

# Relational Database Design Theory

CompSci 316  
Introduction to Database Systems

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## Announcements (Thu. Sep. 12) <sup>2</sup>

- ❖ Homework #1 due next Tuesday
  - If you haven't activated Azure, do it now!
  - All-electronic submission
- ❖ Piazza is up—use it more
  - There is also a thread for forming project teams
- ❖ Location for Rishi's office hours has changed

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## Motivation <sup>3</sup>

SID	name	CID
142	Bart	CPS316
142	Bart	CPS310
857	Lisa	CPS316
857	Lisa	CPS330
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- ❖ How do we tell if a design is bad, e.g., *StudentEnroll* (SID, name, CID)?
  - This design has redundancy, because the name of a student is recorded multiple times, once for each course the student is taking
    - Update, insertion, deletion anomalies
- ❖ How about a systematic approach to detecting and removing redundancy in designs?
  - Dependencies, decompositions, and normal forms

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## Functional dependencies

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- ❖ A functional dependency (FD) has the form  $X \rightarrow Y$ , where  $X$  and  $Y$  are sets of attributes in a relation  $R$
- ❖  $X \rightarrow Y$  means that whenever two tuples in  $R$  agree on all the attributes in  $X$ , they must also agree on all attributes in  $Y$

$X$	$Y$	$Z$
$a$	$b$	$c$
$a$	$b$	?
...	...	...

Must be  $b$  ←      ← Could be anything

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## FD examples

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*Address (street\_address, city, state, zip)*

- ❖  $street\_address, city, state \rightarrow zip$
- ❖  $zip \rightarrow city, state$
- ❖  $zip, state \rightarrow zip?$ 
  - This is a trivial FD
  - Trivial FD:  $LHS \supseteq RHS$
- ❖  $zip \rightarrow state, zip?$ 
  - This is non-trivial, but not completely non-trivial
  - Completely non-trivial FD:  $LHS \cap RHS = \emptyset$

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## Keys redefined using FD's

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A set of attributes  $K$  is a key for a relation  $R$  if

- ❖  $K \rightarrow$  all (other) attributes of  $R$ 
  - That is,  $K$  is a "super key"
- ❖ No proper subset of  $K$  satisfies the above condition
  - That is,  $K$  is minimal

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## Reasoning with FD's

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Given a relation  $R$  and a set of FD's  $\mathcal{F}$

- ❖ Does another FD follow from  $\mathcal{F}$ ?
  - Are some of the FD's in  $\mathcal{F}$  redundant (i.e., they follow from the others)?
- ❖ Is  $K$  a key of  $R$ ?
  - What are all the keys of  $R$ ?

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## Attribute closure

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❖ Given  $R$ , a set of FD's  $\mathcal{F}$  that hold in  $R$ , and a set of attributes  $Z$  in  $R$ :

The closure of  $Z$  (denoted  $Z^+$ ) with respect to  $\mathcal{F}$  is the set of all attributes  $\{A_1, A_2, \dots\}$  functionally determined by  $Z$  (that is,  $Z \rightarrow A_1 A_2 \dots$ )

- ❖ Algorithm for computing the closure
  - Start with closure =  $Z$
  - If  $X \rightarrow Y$  is in  $\mathcal{F}$  and  $X$  is already in the closure, then also add  $Y$  to the closure
  - Repeat until no more attributes can be added

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## A more complex example

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*StudentGrade* (*SID*, *name*, *email*, *CID*, *grade*)

- ❖  $SID \rightarrow name, email$
- ❖  $email \rightarrow SID$
- ❖  $SID, CID \rightarrow grade$

(Not a good design, and we will see why later)

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## Example of computing closure

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- ❖  $\mathcal{F}$  includes:
  - $SID \rightarrow name, email$
  - $email \rightarrow SID$
  - $SID, CID \rightarrow grade$
- ❖  $\{CID, email\}^+ = ?$
- ❖  $email \rightarrow SID$ 
  - Add  $SID$ ; closure is now  $\{CID, email, SID\}$
- ❖  $SID \rightarrow name, email$ 
  - Add  $name, email$ ; closure is now  $\{CID, email, SID, name\}$
- ❖  $SID, CID \rightarrow grade$ 
  - Add  $grade$ ; closure is now all the attributes in  $StudentGrade$

