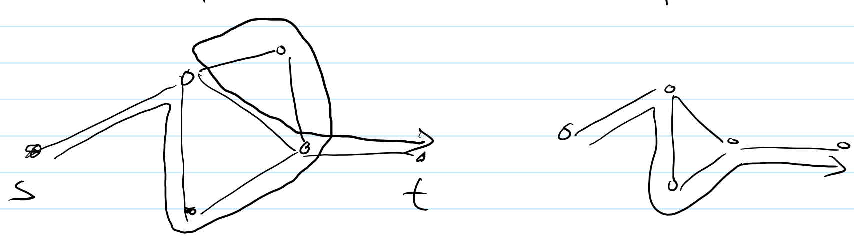
COMPSCI 330 Lecture 24 Complexity Classes and Polynomial Time Reductions

Monday, November 28, 2016 2:55 PM

- Easy problems vs. Hard Problems
- P vs. NP
- Cook-Levin Theorem

- easy problems vs. hard problems
  - easy problem: solvable in polynomial time ($O(n)$, $O(n\log n)$ $O(n^{10})$)
  - hard problem: problems that (we believe) cannot be solved in polynomial time.
- Eulerian path vs. Hamiltonian path
  - Given graph G (undirected), decide whether there is a path from s to t that uses every (a) edge exactly once.
    (b) vertex

(a) Eulerian path (b) Hamiltonian path

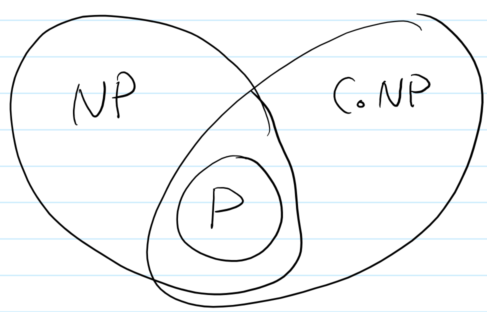


  - Claim: There is an Eulerian path iff graph G is connected and every vertex (except for s, t) has even degree, s, t, have odd degree.
    $\Longrightarrow$ Eulerian Path is easy.
  - Hamiltonian path is hard (formalized later).
- Decision Problem
  - Problems that require a yes/no answer.

    decision problems vs. optimization problems

    Is there a spanning tree with cost ≤ 100? MST
  - approach: classify decision problems into complexity classes.

- P : easy problems: can be solved in polynomial time.
- NP (nondeterministic polynomial time) Given an input x, there is a polynomial time verifier V s.t. given a "proof" C,
  if answer to x is yes, there is a proof C s.t. $V(x, C) = \text{true}$
  if answer to x is no, then for any proof C, $V(x, C) = \text{false}$.
- NP: easy to verify whether a solution is correct,
  but potentially hard to find a solution.
  - $P \subseteq NP$ (need no proof, V is just the algorithm)
- co NP: "negation" of NP, can verify the answer is no.
  - Example: Whether x is a prime.
    can "prove" x is not a prime by writing $x = y \cdot z$, $y, z \neq 1$
    "not hamiltonian path"



- P vs. NP : Is $P = NP$?
  - Solving homework problem / finding a proof are in NP.
- harder problems
  - example: chess (PSPACE)

- Q: How do we decide problem A is harder than problem B?
- A: Reductions
- We say there is a polynomial time reduction from B to A, if for any instance x of B, there is a polynomial time algorithm f that maps x to $y = f(x)$, y is an instance of A, and answer for y is the same as answer for x.

- In this case we say A is harder (or at least not simpler) than B

$(A \geq B)$

$$\begin{array}{ccc} A & & B \\ f(x) & \xleftarrow{f} & x \end{array}$$

if we have an algorithm for A (Q), then $Q(f(x))$ is an algorithm for B.

- NP-hardness: problem A is NP-hard, if for any NP problem B, can reduce B to A.

  $A \geq B$ for any $B \in NP$

  if A is also in NP, then we call A NP-complete.

- if A can be solved in poly time, then $P = NP$.
  - Hamiltonian Path is an NP complete problem.

- Cook-Levin Theorem: For any problem L in NP, there is a polynomial time reduction from L to CIRCUIT-SAT (SAT, 3-SAT).