- Problem 1 Use dynamic Programming

want to compute $Pr[D \geq \frac{m}{2}]$

if only the last state (with v_n votes) is undecided

$Pr[D_n = x]$ — with Probability P_n → $Pr[D_{n-1} = x - v_n]$ in first n-1 states

$Pr[D_n = x]$ — $1-P_n$ → $Pr[D_{n-1} = x]$ in first n-1 states

D_i : # of votes D gets in first i states

$$Pr[D_n = x] = P_n \, Pr[D_{n-1} = x - v_n] + (1-P_n) \, Pr[D_{n-1} = x]$$

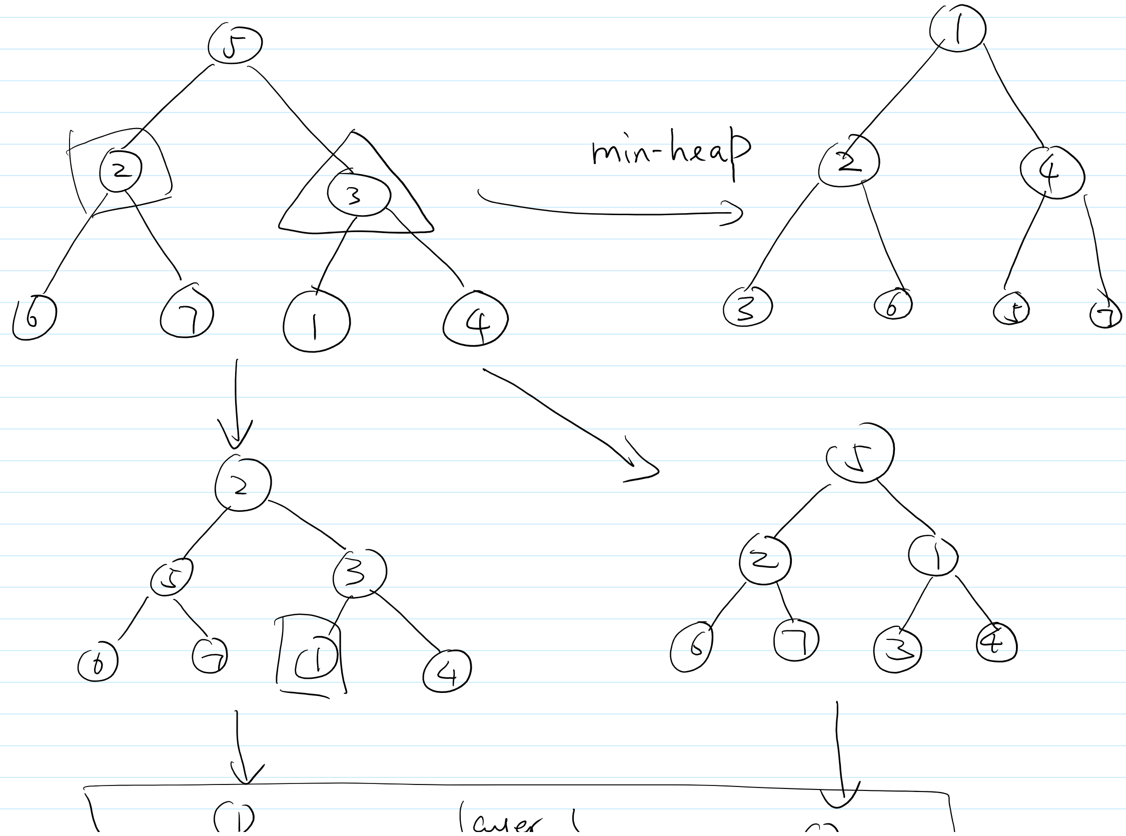
$A[i,j] = Pr[D_i = j]$ (D gets exactly j votes in first i states]

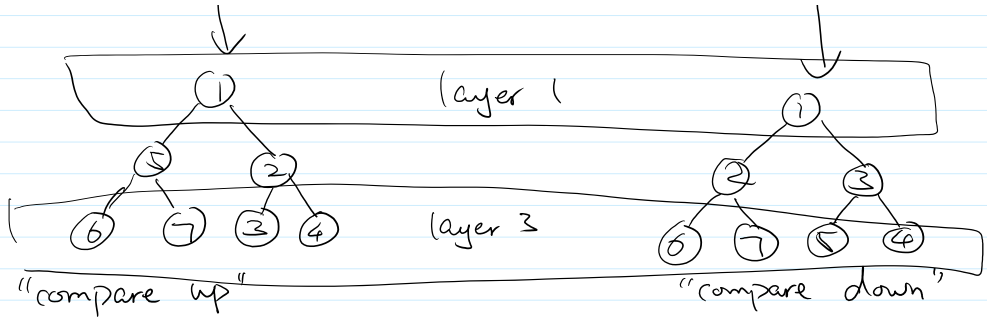
$$A[i,j] = P_i \, A[i-1, j-v_i] + (1-P_i) \, A[i-1, j]$$

$A[0,0] = 1 \quad A[0,j] = 0$ for all $j \neq 0$

Find output $\sum_{j=\frac{m}{2}}^{m} A[n,j]$ $\square$

2.





