CompSci 516
Data Intensive Computing Systems

Lecture 4
Relational Algebra
and
Relational Calculus

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Today's topics

- Finish NULLs and Views in SQL from Lecture 3
- Relational Algebra (RA) and Relational Calculus (RC)
- Reading material
  - [RG] Chapter 4 (RA, RC)
  - [GUW] Chapters 2.4, 5.1, 5.2

Acknowledgement:
The following slides have been created adapting the instructor material of the [RG] book provided by the authors Dr. Ramakrishnan and Dr. Gehrke.

Nulls and Views in SQL

Null Values

- Field values in a tuple are sometimes
  - unknown, e.g., a rating has not been assigned, or
  - inapplicable, e.g., no spouse's name
  - SQL provides a special value null for such situations.

Standard Boolean 2-valued logic

- True = 1, False = 0
- Suppose X = 5
  - (X < 100) AND (X >= 1) is …
  - (X > 100) OR (X >= 1) is …
  - (X > 100) AND (X >= 1) is …
  - NOT(X = 5) is …
- Intuitively,
  - T = 1, F = 0
  - For V1, V2 $\in$ {1, 0}
  - V1 $\wedge$ V2 = MIN (V1, V2)
  - V1 $\vee$ V2 = MAX(V1, V2)
  - $\neg$(V1) = 1 – V1

2-valued logic does not work for nulls

- Suppose rating = null, X = 5
- Is rating>8 true or false?
- What about AND, OR and NOT connectives?
  - (rating > 8) AND (X = 5)?
- What if we have such a condition in the WHERE clause?

3-Valued Logic For Null

- TRUE (= 1), FALSE (= 0), UNKNOWN (= 0.5)
  - unknown is treated as 0.5
- Now you can apply rules from 2-valued logic!
  - For V1, V2 ∈ {1, 0, 0.5}
  - V1 ⋀ V2 = MIN (V1, V2)
  - V1 ⋁ V2 = MAX(V1, V2)
  - ¬(V1) = 1 – V1
- Therefore,
  - NOT UNKNOWN = UNKNOWN
  - UNKNOWN OR TRUE = TRUE
  - UNKNOWN AND TRUE = UNKNOWN
  - UNKNOWN AND FALSE = FALSE
  - UNKNOWN OR FALSE = UNKNOWN

New issues for Null

- The presence of null complicates many issues. E.g.:
  - Special operators needed to check if value IS/IS NOT NULL
  - Be careful!
  - "WHERE X = NULL" does not work!
  - Need to write "WHERE X IS NULL"
- Meaning of constructs must be defined carefully
  - e.g., WHERE clause eliminates rows that don't evaluate to true
  - So not only FALSE, but UNKNOWNs are eliminated too
  - very important to remember!
- But NULL allows new operators (e.g. outer joins)
- Arithmetic with NULL
  - all of +, -, *, / return null if any argument is null
- Can force "no nulls" while creating a table
  - sname char(20) NOT NULL
  - primary key is always not null

Aggregates with NULL

sid	sname	rating	age
22	dustin	7	45
31	lubber	8	55
58	rusty	10	35

R1

- What do you get for
- SELECT count(*) from R1?
- SELECT count(rating) from R1?

Aggregates with NULL

sid	sname	rating	age
22	dustin	7	45
31	lubber	8	55
58	rusty	10	35

R1

- What do you get for
- SELECT count(*) from R1?
- SELECT count(rating) from R1?

sid	sname	rating	age
22	dustin	7	45
31	lubber	null	55
58	rusty	10	35

R2

- What do you get for
- SELECT count(*) from R2?
- SELECT count(rating) from R2?

