

COMPSCI330 Design and Analysis of Algorithms

Assignment 0

Just an Example, No Due Date

Problem 1 (Induction). Prove the following formulas using induction

1. $\sum_{i=0}^n 2^i = 2^{n+1} - 1$.
2. $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$.

Problem 2 (Euclid's Algorithm). For two nonnegative integers a and b , their Greatest Common Divisor (GCD) is the largest integer c such that c divides both a, b (both $a/c, b/c$ are integers). Euclid's algorithm is a 2000+ year-old algorithm that can compute the GCD efficiently. The algorithm can be simply described as below:

```
GCD(a,b)
  If b == 0 then
    return a
  Else
    return GCD(b, a % b)
  End If
```

Here $a\%b$ computes the *remainder* of a being divided by b .

- (a) Show that $GCD(a, 0) = a$ if a is a positive integer.
- (b) Show that for any two positive integers a, b , $GCD(a, b) = GCD(b, a\%b)$.
- (c) Show that Euclid's algorithm computes $GCD(a, b)$ in time $O(\log(a+b))$ assuming computing $a\%b$ takes 1 unit of time. (Hint: Do an induction on $a + b$.)