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## Using attribute closure

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Given a relation  $R$  and set of FD's  $\mathcal{F}$

- ❖ Does another FD  $X \rightarrow Y$  follow from  $\mathcal{F}$ ?
  - Compute  $X^+$  with respect to  $\mathcal{F}$
  - If  $Y \subseteq X^+$ , then  $X \rightarrow Y$  follow from  $\mathcal{F}$
- ❖ Is  $K$  a key of  $R$ ?
  - Compute  $K^+$  with respect to  $\mathcal{F}$
  - If  $K^+$  contains all the attributes of  $R$ ,  $K$  is a super key
  - Still need to verify that  $K$  is *minimal* (how?)

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## Rules of FD's

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- ❖ Armstrong's axioms
  - Reflexivity: If  $Y \subseteq X$ , then  $X \rightarrow Y$
  - Augmentation: If  $X \rightarrow Y$ , then  $XZ \rightarrow YZ$  for any  $Z$
  - Transitivity: If  $X \rightarrow Y$  and  $Y \rightarrow Z$ , then  $X \rightarrow Z$
- ❖ Rules derived from axioms
  - Splitting: If  $X \rightarrow YZ$ , then  $X \rightarrow Y$  and  $X \rightarrow Z$
  - Combining: If  $X \rightarrow Y$  and  $X \rightarrow Z$ , then  $X \rightarrow YZ$
- ☞ Using these rules, you can prove or disprove an FD given a set of FDs

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## Non-key FD's

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- ❖ Consider a non-trivial FD  $X \rightarrow Y$  where  $X$  is not a super key
  - Since  $X$  is not a super key, there are some attributes (say  $Z$ ) that are not functionally determined by  $X$

$X$	$Y$	$Z$
$a$	$b$	$c_1$
$a$	$b$	$c_2$
...	...	...

That  $b$  is always associated with  $a$  is recorded by multiple rows:  
 redundancy, update/insertion/deletion anomaly

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## Example of redundancy

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- ❖ *StudentGrade* ( $SID$ ,  $name$ ,  $email$ ,  $CID$ ,  $grade$ )
- ❖  $SID \rightarrow name, email$

$SID$	$name$	$email$	$CID$	$grade$
142	Bart	bart@fox.com	CPS316	B-
142	Bart	bart@fox.com	CPS310	B
123	Milhouse	milhouse@fox.com	CPS316	B+
857	Lisa	lisa@fox.com	CPS316	A+
857	Lisa	lisa@fox.com	CPS330	A+
456	Ralph	ralph@fox.com	CPS310	C
...	...	...	...	...

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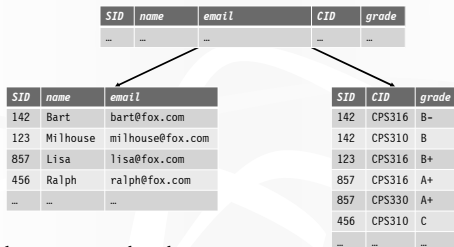
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## Decomposition

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- ❖ Eliminates redundancy
- ❖ To get back to the original relation:

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## Unnecessary decomposition

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SID	name	email
142	Bart	bart@fox.com
123	Milhouse	milhouse@fox.com
857	Lisa	lisa@fox.com
456	Ralph	ralph@fox.com
...	...	...

SID	name
142	Bart
123	Milhouse
857	Lisa
456	Ralph
...	...

SID	email
142	bart@fox.com
123	milhouse@fox.com
857	lisa@fox.com
456	ralph@fox.com
...	...

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## Bad decomposition

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SID	CID	grade
142	CPS316	B-
142	CPS310	B
123	CPS316	B+
857	CPS316	A+
857	CPS330	A+
456	CPS310	C
...	...	...

SID	CID
142	CPS316
142	CPS310
123	CPS316
857	CPS316
857	CPS330
456	CPS310
...	...

