## Due Date: November 20, 2014

Problem 1: Given an undirected graph $G=(V, E)$, the maximum cut problem asks for a partition of $V$ into sets $S$ and $V \backslash S$ so that the number of edges between these sets is maximized. Give a deterministic greedy algorithm that achieves an approximation factor of $1 / 2$, i.e., computes a cut whose value is at least $1 / 2$ times that of the maximum cut.

Problem 2: Consider the following problem : given a set system $(X, \mathcal{R})$ and a positive integer $k$, partition $\mathcal{R}$ into sets $\left\{R_{1}, R_{2}, \ldots, R_{k}\right\}$ such that the sum of the elements covered by each set of the partition, i.e., $\sum_{i=1}^{k}\left|\bigcup_{R \in R_{i}} R\right|$, is maximized. Formulate this problem as a linear program and give a randomized rounding scheme that computes a solution whose expected value is within $(1-1 / e)$ factor of the optimum.

Problem 3: Consider a randomized rounding algorithm for MAX SAT but use the linear function $f\left(y_{i}\right)=y_{i} / 2+1 / 4$ for rounding. Show that this gives a $3 / 4$-approximation algorithm for MAX SAT with probability at least $1 / 2$.

Problem 4: Consider a set system $H=(X, \mathcal{R})$ and a function $\chi: X \rightarrow\{+1,-1\}$. For $R \in \mathcal{R}$, define $\chi(R)=\sum_{x \in R} \chi(x)$ and $\operatorname{disc}(H, \chi)=\max _{R \in \mathcal{R}}|\chi(R)|$. The discrepancy of $H=(X, \mathcal{R})$ is then defined as

$$
\operatorname{disc}(H)=\min _{\chi} \operatorname{disc}(H, \chi)
$$

Prove that $\operatorname{disc}(H) \leq \sqrt{2 n \ln 2 m}$, where $n=|X|, m=|\mathcal{R}|$.
You can use the following version of Chernoff's inequality : let $Y=Y_{1}+\ldots+Y_{N}$, where the $Y_{i}{ }^{\prime}$ s are mutually independent random variables with $\operatorname{Pr}\left[Y_{i}=+1\right]=\operatorname{Pr}\left[Y_{i}=-1\right]=1 / 2$. Then, for any $t>0$ we have

$$
\operatorname{Pr}[Y>t]<\exp \left(-\frac{t^{2}}{2 N}\right) .
$$

Problem 5: Consider the MAX 2SAT problem, in which every clause has at most two literals.
(i) As in the case of the maximum cut problem, express the MAX 2SAT problem as an "integer quadratic program" in which the only constraints are $y_{i} \in\{-1,1\}$ and the objective function is quadratic in the $y_{i}$. (Hint: it may help to introduce a variable $y_{0}$ which indicates whether the value - 1 or 1 is "TRUE").
(ii) Derive a .878-approximation algorithm for the MAX 2SAT problem.

