

CompSci 516

Data Intensive Computing Systems

Lecture 4

Relational Algebra and Relational Calculus

Instructor: Sudeepa Roy

Announcements

- **HW1 reminder:**
 - Due on 09/21 (Thurs), 11:55 pm, no late days
- **Project proposal deadline extended:**
 - Preliminary idea and team members due by 09/18 (Mon) by email to the instructor
 - Proposal due on sakai by 09/25 (Mon), 11:55 pm
 - One report per group
 - Form your group soon
 - Look at recent research papers/demonstrations in top db conferences for ideas (see the project idea file on sakai)
- **Your piazza, sakai, gradiance accounts should be active**
 - Occasional Pop up quizzes will start from next week
 - Bring a laptop in class

Today's topics

- Finish NULLs and Views in SQL from Lecture 3
- Relational Algebra (RA) and Relational Calculus (RC)
- Reading material
 - [RG] Chapter 4 (RA, RC)
 - [GUW] Chapters 2.4, 5.1, 5.2

Acknowledgement:

The following slides have been created adapting the instructor material of the [RG] book provided by the authors Dr. Ramakrishnan and Dr. Gehrke.

Nulls and Views in SQL

Null Values

- Field values in a tuple are sometimes
 - **unknown**, e.g., a rating has not been assigned, or
 - **inapplicable**, e.g., no spouse's name
 - SQL provides a special value **null** for such situations.

Standard Boolean 2-valued logic

- True = 1, False = 0
- Suppose $X = 5$
 - $(X < 100) \text{ AND } (X \geq 1)$ is $T \wedge T = T$
 - $(X > 100) \text{ OR } (X \geq 1)$ is $F \vee T = T$
 - $(X > 100) \text{ AND } (X \geq 1)$ is $F \wedge T = F$
 - $\text{NOT}(X = 5)$ is $\neg T = F$
- Intuitively,
 - $T = 1, F = 0$
 - For $V1, V2 \in \{1, 0\}$
 - $V1 \wedge V2 = \text{MIN}(V1, V2)$
 - $V1 \vee V2 = \text{MAX}(V1, V2)$
 - $\neg(V1) = 1 - V1$

2-valued logic does not work for nulls

- Suppose rating = null, X = 5
- Is rating > 8 true or false?
- What about **AND**, **OR** and **NOT** connectives?
 - (rating > 8) AND (X = 5)?
- What if we have such a condition in the WHERE clause?

3-Valued Logic For Null

- TRUE (= 1), FALSE (= 0), UNKNOWN (= 0.5)
 - unknown is treated as 0.5
- Now you can apply rules from 2-valued logic!
 - For $V1, V2 \in \{1, 0, 0.5\}$
 - $V1 \wedge V2 = \text{MIN}(V1, V2)$
 - $V1 \vee V2 = \text{MAX}(V1, V2)$
 - $\neg(V1) = 1 - V1$
- Therefore,
 - NOT UNKNOWN = UNKNOWN
 - UNKNOWN OR TRUE = TRUE
 - UNKNOWN AND TRUE = UNKNOWN
 - UNKNOWN AND FALSE = FALSE
 - UNKNOWN OR FALSE = UNKNOWN

New issues for Null

- The presence of **null** complicates many issues. E.g.:
 - Special operators needed to check if value **IS/IS NOT NULL**
 - Be careful!
 - “WHERE X = NULL” does not work!
 - Need to write “WHERE X IS NULL”
- Meaning of constructs must be defined carefully
 - e.g., **WHERE clause eliminates rows that don't evaluate to true**
 - So not only FALSE, but UNKNOWNs are eliminated too
 - **very important to remember!**
- But NULL allows new operators (e.g. **outer joins**)
- Arithmetic with NULL
 - all of +, -, *, / return null if any argument is null
- Can force “no nulls” while creating a table
 - **sname char(20) NOT NULL**
 - **primary key is always not null**