SID	grade
142	B-
142	B
123	B+
857	A+
456	C
...	...

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## Lossless join decomposition

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- ❖ Decompose relation  $R$  into relations  $S$  and  $T$ 
  - $attrs(R) = attrs(S) \cup attrs(T)$
  - $S = \pi_{attrs(S)}(R)$
  - $T = \pi_{attrs(T)}(R)$
- ❖ The decomposition is a lossless join decomposition if, given known constraints such as FD's, we can guarantee that  $R = S \bowtie T$
- ❖ Any decomposition gives  $R \subseteq S \bowtie T$  (why?)
  - A lossy decomposition is one with  $R \subset S \bowtie T$

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## Loss? But I got more rows!

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❖ “Loss” refers not to the loss of tuples, but to the loss of information

- Or, the ability to distinguish different original relations

<i>SID</i>	<i>CID</i>
142	CPS316
142	CPS310
123	CPS316
857	CPS316
857	CPS330
456	CPS310
...	...

<i>SID</i>	<i>CID</i>	<i>grade</i>
142	CPS316	B
142	CPS310	B-
123	CPS316	B+
857	CPS316	A+
857	CPS330	A+
456	CPS310	C
...	...	...

No way to tell which is the original relation

<i>SID</i>	<i>grade</i>
142	B-
142	B
123	B+
857	A+
456	C
...	...

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## Questions about decomposition

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❖ When to decompose

❖ How to come up with a correct decomposition (i.e., lossless join decomposition)

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## An answer: BCNF

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❖ A relation  $R$  is in Boyce-Codd Normal Form if

- For every non-trivial FD  $X \rightarrow Y$  in  $R$ ,  $X$  is a super key
- That is, all FDs follow from “key  $\rightarrow$  other attributes”

❖ When to decompose

- As long as some relation is not in BCNF

❖ How to come up with a correct decomposition

- Always decompose on a BCNF violation (details next)
- ☞ Then it is guaranteed to be a lossless join decomposition!

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## BCNF decomposition algorithm

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- ❖ Find a BCNF violation
  - That is, a non-trivial FD  $X \rightarrow Y$  in  $R$  where  $X$  is not a super key of  $R$
- ❖ Decompose  $R$  into  $R_1$  and  $R_2$ , where
  - $R_1$  has attributes  $X \cup Y$
  - $R_2$  has attributes  $X \cup Z$ , where  $Z$  contains all attributes of  $R$  that are in neither  $X$  nor  $Y$
- ❖ Repeat until all relations are in BCNF

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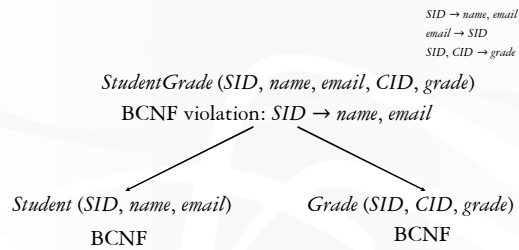
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## BCNF decomposition example

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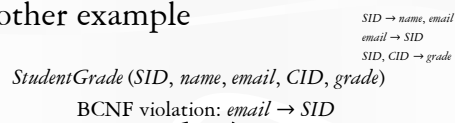
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## Another example

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## Why is BCNF decomposition lossless <sup>25</sup>

Given non-trivial  $X \rightarrow Y$  in  $R$  where  $X$  is not a super key of  $R$ , need to prove:

- ❖ Anything we project always comes back in the join:

$$R \subseteq \pi_{XY}(R) \bowtie \pi_{XZ}(R)$$

- Sure; and it doesn't depend on the FD

- ❖ Anything that comes back in the join must be in the original relation:

$$R \supseteq \pi_{XY}(R) \bowtie \pi_{XZ}(R)$$

- Proof will make use of the fact that  $X \rightarrow Y$

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## Recap <sup>26</sup>

- ❖ Functional dependencies: a generalization of the key concept
- ❖ Non-key functional dependencies: a source of redundancy
- ❖ BCNF decomposition: a method for removing redundancies
  - BCNF decomposition is a lossless join decomposition
- ❖ BCNF: schema in this normal form has no redundancy due to FD's

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## BCNF = no redundancy? <sup>27</sup>

- ❖ *Student (SID, CID, club)*

- Suppose your classes have nothing to do with the clubs you join

- FD's?

