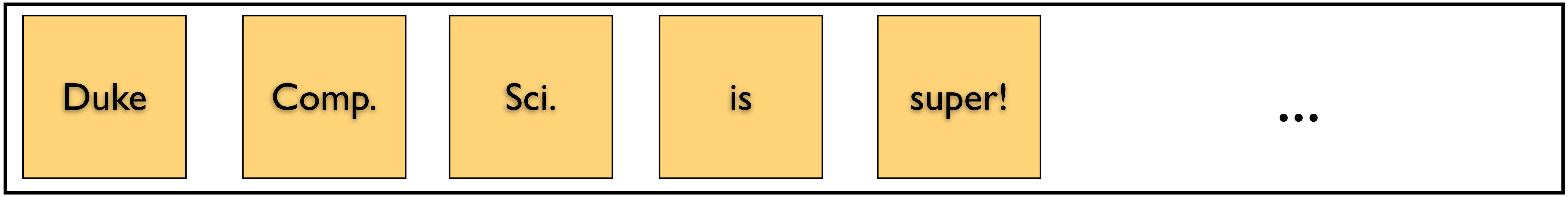
Operations: 12

Adds: 5

Adding didn’t work...



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How expensive is this strategy?

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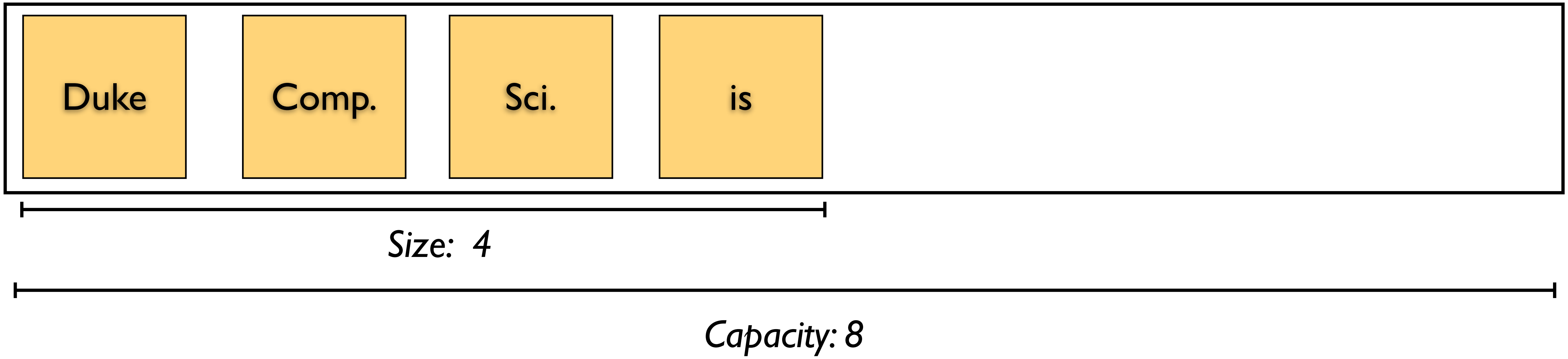