Aggregates with NULL

- COUNT, SUM, AVG, MIN, MAX (with or without DISTINCT)
  - Discards null values first
  - Then applies the aggregate
  - Except count(*)
- If only applied to null values, the result is null

sid	sname	rating	age
22	dustin	7	45
31	lubber	null	55
58	rusty	10	35

R2

- SELECT sum(rating) from R2?
- Ans: …

sid	sname	rating	age
22	dustin	null	45
31	lubber	null	55
58	rusty	null	35

R3

- SELECT sum(rating) from R3?
- Ans: …

Overview: General Constraints

- Useful when more general ICs than keys are involved
- There are also ASSERTIONS to specify constraints that span across multiple tables
- There are TRIGGERS too : procedure that starts automatically if specified changes occur to the DBMS
  - see additional slides at the end

```
CREATE TABLE Sailors
    ( sid INTEGER,
    sname CHAR(10),
    rating INTEGER,
    age REAL,
    PRIMARY KEY (sid),
    CHECK ( rating >= 1
        AND rating <= 10 )
```

```
CREATE TABLE Reserves
    ( sname CHAR(10),
    bid INTEGER,
    day DATE,
    PRIMARY KEY (bid,day),
    CONSTRAINT noInterlakeRes
    CHECK ('Interlake' <>
        ( SELECT B.bname
        FROM Boats B
        WHERE B.bid=bid)))
```

Views

- A view is just a relation, but we store a definition, rather than a set of tuples

```
CREATE VIEW YoungActiveStudents (name, grade)
    AS SELECT S.name, E.grade
    FROM Students S, Enrolled E
    WHERE S.sid = E.sid and S.age<21
```

- Views can be dropped using the DROP VIEW command
- Views and Security: Views can be used to present necessary information (or a summary), while hiding details in underlying relation(s)
  - the above view hides courses "cid" from E

Can create a new table from a query on other tables too

SELECT… INTO…. FROM…. WHERE

```
SELECT S.name, E.grade
INTO YoungActiveStudents
 FROM Students S, Enrolled E
WHERE S.sid = E.sid and S.age<21
```


"WITH" clause – very useful!

- You will find "WITH" clause very useful!

```
WITH Temp1 AS
    (SELECT ….. ..),
    Temp2 AS
    (SELECT ….. ..)
SELECT X, Y
FROM TEMP1, TEMP2
WHERE….
```

- Can simplify complex nested queries

Summary

- SQL has a huge number of constructs and possibilities
  - You need to learn and practice it on your own
  - Given a problem, you should be able to write a SQL query and verify whether a given one is correct
- Pay attention to NULLs
- Can limit answers using "LIMIT" or "TOP" clauses
  - e.g. to output TOP 20 results according to an aggregate
  - also can sort using ASC or DESC keywords

Relational Query Languages

Relational Query Languages

- Query languages: Allow manipulation and retrieval of data from a database
- Relational model supports simple, powerful QLs:
  - Strong formal foundation based on logic
  - Allows for much optimization
- Query Languages != programming languages
  - QLs not intended to be used for complex calculations
  - QLs support easy, efficient access to large data sets

Formal Relational Query Languages

- Two "mathematical" Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:
  - Relational Algebra: More operational, very useful for representing execution plans
  - Relational Calculus: Lets users describe what they want, rather than how to compute it (Non-operational, declarative)
- Note: Declarative (RC, SQL) vs. Operational (RA)

Preliminaries

- A query is applied to relation instances, and the result of a query is also a relation instance.
  - Schemas of input relations for a query are fixed
    - query will run regardless of instance
  - The schema for the result of a given query is also fixed
    - Determined by definition of query language constructs
- Positional vs. named-field notation:
  - Positional notation easier for formal definitions, named-field notation more readable

Example Schema and Instances

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

R1

sid	bid	day
22	101	10/10/96
58	103	11/12/96

Logic Notations

- ∃ There exists
- ∀ For all
- ∧ Logical AND
- ∨ Logical OR
- ¬ NOT

Relational Algebra (RA)

Relational Algebra

- Takes one or more relations as input, and produces a relation as output
  - operator
  - operand
  - semantic
  - so an algebra!
- Since each operation returns a relation, operations can be composed
  - Algebra is "closed"

Relational Algebra

- Basic operations:
  - Selection (σ) Selects a subset of rows from relation
  - Projection (π) Deletes unwanted columns from relation.
  - Cross-product (x) Allows us to combine two relations.
  - Set-difference (-) Tuples in reln. 1, but not in reln. 2.
  - Union (∪) Tuples in reln. 1 or in reln. 2.
- Additional operations:
  - Intersection (∩)
  - join ⋈
  - division(/)
  - renaming (ρ)
  - Not essential, but (very) useful.

Projection

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

- Deletes attributes that are not in projection list.
- Schema of result contains exactly the fields in the projection list, with the same names that they had in the (only) input relation.

