

Due Date : 9/18/2014

Problem 1: [10pts] Show that the set of optimal solutions of an LP is a convex set.

Problem 2: [15pts] **[Complementary Slackness]** Consider a canonical LP instance P : $\max c^T x$ such that $Ax \leq b$ and $x \geq 0$, and let D be its dual. Show that a pair x^*, y^* of feasible solutions in P and D , respectively are optimal if and only if

$$\begin{aligned} y_i^* \left(b_i - \sum_{j=1}^n a_{ij} x_j^* \right) &= 0, \\ x_j^* \left(c_j - \sum_{i=1}^m a_{ij} y_i^* \right) &= 0. \end{aligned}$$

(**Hint:** Use duality theorem.)

Problem 3: [15pts] Show that the primal-dual algorithm for the shortest-path problem described in the class is equivalent to Dijkstra's algorithm, i.e., the sequence of edges chosen by the former is the same as that by the latter (assume there are no ties at any stage).

Problem 4: [15pts] Let $G = (A \cup B, E)$ with $|A| = |B|$ and $E \subseteq A \times B$ be a bipartite graph, and let $w : E \rightarrow \mathbb{R}_{\geq 0}$ be the edge-cost function. A subset $M \subseteq E$ is a *perfect matching* if every vertex is incident on exactly one edge of M . The minimum-weight perfect matching problem asks for computing a perfect matching of the minimum weight. Formulate this as an LP problem and write its dual.

Problem 5: [15pts] Show that if a bipartite graph G has a perfect matching, then there is an optimal solution for the LP of the minimum-weight bipartite matching that is integral. (**Hint:** Show that if there is a fractional optimal solution, then there exists a cycle in G such that every edge in the cycle has a fractional value. Now compute another optimal solution with fewer fractional values.)

Show that this is not true if G is not a bipartite graph. How will you modify the LP formulation so that it works for non-bipartite graphs?