Aggregates with NULL

| sid | sname | rating | age |
|-----|--------|--------|-----|
| 22 | dustin | 7 | 45 |
| 31 | lubber | 8 | 55 |
| 58 | rusty | 10 | 35 |

R1

- What do you get for
- `SELECT count(*) from R1?`
- `SELECT count(rating) from R1?`

Aggregates with NULL

| sid | sname | rating | age |
|-----|--------|--------|-----|
| 22 | dustin | 7 | 45 |
| 31 | lubber | 8 | 55 |
| 58 | rusty | 10 | 35 |

R1

- What do you get for
- `SELECT count(*) from R1?`
- `SELECT count(rating) from R1?`
- **Ans: 3 for both**

Aggregates with NULL

| sid | sname | rating | age |
|-----|--------|--------|-----|
| 22 | dustin | 7 | 45 |
| 31 | lubber | 8 | 55 |
| 58 | rusty | 10 | 35 |

R1

| sid | sname | rating | age |
|-----|--------|--------|-----|
| 22 | dustin | 7 | 45 |
| 31 | lubber | null | 55 |
| 58 | rusty | 10 | 35 |

R2

- What do you get for
- SELECT count(*) from R1?
- SELECT count(rating) from R1?
- **Ans: 3 for both**

- What do you get for
- SELECT count(*) from R2?
- SELECT count(rating) from R2?

Aggregates with NULL

| sid | sname | rating | age |
|-----|--------|--------|-----|
| 22 | dustin | 7 | 45 |
| 31 | lubber | 8 | 55 |
| 58 | rusty | 10 | 35 |

R1

| sid | sname | rating | age |
|-----|--------|--------|-----|
| 22 | dustin | 7 | 45 |
| 31 | lubber | null | 55 |
| 58 | rusty | 10 | 35 |

R2

- What do you get for
- SELECT count(*) from R1?
- SELECT count(rating) from R1?
- **Ans: 3 for both**

- What do you get for
- SELECT count(*) from R2?
- SELECT count(rating) from R2?
- **Ans: First 3, then 2**

Aggregates with NULL

- COUNT, SUM, AVG, MIN, MAX (with or without DISTINCT)
 - Discards null values first
 - Then applies the aggregate
 - Except count(*)
- If only applied to null values, the result is null

| sid | sname | rating | age |
|-----|--------|--------|-----|
| 22 | dustin | 7 | 45 |
| 31 | lubber | null | 55 |
| 58 | rusty | 10 | 35 |

R2

- SELECT sum(rating) from R2?
- **Ans: 17**

| sid | sname | rating | age |
|-----|--------|--------|-----|
| 22 | dustin | null | 45 |
| 31 | lubber | null | 55 |
| 58 | rusty | null | 35 |

R3

- SELECT sum(rating) from R3?
- **Ans: null**

Overview: General Constraints

- Useful when more general ICs than keys are involved
- There are also **ASSERTIONS** to specify constraints that span across multiple tables
- There are **TRIGGERS** too : procedure that starts automatically if specified changes occur to the DBMS
 - see additional slides at the end

```
CREATE TABLE Sailors
  ( sid INTEGER,
    sname CHAR(10),
    rating INTEGER,
    age REAL,
    PRIMARY KEY (sid),
    CHECK ( rating >= 1
           AND rating <= 10 )
```

```
CREATE TABLE Reserves
  ( sname CHAR(10),
    bid INTEGER,
    day DATE,
    PRIMARY KEY (bid,day),
    CONSTRAINT noInterlakeRes
    CHECK ( `Interlake' <>
           ( SELECT B.bname
             FROM Boats B
             WHERE B.bid=bid)))
```

Views

- A **view** is just a relation, but we store a **definition**, rather than a set of tuples

```
CREATE VIEW YoungActiveStudents (name, grade)
  AS SELECT S.name, E.grade
  FROM Students S, Enrolled E
  WHERE S.sid = E.sid and S.age<21
```

- Views can be dropped using the **DROP VIEW** command
- **Views and Security:** Views can be used to present necessary information (or a summary), while hiding details in underlying relation(s)
 - the above view hides courses “cid” from E

Can create a new table from a query on other tables too

SELECT... INTO.... FROM.... WHERE

```
SELECT S.name, E.grade  
INTO YoungActiveStudents  
FROM Students S, Enrolled E  
WHERE S.sid = E.sid and S.age<21
```

“WITH” clause – very useful!

