Graph Algorithms & Iteration on Map-Reduce

CompSci 590.04

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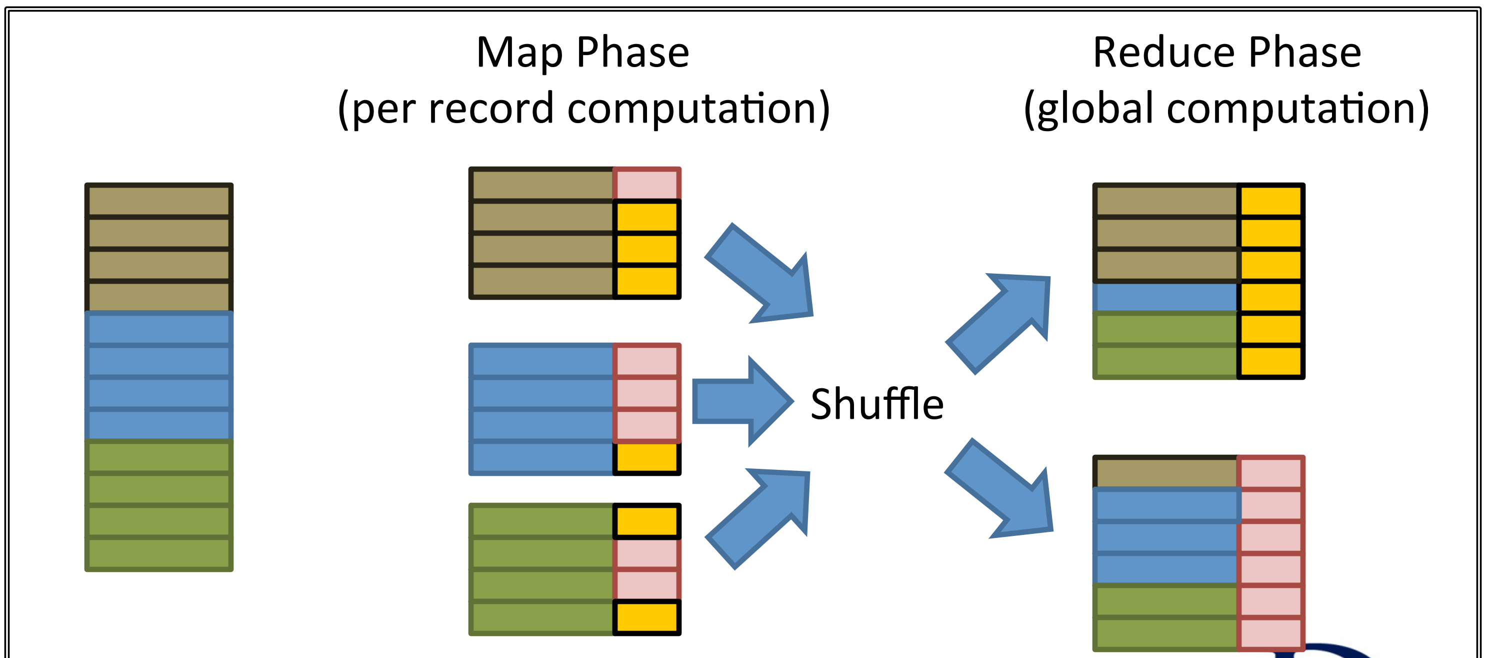
Recap: Map-Reduce

```
map     (k1,v1)        → list(k2,v2);
reduce  (k2,list(v2))  → list(k3,v3).
```



This Class

- Graph Processing
- Iterative-aware Map Reduce

GRAPH PROCESSING

Graph Algorithms

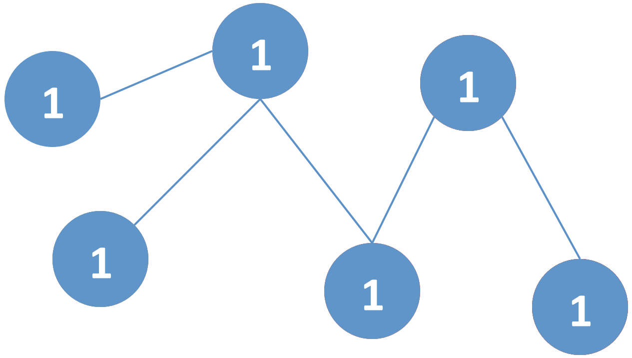
- Diameter Estimation
  - Length of the longest shortest path in the graph
- Connected Components
  - Undirected s-t connectivity (USTCON): check whether two nodes are connected.
- PageRank
  - Calculate importance of nodes in a graph
- Random Walks with Restarts
  - Similarity function that encodes proximity of nodes in a graph

Connected Components

- What is an efficient algorithm for computing the connected components in a graph?