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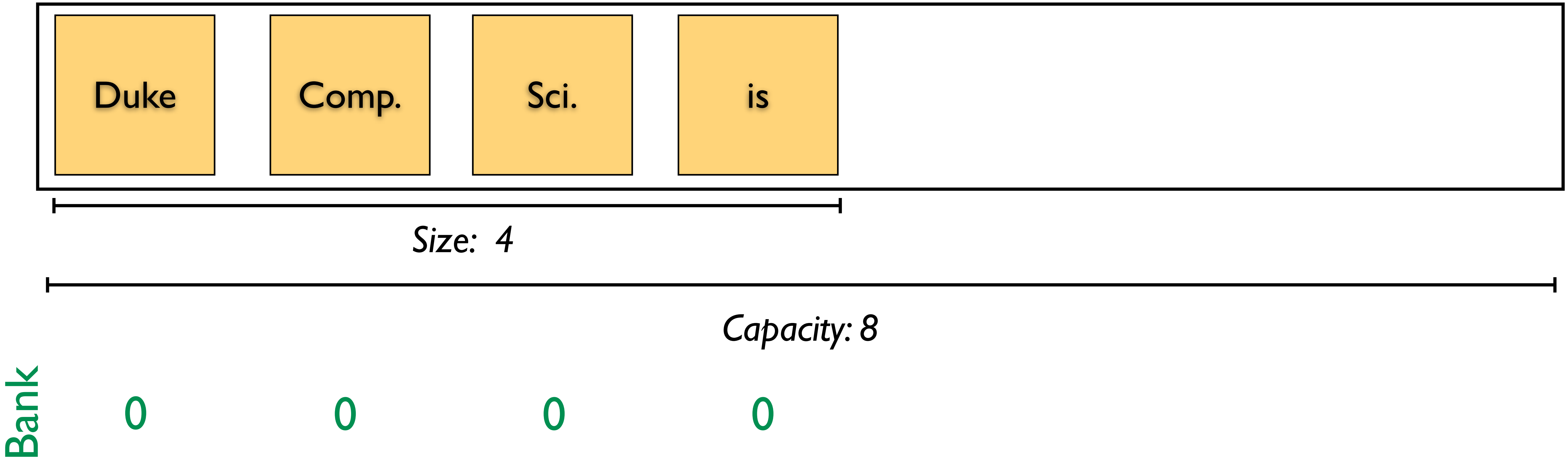
Size: 4

Capacity: 8

Suppose you’ve just doubled your array.

How expensive is this strategy?



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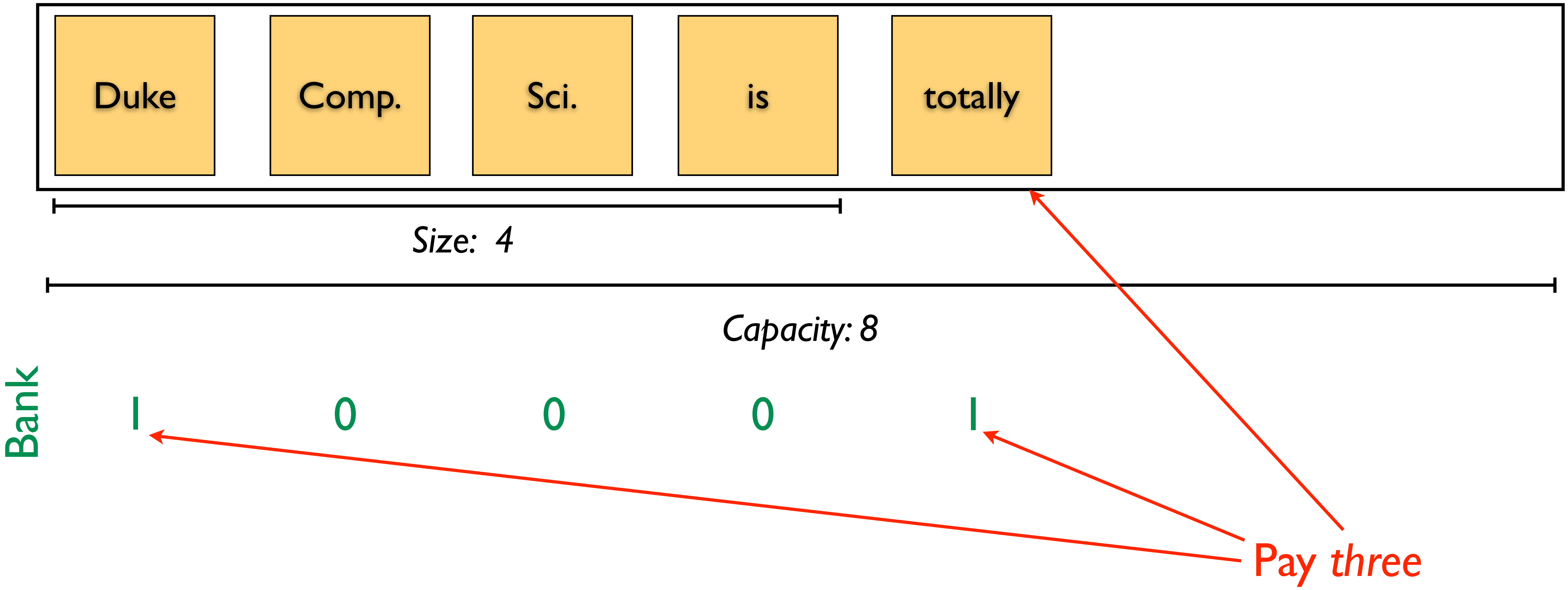
Size: 4

Capacity: 8

Bank

0 0 0 0

How expensive is this strategy?



