Online Machine Scheduling for Unrelated Machines

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<u>Problem</u>: A set of offline machines M (we will index by i)

A set of online jobs J (we will index by j)

p_{ij} : processing time of job j on machine i

Goal: Assign jobs to machines to minimize <u>makespan</u>.

Makespan = Max over all machines $\{$ Load on machine i $\}$

Load on machine i = Sum of p_{ij} of all jobs j assigned to i.

We will use the potential function

$\phi_i = a^{L_i}$ where $a > 1$ is a constant we will fix later and L_i is the load on machine i

$\phi = \sum_{i \in M} \phi_i$ is the overall potential

We will also assume that optimal makespan is 1 (by guessing and scaling)

<u>Algorithm</u>: Assign j to the machine that minimize the increase of ϕ, i.e. assign to $\underset{i \in M}{\operatorname{argmin}} \left(a^{L_i + p_{ij}} - a^{L_i}\right)$.

Let $L_i(j)$ = Load on machine i after job j is assigned by algo

$OPT(j)$ = machine that a fixed optimal solution (OPT) assign job j to.

Then, change in potential in algo due to job j

$$\Delta\phi(j) \leq a^{L_{OPT(j)}(j-1)}\left(a^{p_{OPT(j)j}} - 1\right)$$

$$\leq a^{L_{OPT(j)}(n)}\left(a^{p_{OPT(j)j}} - 1\right)$$

$$\leq a^{L_{OPT(j)}(n) + \sum_{j' < j :\, OPT(j') = OPT(j)} p_{OPT(j)j'}}\left(a^{p_{OPT(j)j}} - 1\right)$$

$$= a^{L_{OPT(j)}(n) + \sum_{j' \le j:\ OPT(j')=OPT(j)} P_{OPT(j)j'}} - a^{L_{OPT(j)}(n) + \sum_{j' < j:\ OPT(j')=OPT(j)} P_{OPT(j)j'}}$$

For some $i \in M$, by telescoping

$$\sum_{j:\ OPT(j)=i} \Delta\phi(j) \le a^{L_i(n) + L_i^{OPT}} - a^{L_i(n)}$$

$$= a^{L_i(n)}\left(a^{L_i^{OPT}} - 1\right)$$

$$\le a^{L_i(n)}\left(a - 1\right)$$

$$\sum_{i \in M} \sum_{j:\ OPT(j)=i} \Delta\phi(j) \le (a-1) \sum_{i \in M} a^{L_i(n)} = (a-1)\phi$$

$$\Rightarrow \phi - \phi_0 \le (a-1)\phi$$, where ϕ_0 = initial value of ϕ $= m$

$$\Rightarrow (2-a)\phi \le m$$

$$\Rightarrow \phi = \frac{m}{2-a} = O(m)$$ (choose $1 < a < 2$ and bounded away from 2)

$$\Rightarrow \max_i L_i(n) = O(\log m)$$