HCC [Kang et al ICDM '09]

- Each node's label l(v) is initialized to itself
- In each iteration
  $l(v) = \min \{l(v), \min_{y \,\varepsilon\, neigh(v)} l(y)\}$



- O(d) iterations (d = diameter of the graph)
  O(|V| + |E|) communication per iteration

GIM-V

- Generalized Iterative Matrix-Vector Multiplication

Connected Components

- Let c^h denote the component-id of a vertex in iteration h

- $c^{h+1} = M \times_G c^h$
  - $c^{new}[i] = \min_j(m[i,j] \times c^h[j])$
  - $c^{h+1}[i] = \min(c^h[i], c^{new}[i])$

Step 1: Generate m[i,j] x c[j]
Step 2: Aggregate to find the min for each node

- Keep iterating till $c^{h+1} = c^h$.

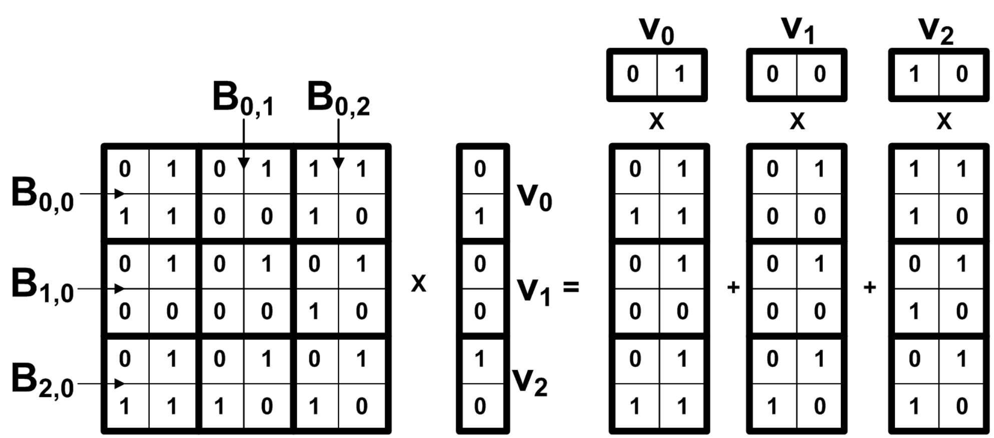
GIM-V and Page Rank

$$p = (cE^T + (1-c)U)p$$

- $p^{next} = M \times_G p^{cur}$
- $p^{next}[i] = (1-c)/n + \text{sum}_j\,(c \times m[i,j] \times p^{cur}[j])$

GIM-V BL

- We assumed each edge in the graph is represented using a different row.
- Can speed up processing if each row represents a bxb sub matrix



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Connected Components

- Iterative Matrix Vector products need O(d) map reduce steps to find the connected components in a graph.

- Diameter of a graph can be large.
  - > 20 for many real world graphs.

- Each map reduce step requires writing data to disk + remotely reading data from disk (I/O + communication)

- Can we find connected components using a smaller number of iterations?

Hash-to-all

- Maintain a cluster at each node
  - Current estimate of connected component

- Initialize cluster(v) = Neighbors(v) U {v}

- Each node sends its cluster to all nodes in the cluster
  - Map: (v, C(v)) → {(u, C(v))} for all u in C(v)

- Union all the clusters sent to a node v
  - Reduce: (u, {C1, C2, …, Ck}) → (u, C1 U C2 U … U Ck)

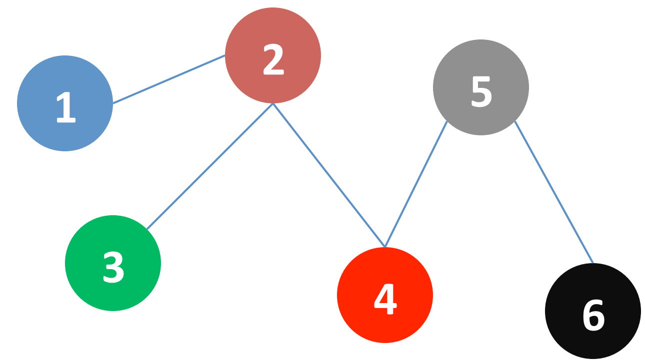
Hash-to-all

- Number of rounds = log d
  - Proof?

- Communication per round = O(n|V| + |E|)
  - Each node is replicated at most n times, where n is the maximum size of a connected component.

