Due Date: October 6, 2012

Remark: Prove the correctness of every algorithm and analyze its running time.

Problem 1: For any edge e in any graph $G = (V, E)$, let $G \setminus e$ denote the graph obtained by deleting e from G. Let $|V| = n$ and $|E| = m$.

Suppose you are given a directed graph G, in which the shortest path from vertex u to vertex v passes through all vertices in G. Give an $O(n \log n + m)$-time algorithm to compute the shortest path from u to v in $G \setminus e$, for every edge e of G. The algorithm should output a set of $|E|$ shortest-path distances, one for each edge of the input graph. All edge weights are non-negative. (**Hint:** *If an edge of the original shortest path is deleted, how do the old and new shortest paths overlap?*)

Problem 2: Formulate the max st-flow as an LP in a matrix form. Write its dual and argue that it is the same as computing a min st-cut.

Problem 3: A *cycle cover* of a given directed graph $G = (V, E)$ is a set of vertex-disjoint cycles that cover all the vertices. Describe and analyze an efficient algorithm to find a cycle cover for a given graph, or correctly report that no cycle cover exists. (**Hint:** *Use bipartite matching.*)

Problem 4: Suppose we are given an $n \times n$ square grid, some of whose squares are colored black and the rest white. Describe and analyze an efficient algorithm to determine whether tokens can be placed on the grid so that: (i) every token is on a white square; (ii) every row of the grid contains exactly one token; and (iii) every column of the grid contains exactly one token.

Your input is a two dimensional array ISWHITE $[1 \ldots n, 1 \ldots n]$ of booleans, indicating which squares are white. Your output is a single boolean.

Problem 5: Given two graphs $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$ and a budget b, the *maximum common subgraph* problem asks for computing two sets of nodes $V_1' \subseteq V_1$ and $V_2' \subseteq V_2$ whose deletion leaves at least b nodes in each graph and makes the two subgraphs identical. Show that this problem is NP-Complete.