SID	CID	club
142	CPS316	ballet
142	CPS316	sumo
142	CPS310	ballet
142	CPS310	sumo
123	CPS316	chess
123	CPS316	golf
...	...	...

- BCNF?

- Redundancies?

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## Multivalued dependencies

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- ❖ A multivalued dependency (MVD) has the form  $X \twoheadrightarrow Y$ , where  $X$  and  $Y$  are sets of attributes in a relation  $R$
- ❖  $X \twoheadrightarrow Y$  means that whenever two rows in  $R$  agree on all the attributes of  $X$ , then we can swap their  $Y$  components and get two new rows that are also in  $R$

$X$	$Y$	$Z$
$a$	$b1$	$c1$
$a$	$b2$	$c2$
$a$	$b1$	$c2$
$a$	$b2$	$c1$
...	...	...

} Must be in  $R$  too

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## MVD examples

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*Student* ( $SID$ ,  $CID$ ,  $club$ )

- ❖  $SID \twoheadrightarrow CID$
- ❖  $SID, CID \twoheadrightarrow club$ 
  - Trivial:  $LHS \cup RHS = \text{all attributes of } R$
- ❖  $SID, CID \twoheadrightarrow SID$ 
  - Trivial:  $LHS \supseteq RHS$

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## Complete MVD + FD rules

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- ❖ FD reflexivity, augmentation, and transitivity
- ❖ MVD complementation:  
If  $X \twoheadrightarrow Y$ , then  $X \twoheadrightarrow attrs(R) - X - Y$
- ❖ MVD augmentation:  
If  $X \twoheadrightarrow Y$  and  $V \subseteq W$ , then  $XW \twoheadrightarrow YV$
- ❖ MVD transitivity:  
If  $X \twoheadrightarrow Y$  and  $Y \twoheadrightarrow Z$ , then  $X \twoheadrightarrow Z - Y$
- ❖ Replication (FD is MVD):  
If  $X \rightarrow Y$ , then  $X \twoheadrightarrow Y$       Try proving things using these!?
- ❖ Coalescence:  
If  $X \twoheadrightarrow Y$  and  $Z \subseteq Y$  and there is some  $W$  disjoint from  $Y$  such that  $W \rightarrow Z$ , then  $X \rightarrow Z$

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## An elegant solution: chase

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- ❖ Given a set of FD's and MVD's  $\mathcal{D}$ , does another dependency  $d$  (FD or MVD) follow from  $\mathcal{D}$ ?
- ❖ Procedure
  - Start with the hypothesis of  $d$ , and treat them as "seed" tuples in a relation
  - Apply the given dependencies in  $\mathcal{D}$  repeatedly
    - If we apply an FD, we infer equality of two symbols
    - If we apply an MVD, we infer more tuples
  - If we infer the conclusion of  $d$ , we have a proof
  - Otherwise, if nothing more can be inferred, we have a counterexample

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## Proof by chase

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- ❖ In  $R(A, B, C, D)$ , does  $A \twoheadrightarrow B$  and  $B \twoheadrightarrow C$  imply that  $A \twoheadrightarrow C$ ?

	Have				Need			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
	<i>a</i>	<i>b</i> 1	<i>c</i> 1	<i>d</i> 1	<i>a</i>	<i>b</i> 1	<i>c</i> 2	<i>d</i> 1
	<i>a</i>	<i>b</i> 2	<i>c</i> 2	<i>d</i> 2	<i>a</i>	<i>b</i> 2	<i>c</i> 1	<i>d</i> 2

$A \twoheadrightarrow B$

<i>a</i>	<i>b</i> 2	<i>c</i> 1	<i>d</i> 1
<i>a</i>	<i>b</i> 1	<i>c</i> 2	<i>d</i> 2

