What can be programmed?

- What class of problems can be *solved*?
  - G5, 1000Mhz Pentium III, Cray, pencil?
  - Alan Turing proved some things, hypothesized others
    - Halting problem, Church-Turing thesis

- What class of problems can be *solved efficiently*?
  - Problems with no practical solution
    - What does practical mean?
  - Problems for which we can’t find a practical solution
    - Solving one solves them all
    - Would you rather be rich or famous?

Schedule students, minimize conflicts

- Given student requests, available teachers
  - write a program that schedules classes
  - Minimize conflicts
- Add a GUI too
  - Web interface
  - …
  - …

One better scenario

I can't write this program because it's provably impossible

Another possible scenario

I can't write this program but neither can all these famous people

The halting problem: writing `doesHalt`

```
public class ProgramUtils
    /**
     * Returns true if progname halts on input,
     * otherwise returns false (progname loops)
     */
    public static boolean doesHalt(String progname,
                                   String input){
    }
}
```

- A compiler is a program that reads other programs as input
  - Can a word counting program count its own words?
- The `doesHalt` method might simulate, analyze, …
  - One program/function that works for *any* program/input

How to tell if Foo stops on 123 456

```
public static void main(String[] args) {
    String prog = "Foo.java";
    String input = "123 456"
    if (ProgramUtils.doesHalt(prog,input)){
        System.out.println(prog+" stops");
    }
    else {
        System.out.println(prog+" 4ever");
    }
}
```

- Can user enter name of program? Input?
  - What's the problem with this program?

Consider the class *`Confuse.java`*

```
public static void main(String[] args){
    String prog = "Foo.java";
    if (ProgramUtils.doesHalt(prog,prog)) {
        while (true) {
            // do nothing forever
        }
    }
}
```

- We want to show writing `doesHalt` is impossible
  - Proof by contradiction:
  - Assume possible, show impossible situation results

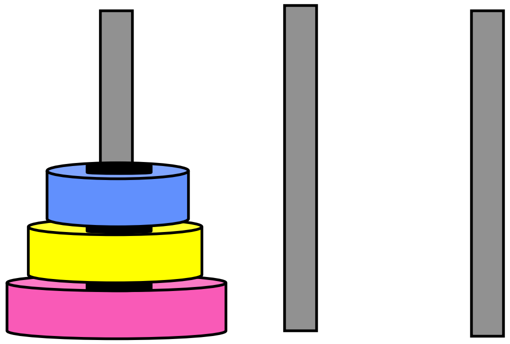
- Can a program read a program? Itself?

What's a meta catalog? Top 10 sites?

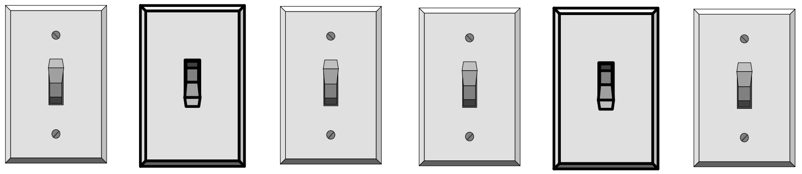
- Consider a website of interesting sites
  - Does the website list itself? Is this a problem?
- Consider a website that lists every useless website
  - Would this be a useful resource?
  - Does the website list itself?
- What about a site of all the sites that list themselves?
  - What about sites that don't list themselves? ***nolist.com***

Not impossible, but impractical

- Towers of Hanoi
  - How long to move n disks?

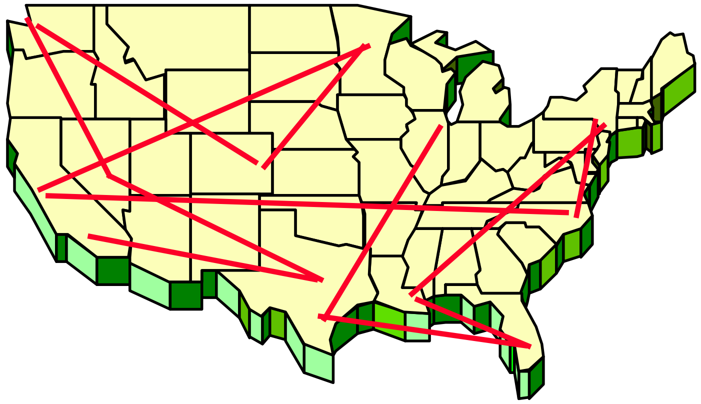


- What combination of switches turns the light on?
  - Try all combinations, how many are there?
  - Is there a better way?



Travelling Salesperson

- Visit every city exactly once
- Minimize cost of travel or distance
- Is there a tour for under $2,000 ? less than 6,000 miles?
- Is close good enough?
  - Within 10% of optimal
  - Within 50% of optimal
  - …



Try all paths, from every starting point -- how long does this take?

a, b, c, d, e, f, g
b, a, c, d, e, f, g ...

Are hard problems easy?

- P = easy problems, NP = “hard” problems
  - P means solvable in polynomial time
    - Difference between N, N^2, N^{10} ?
  - NP means non-deterministic, polynomial time
    - *guess a solution and verify it efficiently*

- Question: P = NP ?
  - if yes, a whole class of difficult problems , the NP-complete problems, can be solved efficiently
  - if no, none of the hard problems can be solved efficiently
  - showing the first problem was NP complete was an exercise in intellectual bootstrapping, satisfiability/Cook/(1971)

Theory and Practice

- Number theory: pure mathematics
  - How many prime numbers are there?
  - How do we factor?
  - How do we determine primeness?

- Computer Science
  - Primality is "easy"
  - Factoring is "hard"
  - Encryption is possible

public-key cryptography

randomized primality testing