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is

totally

Size: 4

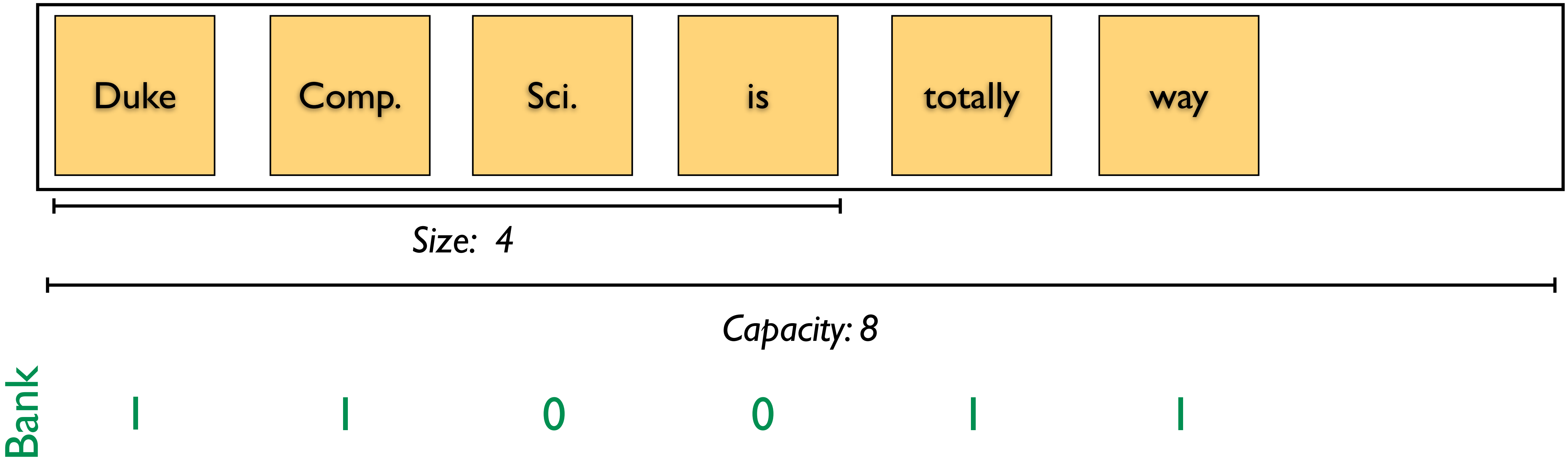
Capacity: 8

Bank

1 0 0 0 1

Pay *three*

How expensive is this strategy?