- You will find “WITH” clause very useful!

```
WITH Temp1 AS  
    (SELECT ..... ..),  
    Temp2 AS  
    (SELECT ..... ..)  
SELECT X, Y  
FROM TEMP1, TEMP2  
WHERE....
```

- Can simplify complex nested queries

Summary

- SQL has a huge number of constructs and possibilities
 - You need to learn and practice it on your own
 - Given a problem, you should be able to write a SQL query and verify whether a given one is correct
- Pay attention to NULLs
- Can limit answers using “LIMIT” or “TOP” clauses
 - e.g. to output TOP 20 results according to an aggregate
 - also can sort using ASC or DESC keywords

Relational Query Languages

Relational Query Languages

- **Query languages:** Allow manipulation and retrieval of data from a database
- Relational model supports simple, powerful QLs:
 - Strong formal foundation based on logic
 - Allows for much optimization
- Query Languages **!=** programming languages
 - QLs not intended to be used for complex calculations
 - QLs support easy, efficient access to large data sets

Formal Relational Query Languages

- Two “mathematical” Query Languages form the basis for “real” languages (e.g. SQL), and for implementation:
 - **Relational Algebra**: More **operational**, very useful for representing execution plans
 - **Relational Calculus**: Lets users describe what they want, rather than how to compute it (**Non-operational, declarative, or procedural**)
- **Note: Declarative (RC, SQL) vs. Operational (RA)**

Preliminaries

- A query is applied to **relation instances**, and the result of a query is also a relation instance.
 - **Schemas of input** relations for a query are **fixed**
 - query will run regardless of instance
 - The **schema for the result** of a given query is also **fixed**
 - Determined by definition of query language constructs
- **Positional vs. named-field notation:**
 - Positional notation easier for formal definitions, named-field notation more readable

Example Schema and Instances

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

S1

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

S2

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

R1

| <u>sid</u> | <u>bid</u> | <u>day</u> |
|------------|------------|------------|
| 22 | 101 | 10/10/96 |
| 58 | 103 | 11/12/96 |

Logic Notations

- \exists There exists
- \forall For all
- \wedge Logical AND
- \vee Logical OR
- \neg NOT

Relational Algebra (RA)

Relational Algebra

- Takes one or more relations as input, and produces a relation as output
 - operator
 - operand
 - semantic
 - so an algebra!
- Since each operation returns a relation, **operations can be composed**
 - Algebra is “closed”

Relational Algebra

- Basic operations:
 - Selection (σ) Selects a subset of rows from relation
 - Projection (π) Deletes unwanted columns from relation.
 - Cross-product (\times) Allows us to combine two relations.
 - Set-difference ($-$) Tuples in reln. 1, but not in reln. 2.
 - Union (\cup) Tuples in reln. 1 or in reln. 2.
- Additional operations:
 - Intersection (\cap)
 - join \bowtie
 - division($/$)
 - renaming (ρ)
 - Not essential, but (very) useful.

Projection

S2

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

- Deletes attributes that are not in projection list.
- Schema** of result contains exactly the fields in the projection list, with the same names that they had in the (only) input relation.
- Projection operator has to **eliminate duplicates** (Why)
 - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it (performance)

| sname | rating |
|--------|--------|
| yuppy | 9 |
| lubber | 8 |
| guppy | 5 |
| rusty | 10 |

$\pi_{sname, rating}(S2)$

| age |
|------|
| 35.0 |
| 55.5 |

$\pi_{age}(S2)$

Selection

- Selects rows that satisfy **selection condition**

- No duplicates in result.
Why?

