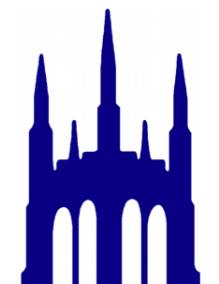


Recurrence Relations

(or: The Big-O of Recursive Functions)



A Big-O Warmup

```
// Assume strings has size n.  
// Assume "robot" appears in strings.  
public int findRobot(String[] strings) {  
    int current = 0;  
    while (!strings[current].equals("robot")) {  
        current++;  
    }  
    return current;  
}
```

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```

- 1. $O(n^2)$
- 2. $O(l)$
- 3. $O(n)$
- 4. $O(\log n)$
- 5. $O(n \log n)$

A (harder!) Big-O Warmup

```
// Return the index of v in the sorted array, or -1 if it's not there.  
// Search between the indices low and high.  
public int findInSorted(int[] sorted, int v, int low, int high) {  
    while (low < high) {  
        int midpoint = (low + high) / 2;  
        if (v < sorted[midpoint]) {  
            high = midpoint; // Search in the lower half.  
        } else if (v > sorted[midpoint]) {  
            low = midpoint + 1; // Search in the upper half.  
        } else {  
            return midpoint;  
        }  
    }  
    return -1;  
}
```

A (harder!) Big-O Warmup

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    return -1;  
}
```

- 1. $O(n^2)$
- 2. $O(l)$
- 3. $O(n)$
- 4. $O(\log n)$
- 5. $O(n \log n)$

A problem

```
// Is the value v contained in the binary search tree rooted at node?  
public boolean BSTcontainsValue(int v, TreeNode node) {  
    if (node == null) {  
        return false;  
    }  
    if (node.value == v) {  
        return true;  
    }  
  
    if (v < node.value) {  
        return BSTcontainsValue(v, node.left);  
    } else {  
        return BSTcontainsValue(v, node.right);  
    }  
}
```

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    }  
}
```

1. What's the Big-O if the tree is balanced?
2. What's the Big-O if it isn't?

What you'll need to do (or not)

1. Write down the recurrence relation.
2. *Solve the recurrence relation.* You won't need to do this in this class
3. Compute the Big-O from that solution.

A problem

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    if (v < node.value) {  
        return BSTcontainsValue(v, node.left);  
    } else {  
        return BSTcontainsValue(v, node.right);  
    }  
}
```

1. What's the Big-O if the tree is balanced?
2. What's the Big-O if it isn't?

A problem

```
public int height(TreeNode node) {  
    if (node == null) {  
        return 0;  
    }  
    int leftHeight = height(node.left);  
    int rightHeight = height(node.right);  
    return Math.max(leftHeight, rightHeight) + 1;  
}
```

<http://goo.gl/UwqvK>

A problem

```
public int height(TreeNode node) {  
    if (node == null) {  
        return 0;  
    }  
    int leftHeight = height(node.left);  
    int rightHeight = height(node.right);  
    return Math.max(leftHeight, rightHeight) + 1;  
}
```

<http://goo.gl/UwqvK>

<http://goo.gl/8V6kS>

A problem

```
public boolean isBalanced(TreeNode node) {  
    int left = computeHeight(node.left);  
    int right = computeHeight(node.right);  
    if (Math.abs(left - right) > 1) {  
        return false;  
    }  
  
    return (isBalanced(node.left) &&  
            isBalanced(node.right));  
}
```

The Table to end all Tables

| Recurrence | An algorithm with this recurrence | Running Time |
|-------------------------|--------------------------------------|---------------|
| $T(n) = T(n/2) + O(1)$ | Binary Search | $O(\log n)$ |
| $T(n) = T(n-1) + O(1)$ | Linear Search | $O(n)$ |
| $T(n) = 2T(n/2) + O(1)$ | Tree traversal | $O(n)$ |
| $T(n) = 2T(n/2) + O(n)$ | QuickSort | $O(n \log n)$ |
| $T(n) = T(n-1) + O(n)$ | BubbleSort | $O(n^2)$ |

Two more

```
public int maximum(int[] values, int low, int high) {  
    if (low == high) {  
        return values[low];  
    }  
    int mid = (low + high) / 2;  
    return Math.max(maximum(values, low, mid),  
                    maximum(values, mid+1, high));  
}
```

```
// Reverse the array values, between the indices low and high.  
public static void reverse(int[] values, int low, int high) {  
    if (low >= high) {  
        return;  
    }  
    int temp = values[low];  
    values[low] = values[high];  
    values[high] = temp;  
    reverse(values, low+1, high-1);  
}
```

<http://goo.gl/YxiK4>