Due on Nov 10 before midnight, 2013

Problem 1
Consider the following problem. Given an undirected graph $G = (V, E)$ with a non-negative weight function $w(\cdot)$ on the vertices V, a set of terminals $T \subseteq V$ and a number K, decide whether there exists a subgraph G' of G such that in G', there exists a path between every pair of terminals and the total weight of nodes in G' is at most K. Show that this problem is NP-Complete. (**Hint:** *Reduce from the set cover problem.*)

Problem 2
In the optimization version of the above problem, the objective is to find such a subgraph G' with minimum total node weight. First show that you can assume, without loss of generality, that all terminals are leaves of weight 0 in G.

We now describe a greedy algorithm for this problem. Define a spider to be a tree such that at most one vertex has degree greater than 2.

i. Initially, let G_0 be the set terminals T with no edge.

ii. At the i-th step, choose a subgraph S of G such that

- S is a spider
- all leaves of S are terminals
- S has the minimum cost-benefit ratio $\frac{C(S)}{B(S)}$. Here the cost $C(S)$ is the total node weight of S and the benefit $B(S) = \alpha(G_i) - \alpha(G_i \cup S)$, where $\alpha(G)$ denotes the number of connected components in graph G.

Let $G_{i+1} = G_i \cup S$. If G_{i+1} connects all terminals, stop the algorithm and output G_{i+1}; else go to ii.

Answer the following questions about this greedy algorithm.

(1) Give a polynomial time implementation of this greedy algorithm.

(2) Show that this algorithm returns an $O(\log \kappa)$-approximate solution, where $\kappa = |T|$.

(**Hint:** *Think of the analysis of the greedy algorithm for the set cover algorithm.*)