node at layer i takes $O(i)$ time

$$\text{total cost} = \sum_{i=1}^{\log_2 n} 2^{i-1} \cdot O(i)$$

nodes in layer i

$$\geq 2^{\log_2 n - 1} \cdot (\log_2 n) \quad \text{(last layer)}$$

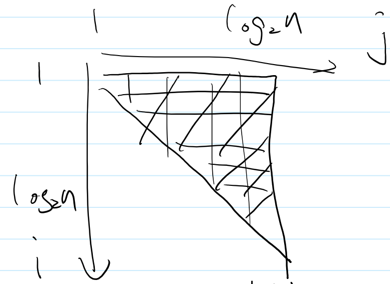
$$= \frac{n}{2} \log_2 n$$

node at layer i takes $\log_2 n - i + 1$

$$\text{total cost} = \sum_{i=1}^{\log_2 n} 2^{i-1} \cdot (\log_2 n - i + 1)$$

$$= \sum_{i=1}^{\log_2 n} 2^{i-1} \left(\sum_{j=i}^{\log_2 n} 1 \right)$$

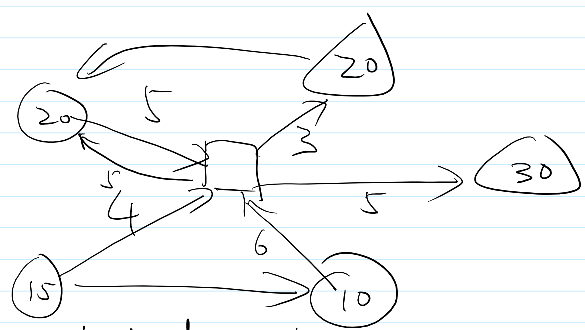
$$= \sum_{i=1}^{\log_2 n} \sum_{j=1}^{i} 2^{j-1}$$



$$= \sum_{i=1}^{\log_2 n} (2^i - 1)$$

$$= 2^{\log_2 n + 1} - \log_2 n - 1$$

$$= 2n - \log_2 n - 1$$



neighborhoods   parking lots

$x_{(u,v)}$ amount of people using edge u,v

for all neighborhoods u

$\sum$ outgoing $- \sum$ incoming $= \#$ people

for all parking lot v

$\sum$ incoming $- \sum$ outgoing $\leq$ capacity

for other nodes

$\sum$ incoming $= \sum$ outgoing

$\sum$ incoming $-$ $\sum$ outgoing $\leq$ capacity

for other nodes

$\sum$ incoming $=$ $\sum$ outgoing $\quad \sum_{v:(u,v)\in E} X_{(u,v)}$ (outgoing edges for u)

for each road

$0 \leq X_{u,v} \leq \text{capacity} \cdot \underline{\underline{t}}$

time

$-$ $\min t$

-4: $(x_1 \vee \overline{x_2} \vee x_3) \wedge (x_2 \vee x_4 \vee \overline{x_5})$

- for each var $x_1^2 - x_1 = 0$

$(1-x_1)\, x_2\, (1-x_3) = 0$ (not quadratic)

$y_1 = (1-x_1)\, x_2$

$y_1 (1-x_3) = 0$

- How to write the solution:

We will reduce 3-SAT to QUADRATIC EQUATIONS

① Given a 3-SAT formula with m clauses and n variables for each variable x_i $(i=1,2,\cdots,n)$, create a variable z_i in quadratic equations and add an equation $z_i^2 - z_i = 0$ (1)

for each literal $l = x_i$ or $\overline{x_i}$, associate a linear term $f(l) = 1 - z_i$ for $l = x_i$ and $f(l) = z_i$ for $l = \overline{x_i}$

for a clause $C_i = l_{i,1} \vee l_{i,2} \vee l_{i,3}$, create a variable y_i, and add equations

$$y_i = f(l_{i,1})\, f(l_{i,2}), \quad y_i f(l_{i,3}) = 0$$

as an example if $C_1 = x_1 \vee \overline{x_2} \vee x_3$, the corresponding equations are

$$y_1 = (1-z_1)\, z_2 \; (2) \quad y_1 (1-z_3) = 0 \; (3)$$

Now we have converted a 3-SAT instance to a QUADRATIC EQUATION instance

② Claim: If the 3-SAT instance has a solution, then the QUADEQ problem also has a solution.

Proof: The solution would be $z_i = x_i$, and for clause $C_j = l_{j,1} \vee l_{j,2} \vee l_{j,3}$

$$y_j = f(l_{j,1}) f(l_{j,2})$$

Clearly, equations (1) (2) are all satisfied. We only need to show equations of the form (3) are also satisfied.

For each clause C_j, we know at least one of the literals is true.

If $l_{j,3}$ is true, then $f(l_{j,3}) = 0$ and $y_j f(l_{j,3}) = 0$

If $l_{j,1}$ or $l_{j,2}$ is true, then $y_j = 0$ and $y_j f(l_{j,3}) = 0$

in any case equations of the form (3) are also satisfied.

③ Claim: If the QUADEQ problem has a solution, then the 3-SAT problem also has a solution.

Proof: The solution is just $x_i = z_i$. This is valid because $z_i = 0$ or 1 by equations of type 1.

For any clause C_j, we know

$$y_j = f(l_{j,1}) f(l_{j,2}) \quad y_j f(l_{j,3}) = 0$$

so $$f(l_{j,1}) f(l_{j,2}) f(l_{j,3}) = 0$$

as a result one of $f(l_{j,p}) = 0$ for $p = 1, 2,$ or 3.

For a literal $f(l) = 0$ if and only if it is satisfied.

Therefore at least one literal is satisfied for every clause $\square$