Hash-to-Min

- Each node *v* maintains a cluster C(*v*) which is initialized to
  {*v*} U Neighbors(*v*)



- In each iteration

  Map:

  v_{min} = min {C(v)}
  Send C(v) to v_{min}
  Send v_{min} to nodes in C(v)

  Reduce:

  C(v) is the union of all incoming clusters

Hash-to-Min

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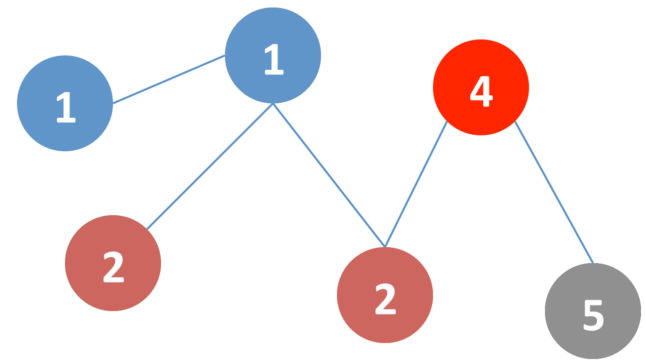

- In each iteration

  Map:

  v_{min} = min {C(v)}
  Send C(v) to v_{min}
  Send v_{min} to nodes in C(v)

  Reduce:

  C(v) is the union of all incoming clusters

v	C(v)
1	1,2
2	1,2,3,4
3	2,3
4	2,4,5
5	4,5,6
6	5,6

Hash-to-Min

- Each node *v* maintains a cluster C(*v*) which is initialized to
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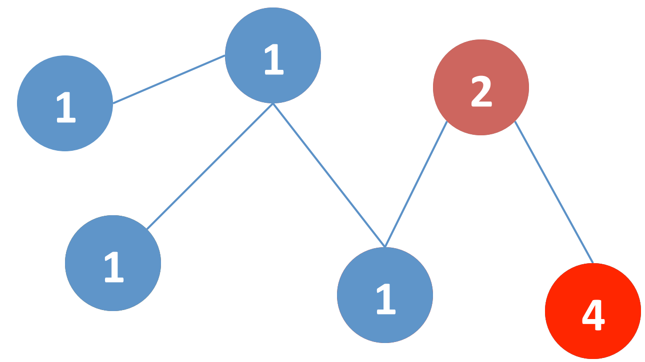

- In each iteration

  Map:

  v_{min} = min {C(v)}
  Send C(v) to v_{min}
  Send v_{min} to nodes in C(v)

  Reduce:

  C(v) is the union of all incoming clusters

v	C(v)
1	1,2,3,4
2	1,2,3,4,5
3	1
4	1,4,5,6
5	2
6	4

Hash-to-Min

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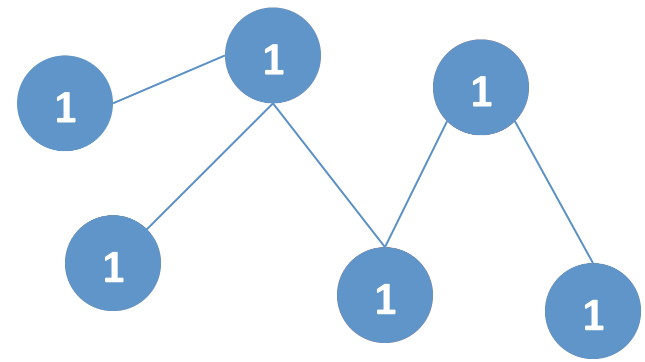
  {*v*} U Neighbors(*v*)



- In each iteration

  Map:

  v_{min} = min {C(v)}
  Send C(v) to v_{min}
  Send v_{min} to nodes in C(v)

  Reduce:

  C(v) is the union of all incoming clusters

v	C(v)
1	1,2,3,4,5,6
2	1
3	1
4	1
5	1
6	1

Hash-to-Min

- In the end, cluster of vertex with minimum id contains the entire connected component.
  Cluster of other vertices in the component is a singleton having the minimum vertex.

- Communication cost: Assuming a random assignment of ids to vertices, expected communication cost is O(k(|V| + |E|)) in iteration k

- Number of iterations: ???
  - On a path graph: 4 log n
  - In a general graph: Can be as big as d

Leader Algorithm

- Let π be an arbitrary total order over the vertices.
- Begin with l(v) = v, and all nodes active

In each iteration:

- Let C(v) be the connected component containing v
- Let Γ(v) be the neighbors of C(v) that are not in C(v)
- Call each active node a leader with probability ½.
- For each active non-leader w, find w* = min(Γ(w))
- If w* is not empty and l(w*) is a leader,
  then mark w as passive, and
  relabel each node with label w by l(w*)

Correctness

- If at any point of time two nodes s and t have the same label, then they are connected in G.

- Consider an iteration, when $l(s) \neq l(t)$ before the iteration, but $l(s) = l(t)$ after.
- This means, $l(s) = w$ (non-leader node), $l(t) = w^*$
- By induction, s is connected to all nodes in $\Gamma(w)$,
  t is connected to all nodes in $\Gamma(w^*)$, and
  w is connected to w^*.
- Therefore, s and t are connected.