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

$\pi_{sname,rating}(S2)$

- Projection operator has to eliminate duplicates (Why)
  - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it (performance)

age
35.0
55.5

$\pi_{age}(S2)$

Selection

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

- Selects rows that satisfy selection condition

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

$\sigma_{rating>8}(S2)$

- No duplicates in result. Why?
- Schema of result identical to schema of (only) input relation

sname	rating
yuppy	9
rusty	10

$\pi_{sname,rating}(\sigma_{rating>8}(S2))$

Composition of Operators

- Result relation can be the input for another relational algebra operation
  - Operator composition

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

$\sigma_{rating>8}(S2)$

sname	rating
yuppy	9
rusty	10

$\pi_{sname,rating}(\sigma_{rating>8}(S2))$

Union, Intersection, Set-Difference

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

- All of these operations take two input relations, which must be union-compatible:
  - Same number of fields.
  - `Corresponding' fields have the same type
  - same schema as the inputs

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

$S1 \cup S2$

Union, Intersection, Set-Difference

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

- Note: no duplicate
  - "Set semantic"
  - SQL: UNION
  - SQL allows "bag semantic" as well: UNION ALL

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

$S1 \cup S2$

Union, Intersection, Set-Difference

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

sid	sname	rating	age
22	dustin	7	45.0

$S1 - S2$

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

$S1 \cap S2$

Cross-Product

- Each row of S1 is paired with each row of R1.
- Result schema has one field per field of S1 and R1, with field names `inherited' if possible.
  - Conflict: Both S1 and R1 have a field called sid.

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/96
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	22	101	10/10/96
31	lubber	8	55.5	58	103	11/12/96
58	rusty	10	35.0	22	101	10/10/96
58	rusty	10	35.0	58	103	11/12/96

Renaming Operator ρ

$(\rho_{\text{sid} \to \text{sid1}}\ S1) \times (\rho_{\text{sid} \to \text{sid1}}\ R1)$

or

$\rho(C(1 \to \text{sid1}, 5 \to \text{sid2}),\ S1 \times R1)$

C is the new relation name

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/96
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	22	101	10/10/96
31	lubber	8	55.5	58	103	11/12/96
58	rusty	10	35.0	22	101	10/10/96
58	rusty	10	35.0	58	103	11/12/96

- In general, can use ρ(<Temp>, <RA-expression>)

Joins

$$R \bowtie_c S = \sigma_c (R \times S)$$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	58	103	11/12/96

$$S1 \bowtie_{S1.sid < R1.sid} R1$$

- Result schema same as that of cross-product.
- Fewer tuples than cross-product, might be able to compute more efficiently

Find names of sailors who've reserved boat #103

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

Find names of sailors who've reserved boat #103

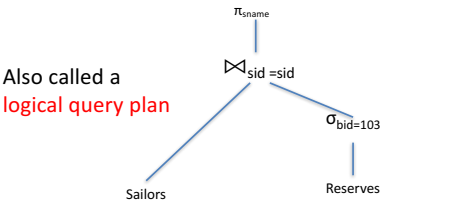
Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

- Solution 1:
- Solution 2:

Expressing an RA expression as a Tree

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

Also called a logical query plan



$\pi_{sname}((\sigma_{bid=103} Reserves) \bowtie Sailors)$

Find sailors who've reserved a red or a green boat

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

Use of rename operation

- Can identify all red or green boats, then find sailors who've reserved one of these boats:

Can also define Tempboats using union
Try the "AND" version yourself

What about aggregates?

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

- Extended relational algebra
- $\gamma_{age,\ avg(rating) \rightarrow avgr}$ Sailors
- Also extended to "bag semantic": allow duplicates
  - Take into account cardinality
  - R and S have tuple t resp. m and n times
  - R ∪ S has t m+n times
  - R ∩ S has t min(m, n) times
  - R – S has t max(0, m-n) times
  - sorting(τ), duplicate removal (δ) operators

Relational Calculus (RC)

Relational Calculus

- RA is procedural
  - $\pi_A(\sigma_{A=a} R)$ and $\sigma_{A=a}(\pi_A R)$ are equivalent but different expressions
- RC
  - non-procedural and declarative
  - describes a set of answers without being explicit about how they should be computed
- TRC (tuple relational calculus)
  - variables take tuples as values
  - we will primarily do TRC
- DRC (domain relational calculus)
  - variables range over field values