- Schema of result identical to schema of (only) input relation

S2

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

| sid | sname | rating | age |
|-----|-------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 58 | rusty | 10 | 35.0 |

$$\sigma_{rating > 8}(S2)$$

| sname | rating |
|-------|--------|
| yuppy | 9 |
| rusty | 10 |

$$\pi_{sname, rating}(\sigma_{rating > 8}(S2))$$

Composition of Operators

- Result relation can be the input for another relational algebra operation
 - Operator composition

| sid | sname | rating | age |
|-----|-------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 58 | rusty | 10 | 35.0 |

$$\sigma_{rating > 8}(S2)$$

| sname | rating |
|-------|--------|
| yuppy | 9 |
| rusty | 10 |

$$\pi_{sname, rating}(\sigma_{rating > 8}(S2))$$

Union, Intersection, Set-Difference

S1

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

S2

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

- All of these operations take two input relations, which must be **union-compatible**:
 - Same number of fields.
 - ‘Corresponding’ fields have the same type
 - same schema as the inputs

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |
| 44 | guppy | 5 | 35.0 |
| 28 | yuppy | 9 | 35.0 |

Union, Intersection, Set-Difference

S1

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

S2

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

- Note: no duplicate
 - “Set semantic”
 - SQL: **UNION**
 - SQL allows “bag semantic” as well:
UNION ALL

| sid | sname | rating | age |
|-----|--------|--------|------|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |
| 44 | guppy | 5 | 35.0 |
| 28 | yuppy | 9 | 35.0 |

Union, Intersection, Set-Difference

S_1

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 22 | dustin | 7 | 45.0 |
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

S_2

| <u>sid</u> | sname | rating | age |
|------------|--------|--------|------|
| 28 | yuppy | 9 | 35.0 |
| 31 | lubber | 8 | 55.5 |
| 44 | guppy | 5 | 35.0 |
| 58 | rusty | 10 | 35.0 |

| sid | sname | rating | age |
|-----|--------|--------|------|
| 22 | dustin | 7 | 45.0 |

$S_1 - S_2$

| sid | sname | rating | age |
|-----|--------|--------|------|
| 31 | lubber | 8 | 55.5 |
| 58 | rusty | 10 | 35.0 |

$S_1 \cap S_2$

Cross-Product

- Each row of S1 is paired with each row of R1.
- **Result schema** has one field per field of S1 and R1, with field names 'inherited' if possible.
 - Conflict: Both S1 and R1 have a field called sid.

| (sid) | sname | rating | age | (sid) | bid | day |
|-------|--------|--------|------|-------|-----|----------|
| 22 | dustin | 7 | 45.0 | 22 | 101 | 10/10/96 |
| 22 | dustin | 7 | 45.0 | 58 | 103 | 11/12/96 |
| 31 | lubber | 8 | 55.5 | 22 | 101 | 10/10/96 |
| 31 | lubber | 8 | 55.5 | 58 | 103 | 11/12/96 |
| 58 | rusty | 10 | 35.0 | 22 | 101 | 10/10/96 |
| 58 | rusty | 10 | 35.0 | 58 | 103 | 11/12/96 |

Renaming Operator ρ

$$(\rho_{\text{sid} \rightarrow \text{sid1}} S1) \times (\rho_{\text{sid} \rightarrow \text{sid1}} R1)$$

or

$$\rho(C(1 \rightarrow \text{sid1}, 5 \rightarrow \text{sid2}), S1 \times R1)$$

C is the
new relation
name

| (sid) | sname | rating | age | (sid) | bid | day |
|-------|--------|--------|------|-------|-----|----------|
| 22 | dustin | 7 | 45.0 | 22 | 101 | 10/10/96 |
| 22 | dustin | 7 | 45.0 | 58 | 103 | 11/12/96 |
| 31 | lubber | 8 | 55.5 | 22 | 101 | 10/10/96 |
| 31 | lubber | 8 | 55.5 | 58 | 103 | 11/12/96 |
| 58 | rusty | 10 | 35.0 | 22 | 101 | 10/10/96 |
| 58 | rusty | 10 | 35.0 | 58 | 103 | 11/12/96 |