Number of Iterations

- Every connected component has a unique label after O(log N) rounds with high probability
- Suppose there is some connected component with two active labels.
- An active label w survives an iteration if:
  1. w is marked a leader
  2. w is not marked a leader and l(w*) is not marked a leader
- Hence, in every iteration, the expected number of active labels reduces by ¼.

ITERATION AWARE MAP-REDUCE

Iterative Computations

PageRank:

do

$p^{next} = (cM + (1\text{-}c)\ U)p^{cur}$

while(p^{next} != p^{cur})

- Loops are not supported in Map-Reduce
  - Need to encode iteration in the launching script
- M is a loop invariant. But needs to written to disk and read from disk in every step.
- M may not be co-located with mappers and reducers running the iterative computation.

HaLoop

- Iterative Programs

$$R_{i+1} = \underbrace{R_0}_{\text{Initial Relation}} \cup (R_i \bowtie \underbrace{L}_{\text{Invariant Relation}})$$

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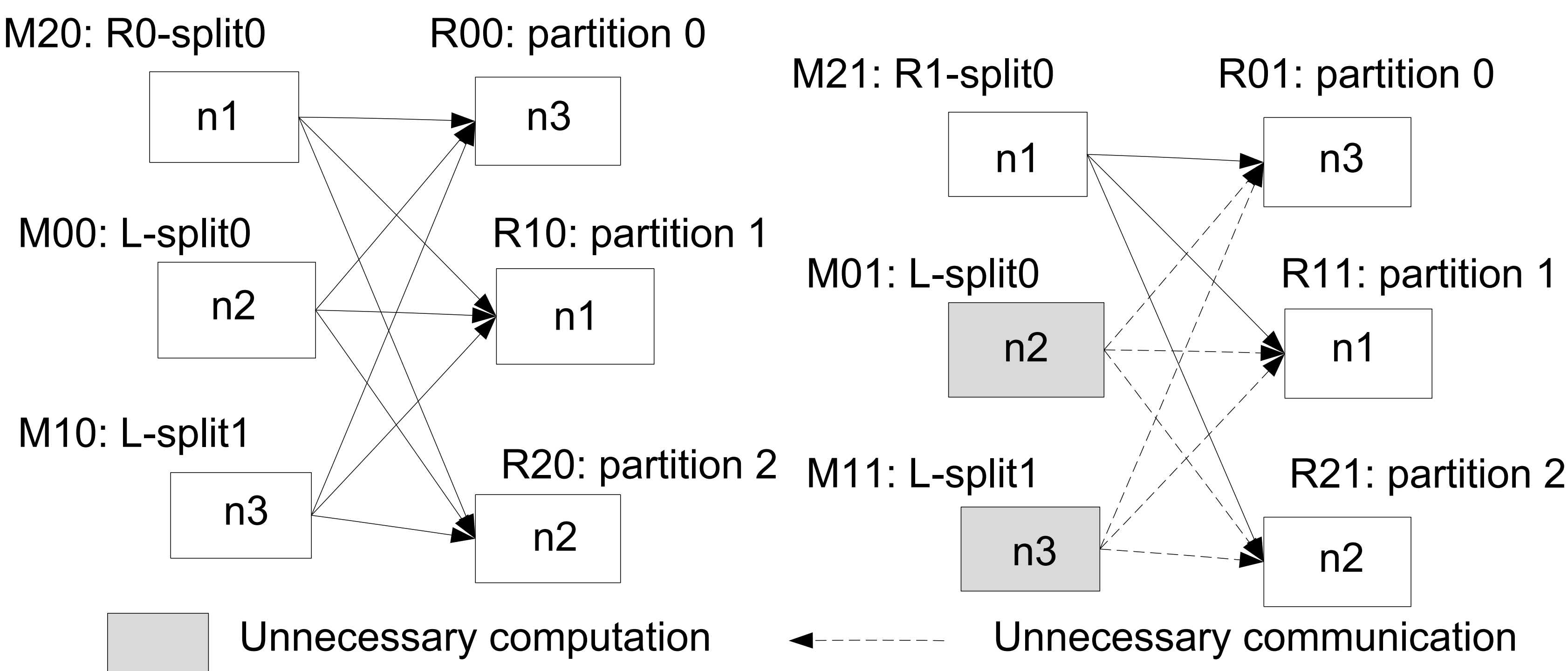
Loop aware task scheduling

- Inter-Iteration Locality
- Caching and Indexing of invariant tables



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Summary

- No native support for iteration in Map-Reduce
  – Each iteration writes/reads data from disk leading to overheads

- Many graph algorithms need iterative computation
  – Need to design algorithms that can minimize number of iterations

- New frameworks that minimize overheads by caching invariant tables in the iterative computation
  – HaLoop