TRC: example

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

- Find the name and age of all sailors with a rating above 7

∃ There exists

{P | ∃ S ∈ Sailors (S.rating > 7 ∧ P.name = S.name ∧ P.age = S.age)}

- P is a tuple variable
  - with exactly two fields name and age (schema of the output relation)
  - P.name = S.name ∧ P.age = S.age gives values to the fields of an answer tuple
- Use parentheses, ∀ ∃ ∨ ∧ > < = ≠ ¬ etc as necessary
- A ⇒ B is very useful too
  - next slide

A ⇒ B

- A "implies" B
- Equivalently, if A is true, B must be true
- Equivalently, ¬ A ∨ B, i.e.
  - either A is false (then B can be anything)
  - otherwise (i.e. A is true) B must be true

Useful Logical Equivalences

∃	There exists
∀	For all
∧	Logical AND
∨	Logical OR
¬	NOT

- ∀x P(x) = ¬∃x [¬P(x)]
- ¬(P ∨ Q) = ¬ P ∧ ¬ Q
- ¬(P ∧ Q) = ¬ P ∨ ¬ Q

de Morgan's laws

  - Similarly, ¬(¬P ∨ Q) = P ∧ ¬ Q etc.

- A ⇒ B = ¬ A ∨ B

TRC: example

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

- Find the names of sailors who have reserved at least two boats

TRC: example

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

- Find the names of sailors who have reserved at least two boats

{P | ∃ S ∈ Sailors (∃ R1 ∈ Reserves ∃ R2 ∈ Reserves ∧ S.sid = R1.sid ∧ S.sid = R2.sid ∧ R1.bid ≠ R2.bid ∧ P.name = S.name)}

TRC: example

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

- Find the names of sailors who have reserved all boats
- Division operation

TRC: example

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

- Find the names of sailors who have reserved all boats
- Division operation in RA!

TRC: example

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

- Find the names of sailors who have reserved all red boats

How will you change the previous TRC expression?

TRC: example

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

- Find the names of sailors who have reserved all red boats

Recall that A ⇒ B is logically equivalent to ¬ A ∨ B
so ⇒ can be avoided, but it is cleaner and more intuitive

DRC: example

Sailors(sid, sname, rating, age)
Boats(bid, bname, color)
Reserves(sid, bid, day)

- Find the name and age of all sailors with a rating above 7

TRC:
{P | ∃ S ϵ Sailors (S.rating > 7 ∧ P.name = S.name ∧ P.age = S.age)}

DRC:
{<N, A> | ∃ <I, N, T, A> ϵ Sailors ∧ T > 7}

- Variables are now domain variables
- We will use use TRC
  - both are equivalent

More Examples: RC

- The famous "Drinker-Beer-Bar" example!

UNDERSTAND THE DIFFERENCE IN ANSWERS
FOR ALL FOUR DRINKERS

Acknowledgement: examples and slides by Profs. Balazinska and Suciu, and the [GUW] book

Likes(drinker, beer)
Frequents(drinker, bar)
Serves(bar, beer)

Drinker Category 1

Find drinkers that frequent some bar that serves some beer they like.

Likes(drinker, beer)
Frequents(drinker, bar)
Serves(bar, beer)

Drinker Category 1

Find drinkers that frequent some bar that serves some beer they like.

Q(x) = ∃y. ∃z. Frequents(x, y) ∧ Serves(y,z) ∧ Likes(x,z)

a shortcut for
{x | ∃Y ϵ Frequents Z ϵ Serves W ϵ Likes (T.drinker = x.drinker ∧ T.bar = Z.bar ∧ W.beer = ……}

The difference is that in the first one, one variable = one attribute
in the second one, one variable = one tuple (Tuple RC)
Both are equivalent and feel free to use the one that is convenient to you

Likes(drinker, beer)
Frequents(drinker, bar)
Serves(bar, beer)

Drinker Category 4

Find drinkers that frequent some bar that serves some beer they like.

Q(x) = ∃y. ∃z. Frequents(x, y)∧Serves(y,z)∧Likes(x,z)

Find drinkers that frequent only bars that serves some beer they like.