Duke Comp. Sci. is totally way

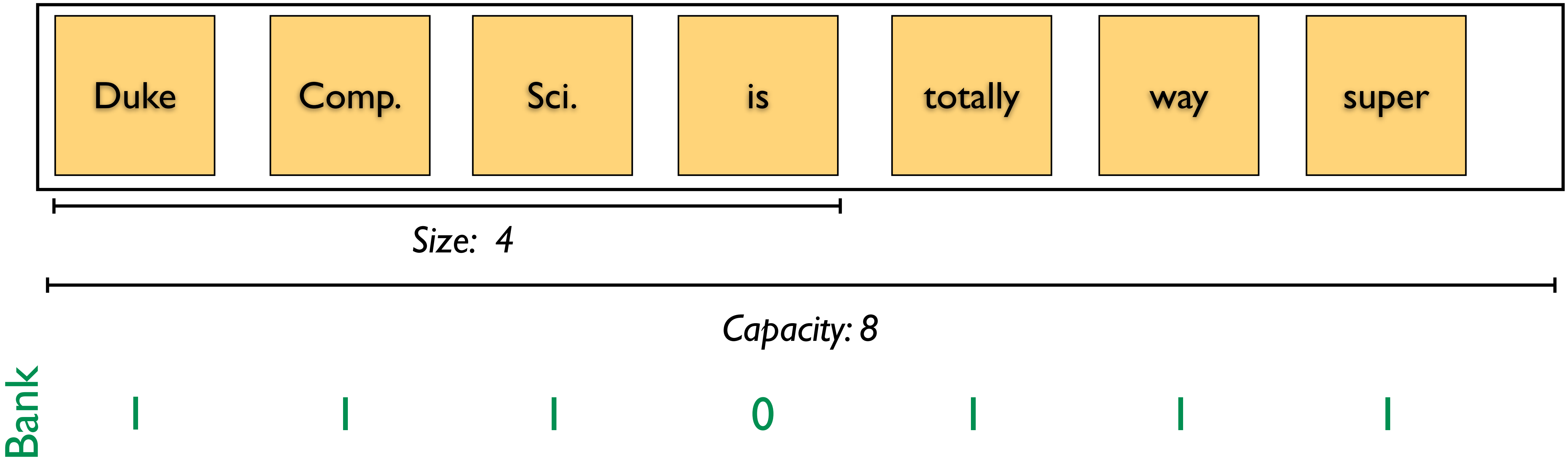
Size: 4

Capacity: 8

Bank

1 1 0 0 1 1

How expensive is this strategy?