- In general, can use $\rho(\langle \text{Temp} \rangle, \langle \text{RA-expression} \rangle)$

Joins

$$R \bowtie_c S = \sigma_c (R \times S)$$

| (sid) | sname | rating | age | (sid) | bid | day |
|-------|--------|--------|------|-------|-----|----------|
| 22 | dustin | 7 | 45.0 | 58 | 103 | 11/12/96 |
| 31 | lubber | 8 | 55.5 | 58 | 103 | 11/12/96 |

$$S1 \bowtie_{S1.sid < R1.sid} R1$$

- Result schema same as that of cross-product.
- Fewer tuples than cross-product, might be able to compute more efficiently

Find names of sailors who've reserved boat #103

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

Find names of sailors who've reserved boat #103

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

- **Solution 1:** $\pi_{sname}((\sigma_{bid=103} Reserves) \bowtie Sailors)$
- **Solution 2:** $\pi_{sname}(\sigma_{bid=103}(Reserves \bowtie Sailors))$

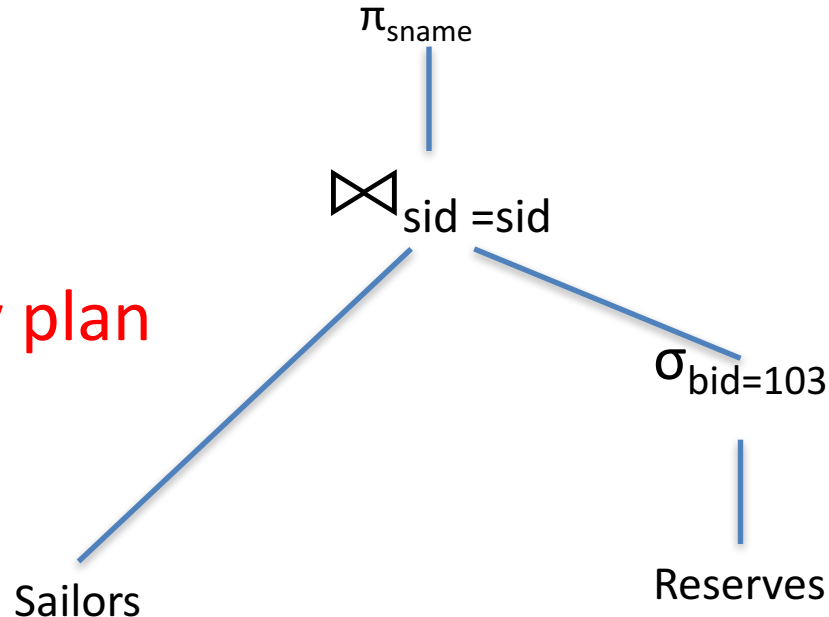
Expressing an RA expression as a Tree

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

Also called a
logical query plan



$\pi_{sname}((\sigma_{bid=103} Reserves) \bowtie Sailors)$

Find sailors who've reserved a red or a green boat

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

Use of rename operation

- Can identify all red or green boats, then find sailors who've reserved one of these boats:

$$\rho (Tempboats, (\sigma_{color='red' \vee color='green'} Boats))$$
$$\pi_{sname}(Tempboats \bowtie Reserves \bowtie Sailors)$$

Can also define Tempboats using union
Try the “AND” version yourself

What about aggregates?