Q(x) =

Find drinkers that frequent some bar that serves only beers they like.

Q(x) =

Find drinkers that frequent only bars that serves only beer they like.

Q(x) =

Why should we care about RC

- RC is declarative, like SQL, and unlike RA (which is operational)
- Gives foundation of database queries in first-order logic
  - you cannot express all aggregates in RC, e.g. cardinality of a relation or sum (possible in extended RA and SQL)
  - still can express conditions like "at least two tuples" (or any constant)
- RC expression may be much simpler than SQL queries
  - and easier to check for correctness than SQL
  - power to use ∀ and ⇒
  - then you can systematically go to a "correct" SQL query

Likes(drinker, beer)
Frequents(drinker, bar)
Serves(bar, beer)

From RC to SQL

Query: Find drinkers that like some beer (so much) that they frequent all bars that serve it

Q(x) = ∃y. Likes(x, y)∧∀z.(Serves(z,y) ⇒ Frequents(x,z))

Likes(drinker, beer)
Frequents(drinker, bar)
Serves(bar, beer)

From RC to SQL

Query: Find drinkers that like some beer so much that they frequent all bars that serve it

Q(x) = ∃y. Likes(x, y)∧∀z.(Serves(z,y) ⇒ Frequents(x,z))

≡ Q(x) = ∃y. Likes(x, y)∧∀z.(¬ Serves(z,y) ∨ Frequents(x,z))

Step 1: Replace ∀ with ∃ using de Morgan's Laws

∀x P(x) same as ¬∃x ¬P(x)

Q(x) = ∃y. Likes(x, y)∧ ¬∃z.(Serves(z,y) ∧ ¬Frequents(x,z))

¬(¬P∨Q) same as P∧ ¬ Q

Likes(drinker, beer)
Frequents(drinker, bar)
Serves(bar, beer)

From RC to SQL

Q(x) = ∃y. Likes(x, y) ∧¬ ∃z.(Serves(z,y)∧¬Frequents(x,z))

Step 2: Translate into SQL

```
SELECT DISTINCT L.drinker
FROM Likes L
WHERE not exists
  (SELECT S.bar
   FROM Serves S
   WHERE L.beer=S.beer
      AND not exists (SELECT *
                      FROM Frequents F
                      WHERE F.drinker=L.drinker
                        AND F.bar=S.bar))
```


this slide can be skipped until we do Datalog will revisit "Safe queries" then

The "correct" intermediate steps

- Write the query in RC
- If you have a variable under "negation", also add the "domain", i.e. where the variable appears without a negation
  - e.g. if you have ¬ H(x, y) for a subquery,
  - where x and y can only come from a relation R(x, y)
  - make it R(x, y) ∧ ¬ H(x, y)
  - This is to make the query "safe" and "domain independent" – we will discuss this when we do Datalog
  - Intuitively, if you are trying to find "sailors that do not satisfy some criteria" you have to specify the domain of sailors, say from the sailor table, otherwise you are looking at an infinite space

this slide can be skipped until we do Datalog
will revisit "Safe queries" then

The "correct" intermediate step

Make all *subqueries* with negation domain independent
i.e. say where x is coming from

$Q(x) = \exists y.\ Likes(x, y) \wedge \neg\exists z.(Likes(x,y) \wedge Serves(z,y) \wedge \neg Frequents(x,z))$

which can be simplified to the previous query

```
SELECT DISTINCT L.drinker FROM Likes L
WHERE not exists
  (SELECT * FROM Likes L2, Serves S
   WHERE L2.drinker=L.drinker and L2.beer=L.beer
         and L2.beer=S.beer
         and not exists (SELECT * FROM Frequents F
                         WHERE F.drinker=L2.drinker
                           and F.bar=S.bar))
```


Summary

- You learnt three query languages for the Relational DB model
  - SQL
  - RA
  - RC
- All have their own purposes
- You should be able to write a query in all three languages and convert from one to another
  - However, you have to be careful, not all "valid" expressions in one may be expressed in another
  - {S | ¬ (S ∈ Sailors)} – infinitely many tuples – an "unsafe" query
  - More when we do "Datalog", also see Ch. 4.4 in [RG]
- Next topic: DBMS internals
  - storage/indexing, query execution, algorithms, optimization