Lecture 3.5

Tuesday, September 10, 2013
1:57 AM

Capacity Scaling (Weakly polynomial Algorithms)

$\Delta \leftarrow U$

while $\Delta \geq 1$ (integer capacities)

  $G(f, \Delta)$: residual network with $u_f \geq \Delta$ edges

  while $\exists$ (s, t) path in $G(f, \Delta)$

    send Δ flow on s-t path

    recompute $G(f, \Delta)$

  end while

  $\Delta \leftarrow \Delta / 2$

endwhile

Lemma: Inner loop executes $\leq 2m$ times

Proof: When inner loop exits, total remaining flow $< \Delta \cdot m$ by flow decomposition

$\Rightarrow$ In next round, at most $2m$ steps

Theorem: Overall: $O(mn \log U)$ time

Proof: Outer loop executes $O(\log U)$ times and inner loop $O(m)$ times in each iteration of outer loop. Finding an s-t path and updating $G(f, \Delta)$ can be done in $O(n)$ amortized time.