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

- Extended relational algebra
- $\gamma_{age, avg(rating)} \rightarrow avg_r$ Sailors
- Also extended to “bag semantic”: allow duplicates
 - Take into account cardinality
 - R and S have tuple t resp. m and n times
 - $R \cup S$ has t m+n times
 - $R \cap S$ has t min(m, n) times
 - $R - S$ has t max(0, m-n) times
 - sorting(τ), duplicate removal (δ) operators

Relational Calculus (RC)

Relational Calculus

- RA is procedural
 - $\pi_A(\sigma_{A=a} R)$ and $\sigma_{A=a}(\pi_A R)$ are equivalent but different expressions
- RC
 - non-procedural and declarative
 - describes a set of answers without being explicit about how they should be computed
- TRC (tuple relational calculus)
 - variables take tuples as values
 - we will primarily do TRC
- DRC (domain relational calculus)
 - variables range over field values

TRC: example

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

- Find the name and age of all sailors with a rating above 7

| |
|------------------------|
| \exists There exists |
|------------------------|

$\{P \mid \exists S \in \text{Sailors} (S.\text{rating} > 7 \wedge P.\text{sname} = S.\text{sname} \wedge P.\text{age} = S.\text{age})\}$

- P is a tuple variable
 - with exactly two fields sname and age (schema of the output relation)
 - $P.\text{sname} = S.\text{sname} \wedge P.\text{age} = S.\text{age}$ gives values to the fields of an answer tuple
- Use parentheses, \forall \exists \vee \wedge $>$ $<$ $=$ \neq \neg etc as necessary
- $A \Rightarrow B$ is very useful too
 - next slide

$$A \Rightarrow B$$

- A “implies” B
- Equivalently, if A is true, B must be true
- Equivalently, $\neg A \vee B$, i.e.
 - either A is false (then B can be anything)
 - otherwise (i.e. A is true) B must be true

Useful Logical Equivalences

- $\forall x P(x) = \neg \exists x [\neg P(x)]$

| | |
|-----------|--------------|
| \exists | There exists |
| \forall | For all |
| \wedge | Logical AND |
| \vee | Logical OR |
| \neg | NOT |

- $\neg(P \vee Q) = \neg P \wedge \neg Q$
- $\neg(P \wedge Q) = \neg P \vee \neg Q$

} de Morgan's laws

– Similarly, $\neg(\neg P \vee Q) = P \wedge \neg Q$ etc.

- $A \Rightarrow B = \neg A \vee B$

TRC: example

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

- Find the names of sailors who have reserved at least two boats

TRC: example

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

- Find the names of sailors who have reserved at least two boats

$\{P \mid \exists S \in \text{Sailors} (\exists R1 \in \text{Reserves} \exists R2 \in \text{Reserves} (S.\text{sid} = R1.\text{sid} \wedge S.\text{sid} = R2.\text{sid} \wedge R1.\text{bid} \neq R2.\text{bid}) \wedge P.\text{sname} = S.\text{sname})\}$

TRC: example

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

- Find the names of sailors who have reserved all boats
- Called the “Division” operation

TRC: example

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

- Find the names of sailors who have reserved all boats
- Division operation in RA!

$\{P \mid \exists S \in \text{Sailors} [\forall B \in \text{Boats} (\exists R \in \text{Reserves} (S.\text{sid} = R.\text{sid} \wedge R.\text{bid} = B.\text{bid}))] \wedge P.\text{sname} = S.\text{sname}\}$

TRC: example

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

- Find the names of sailors who have reserved all red boats

How will you change the previous TRC expression?

TRC: example

Sailors(sid, sname, rating, age)

Boats(bid, bname, color)

Reserves(sid, bid, day)

- Find the names of sailors who have reserved all red boats

$\{P \mid \exists S \in \text{Sailors} (\forall B \in \text{Boats} (B.\text{color} = \text{'red'} \Rightarrow (\exists R \in \text{Reserves} (S.\text{sid} = R.\text{sid} \wedge R.\text{bid} = B.\text{bid}))) \wedge P.\text{sname} = S.\text{sname})\}$

Recall that $A \Rightarrow B$ is logically equivalent to $\neg A \vee B$

so \Rightarrow can be avoided, but it is cleaner and more intuitive