$B \twoheadrightarrow C$

<i>a</i>	<i>b</i> 2	<i>c</i> 1	<i>d</i> 2
<i>a</i>	<i>b</i> 2	<i>c</i> 2	<i>d</i> 1

$B \twoheadrightarrow C$

<i>a</i>	<i>b</i> 1	<i>c</i> 2	<i>d</i> 1
<i>a</i>	<i>b</i> 1	<i>c</i> 1	<i>d</i> 2

$A \twoheadrightarrow C$

<i>a</i>	<i>b</i> 1	<i>c</i> 2	<i>d</i> 1
<i>a</i>	<i>b</i> 2	<i>c</i> 1	<i>d</i> 2

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## Another proof by chase

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- ❖ In  $R(A, B, C, D)$ , does  $A \rightarrow B$  and  $B \rightarrow C$  imply that  $A \rightarrow C$ ?

	Have				Need
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	$c1 = c2$
	<i>a</i>	<i>b</i> 1	<i>c</i> 1	<i>d</i> 1	
	<i>a</i>	<i>b</i> 2	<i>c</i> 2	<i>d</i> 2	

$A \rightarrow B$      $b1 = b2$

$B \rightarrow C$      $c1 = c2$

$A \rightarrow C$

In general, both new tuples and new equalities may be generated

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## Counterexample by chase

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❖ In  $R(A, B, C, D)$ , does  $A \twoheadrightarrow BC$  and  $CD \rightarrow B$  imply that  $A \rightarrow B$ ?

	Have				Need
	A	B	C	D	$b1 = b2$ ❖
$A \twoheadrightarrow BC$	a	b1	c1	d1	
	a	b2	c2	d2	
	a	b2	c2	d1	
	a	b1	c1	d2	

Counterexample!

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## 4NF

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- ❖ A relation  $R$  is in Fourth Normal Form (4NF) if
  - For every non-trivial MVD  $X \twoheadrightarrow Y$  in  $R$ ,  $X$  is a superkey
  - That is, all FD's and MVD's follow from "key  $\rightarrow$  other attributes" (i.e., no MVD's and no FD's besides key functional dependencies)
- ❖ 4NF is stronger than BCNF
  - Because every FD is also a MVD

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## 4NF decomposition algorithm

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- ❖ Find a 4NF violation
  - A non-trivial MVD  $X \twoheadrightarrow Y$  in  $R$  where  $X$  is not a superkey
- ❖ Decompose  $R$  into  $R_1$  and  $R_2$ , where
  - $R_1$  has attributes  $X \cup Y$
  - $R_2$  has attributes  $X \cup Z$  ( $Z$  contains  $R$  attributes not in  $X$  or  $Y$ )
- ❖ Repeat until all relations are in 4NF
- ❖ Almost identical to BCNF decomposition algorithm
- ❖ Any decomposition on a 4NF violation is lossless

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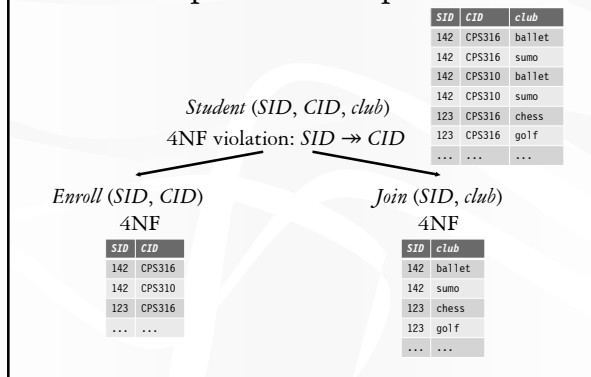
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## 4NF decomposition example

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## Summary

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- ❖ Philosophy behind BCNF, 4NF:
  - Data should depend on the key, the whole key, and nothing but the key!
- ❖ Other normal forms
  - 3NF: More relaxed than BCNF; will not remove redundancy if doing so makes FDs harder to enforce
  - 2NF: Slightly more relaxed than 3NF
  - 1NF: All column values must be atomic

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