Duke Comp. Sci. is totally way super

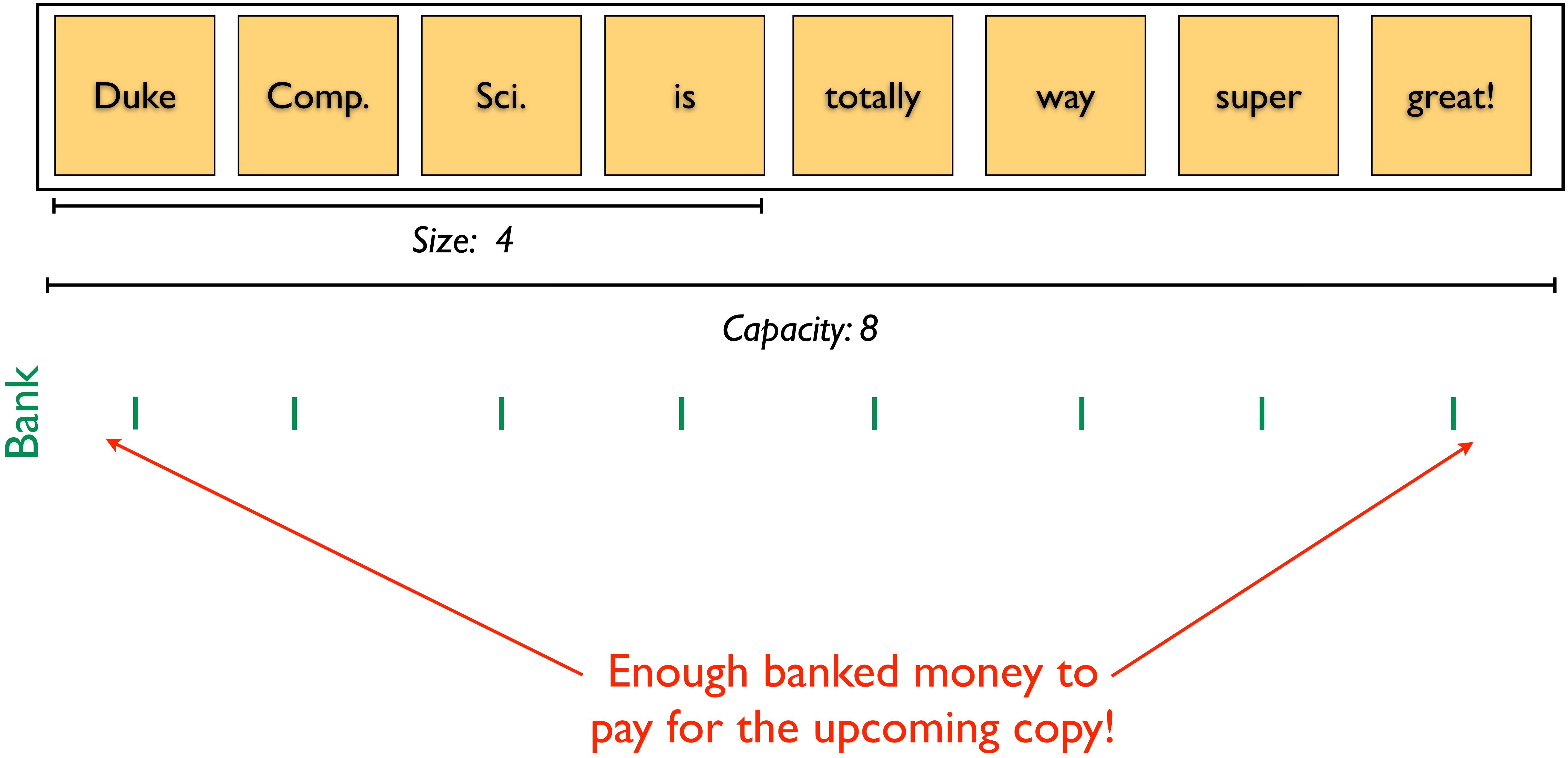
Size: 4

Capacity: 8

Bank

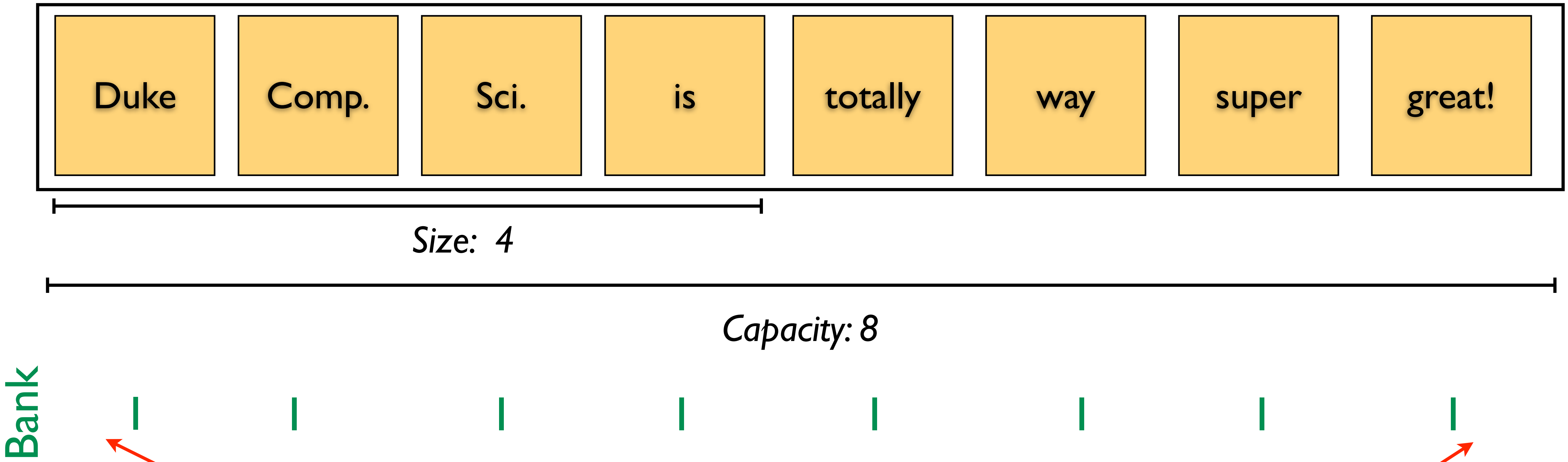
1 1 1 0 1 1 1

How expensive is this strategy?



Enough banked money to pay for the upcoming copy!

How expensive is this strategy?



Enough banked money to pay for the upcoming copy!

Key fact: add() costs a constant *amount of money!*

add has *constant amortized cost.*

Amortization facts

Amortized analysis deals with the cost of *n* operations, not the cost of one operation.

“N calls to add cost *O*(*n*) total.”

“One call to add might be *O*(*n*), too.”

Almost always good enough. So-called “realtime” applications are the exception.

The bank isn’t part of the data structure (no data is stored). It’s just an analytical tool.

A stack in two queues!

A stack in two queues!

Snarf Oct22InClass

http://goo.gl/4o5oN