CPS 570: Artificial Intelligence

Search

Instructor: Vincent Conitzer

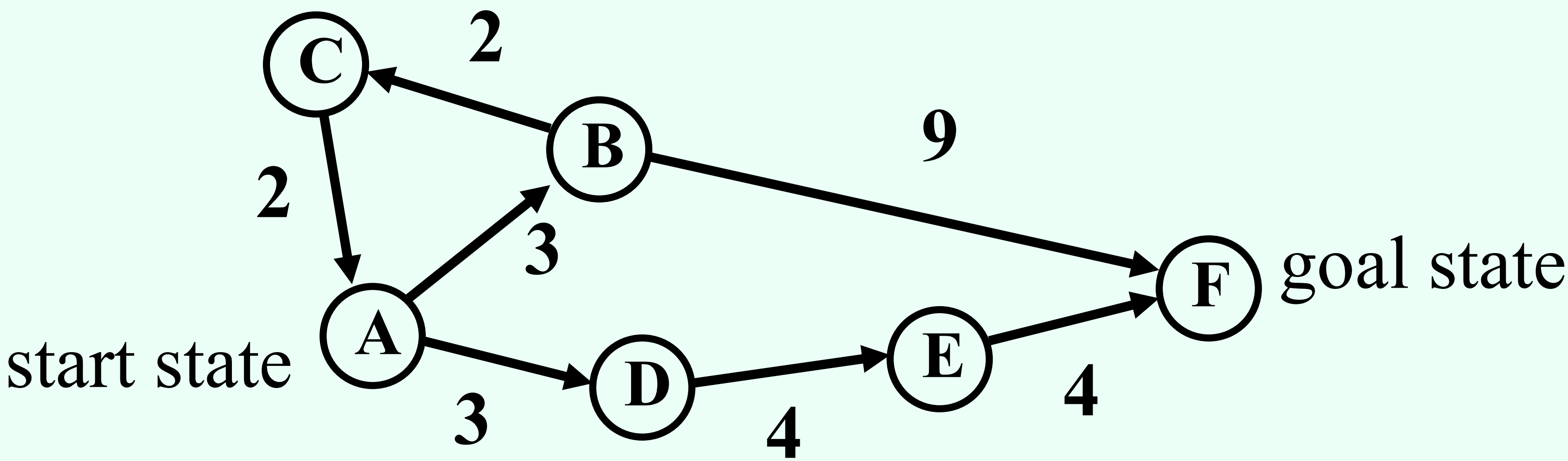
Rubik's Cube robot

- https://www.youtube.com/watch?v=iBE46R-fD6M

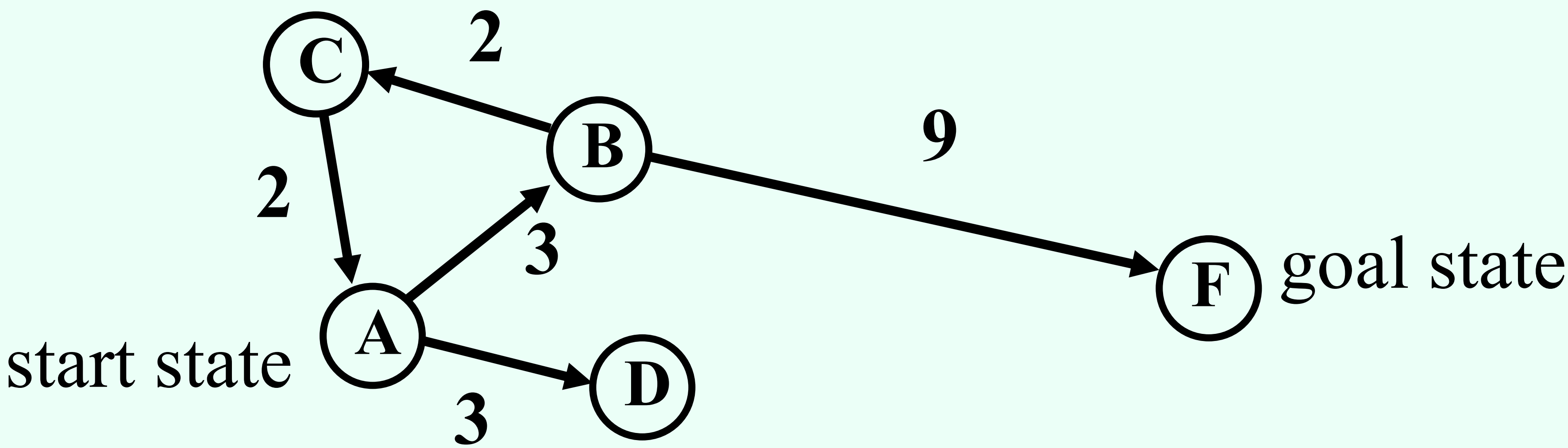
Search

- We have some actions that can change the state of the world
  - Change induced by an action is perfectly predictable
- Try to come up with a sequence of actions that will lead us to a goal state
  - May want to minimize number of actions
  - More generally, may want to minimize total cost of actions
- Do not need to execute actions in real life while searching for solution!
  - Everything perfectly predictable anyway

