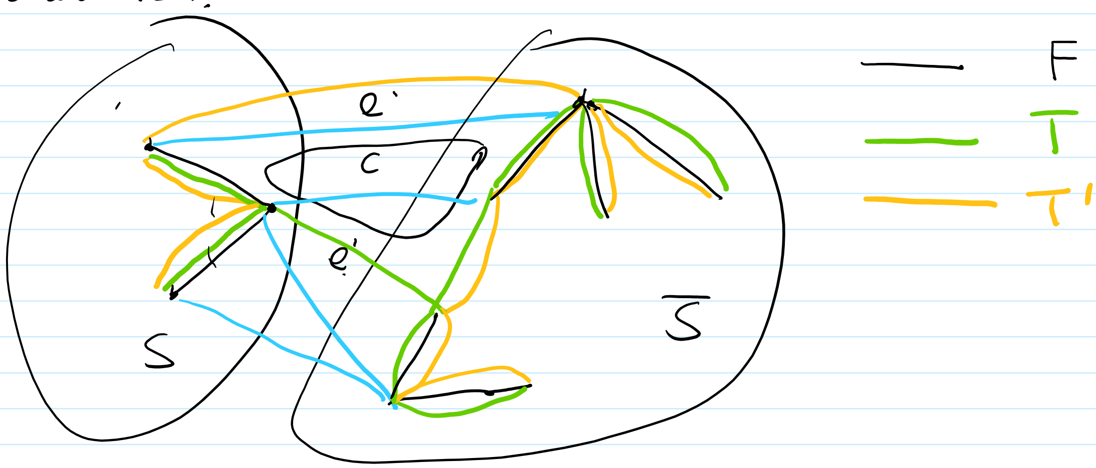
Lecture 15 Minimum Spanning Tree (cont'd)

Thursday, October 26, 2017 2:40 PM

- Key Property: Suppose we have a set of edge F, F is a subset of edges in T, and T is a MST. If there is a cut $(S, \bar{S})$ that does not contain any edge in F, let e be the min cost edge in the cut $(S, \bar{S})$, then $F \cup \{e\}$ is a subset of T', where T' is also an MST.



Proof: if the edge e is in T, then $F \cup \{e\} \subseteq T$
we can choose $T' = T$ and property is true.

if the edge e is not in T (want to find an MST that includes e)

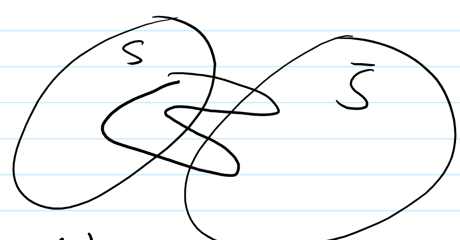
add e to T, this creates a cycle C

cycle C will contain at least one other edge that crosses the cut

assume e' is one of them.

by assumption that $(S, \bar{S})$ does not contain any edge in F

$e' \notin F$



also, since e is the min cost edge $w(e) \leq w(e')$

let $T' = T \cup \{e\} \setminus \{e'\}$

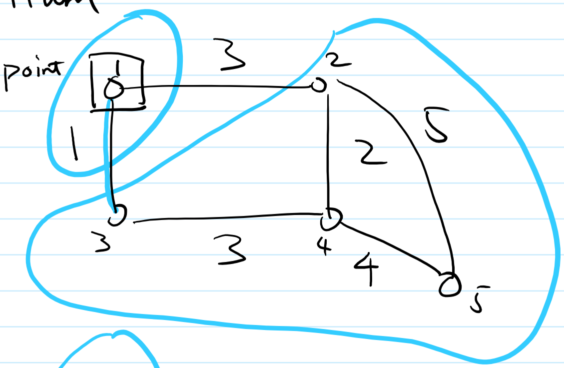
easy to see $F \cup \{e\} \subseteq T'$

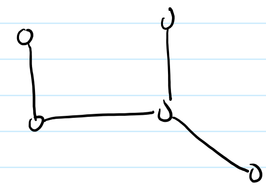
and $w(T') = w(T) + w(e) - w(e') \leq w(T)$

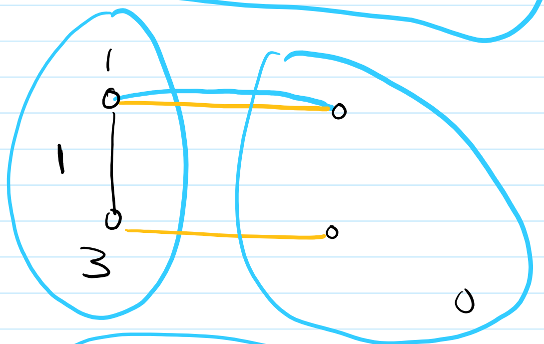
because T is an MST, T' must also be an MST. $\square$

