COMPSCI 330 Lecture 1: Introductions

Monday, August 29, 2016 1:53 PM

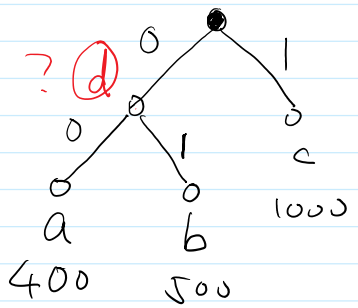
- Design and Analysis of Algorithms
  - Algorithm: precise instructions on how to perform a task.
  - Design: solve hard problem using basic operations
    - abstraction
    - reduction
      - divide and conquer
      - dynamic programming
      - greedy
  - Analysis

    recall: Huffman tree

a b c b a

↓ ↓ ↓ ↓ ↓

00 01 1 01 00



  - Correctness: why can this tree be used for encoding/dec?
  - optimality: why merge two least frequent characters?

    (it produces the shortest encoding)
  - time/space complexity: how much time/memory is needed to run the alg with n characters.

(- robustness - fairness - privacy ...)

- Asymptotic Analysis
  - Def. $f(n) = O(g(n))$, if $\exists$ constant $C>0$ such that $\forall\ n>0\ f(n) \le C\,g(n)$

    $f(n) = \Omega(g(n))$ if $\exists$ constant $C>0$ such that $\forall\ n>0\ f(n) \ge C\,g(n)$

    $f(n) = \Theta(g(n))$ if $\exists$ constants $C_1, C_2>0$ such that $\forall\ n>0\ C_1\,g(n) \le f(n) \le C_2\,g(n)$
  - Property: ① small value n does not matter

    $f(n) = 10,000\,n \quad g(n) = n^2$

    still $f(n) = O(g(n))$

    ② drop the insignificant term

    $3n^3 + 50n^2 + 1000n + 250 = O(n^3)$

    $1 < \log n < \sqrt{n} < n < n^2 < n^3 < 2^n < 3^n < \dots$
  - Why asymptotic?
    - lazy

      $\log n + \log(n-1) + \dots + \log 1 = \Theta(n \log n)$

      $\ge \frac{n}{2} \log \frac{n}{2} = \frac{n}{2}\log n - \frac{n}{2}$
    - robust
    - easy to compare:
- Euclid's Algorithm: Greatest Common Divisor (gcd)
  - Given: a, b nonnegative integers
  - Goal: find the largest c such that c is a divisor of both a, b.

    ($\gcd(0,0) = 0$)
  - $\gcd(100, 30)$

    ↓

    $\gcd(30, 10)$

    ↓

    $\gcd(10, 0) = 10$

- Correctness:

- Lemma: gcd always terminates

Hypothesis: if $a+b \leq n$, then gcd terminates.

Base case: $n = 0$

$a = b = 0$ ✓

induction: assume hypothesis hold for n,

consider $a+b = n+1$ (without loss of generality $a \geq b$)

if $a = n+1$, $b = 0$, ✓

otherwise $b + (a \bmod b) < a+b = n+1$

by induction hypothesis, $\gcd(b, a \bmod b)$ terminates ▨

```
gcd(a,b)
  if a<b then
     swap(a, b)
  if b==0 then
     return a
  else return
     gcd(b, a mod b)
```

- Lemma: gcd(a,b) is correct.

if $b = 0$ then $\gcd(a, 0) = a$

if $b \neq 0$ let $a \bmod b = a - kb$ (k: integer)

if c is common divisor of a, b,

$$\frac{a \bmod b}{c} = \frac{a-kb}{c} = \underbrace{\frac{a}{c} - k \cdot \frac{b}{c}}_{\text{integer}}$$

$\Rightarrow$ c is a c.d. of b, $a \bmod b$

if c is c.d. of b, $a \bmod b = a - kb$

$$\frac{a}{c} = \frac{(a-kb)+kb}{c} = \frac{a-kb}{c} + k \cdot \frac{b}{c}$$

$\Rightarrow$ c is a c.d. of a, b

$\Rightarrow \gcd(a, b) = \gcd(b, a \bmod b)$ ▨