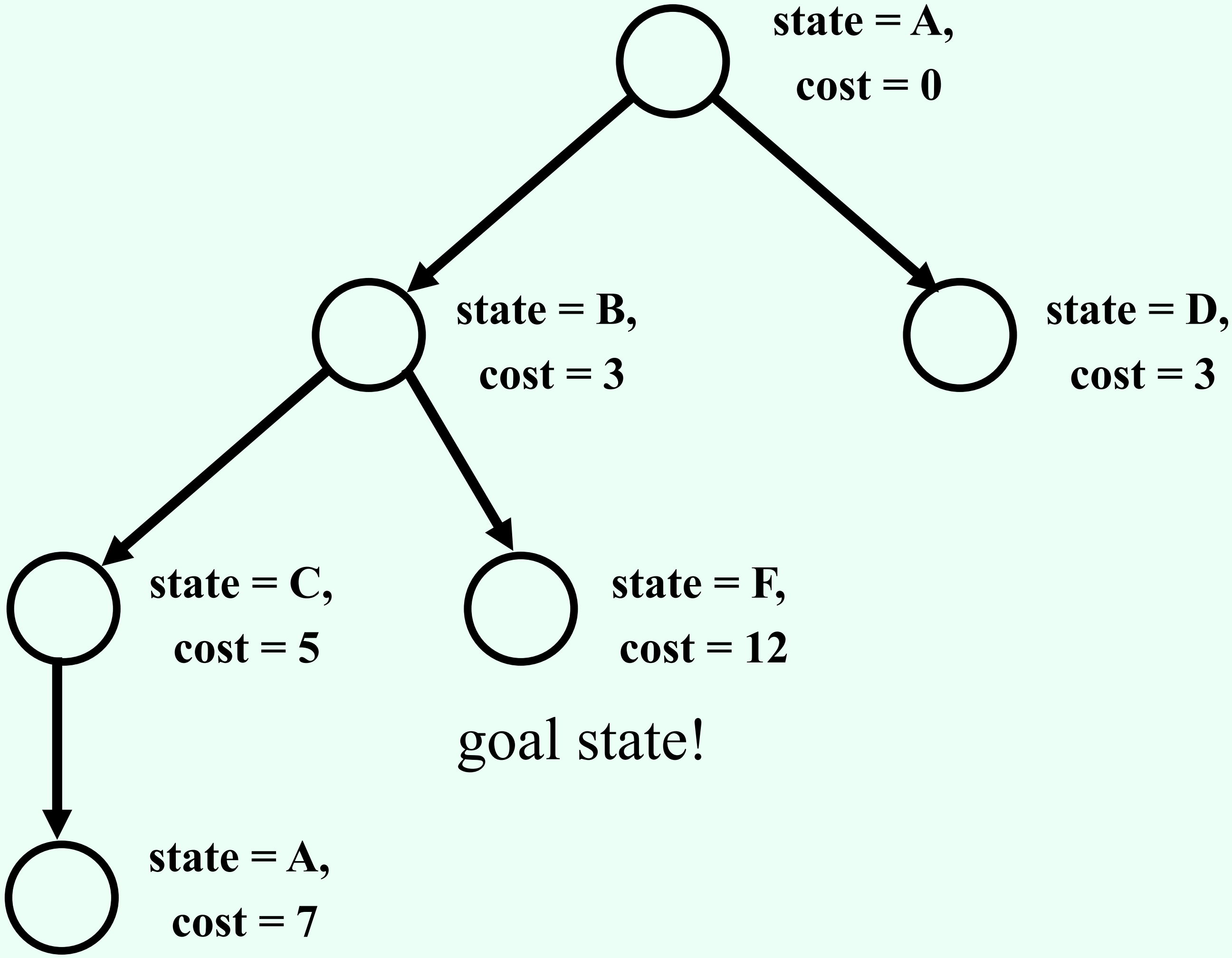
A simple example: traveling on a graph