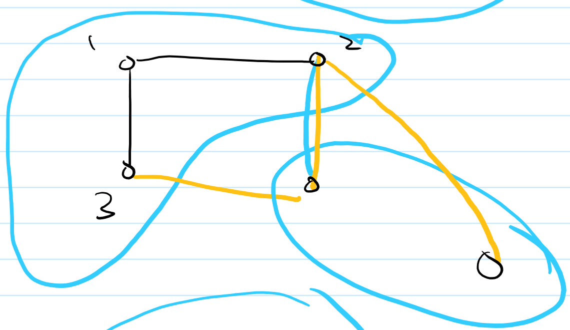
- Prim's algorithm

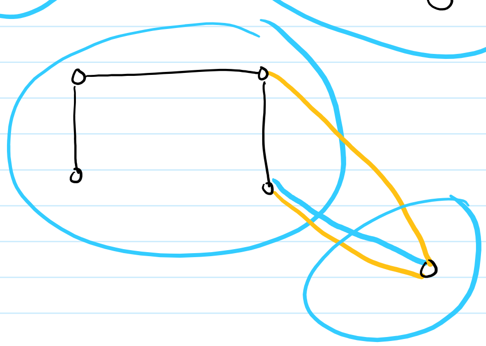
starting point

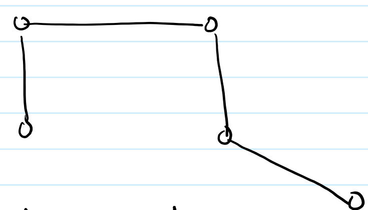








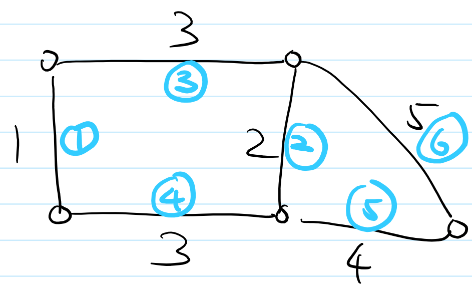




Keep set S of visited vertex

for any $v \notin S$, keep track of $dis[v]$ = min edge cost from v to a vertex $u \in S$

– Kruskal algorithm



1 2 . . . 6

1

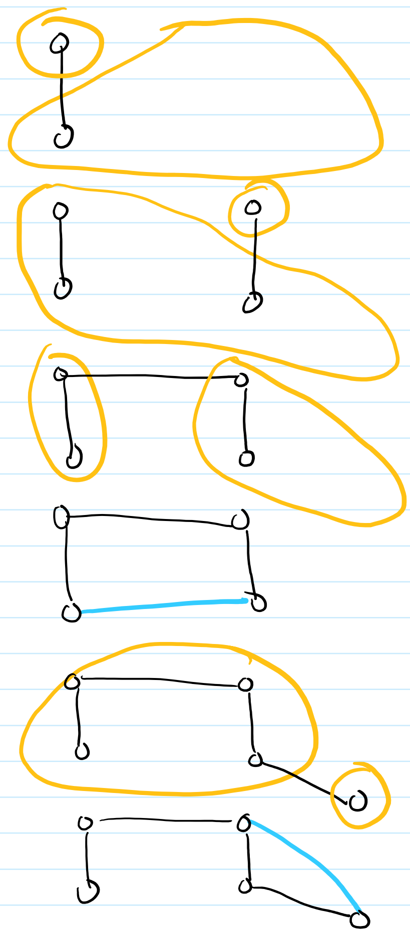
2

3

4

5

6

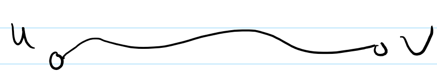


Proof of Kruskal's algorithm:

whenever edge $e=(u,v)$ is added by Kruskal

want to find a cut $(S, \bar{S})$ that doesn't contain previous edges and e is the min cost edge in $(S, \bar{S})$

if (u,v) is added, then u,v are in different components



let S be the connected component (using previous edges) containing u

because S is a connected component, $(S,\bar{S})$ has no previous edges.

assume towards contradiction that there is $e' \in (S,\bar{S})$

$w(e') < w(e)$

e' will be considered before e (sorting)

by Kruskal e' will be added to the tree, contradiction. $\square$