Searching for a solution

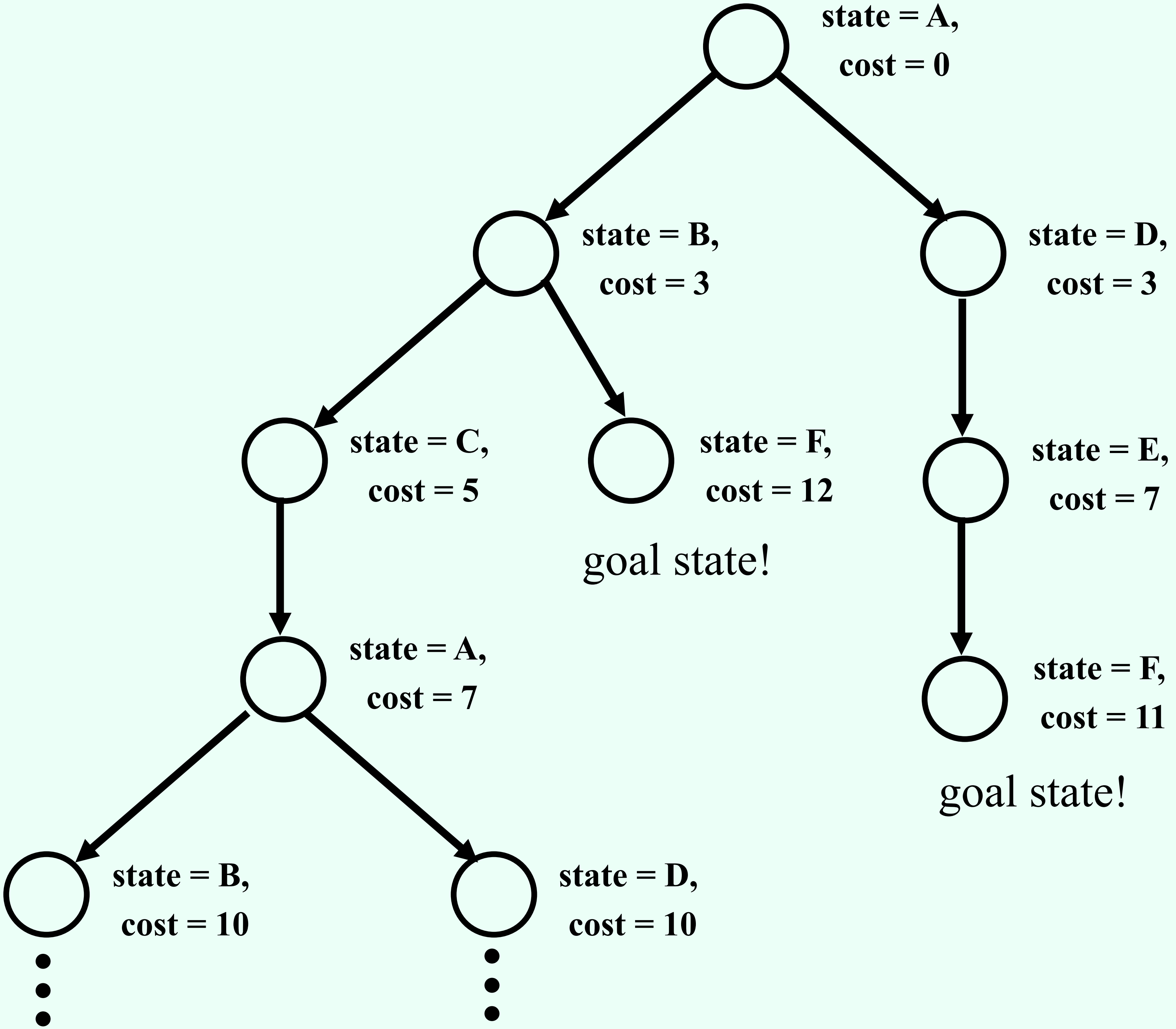


Search tree

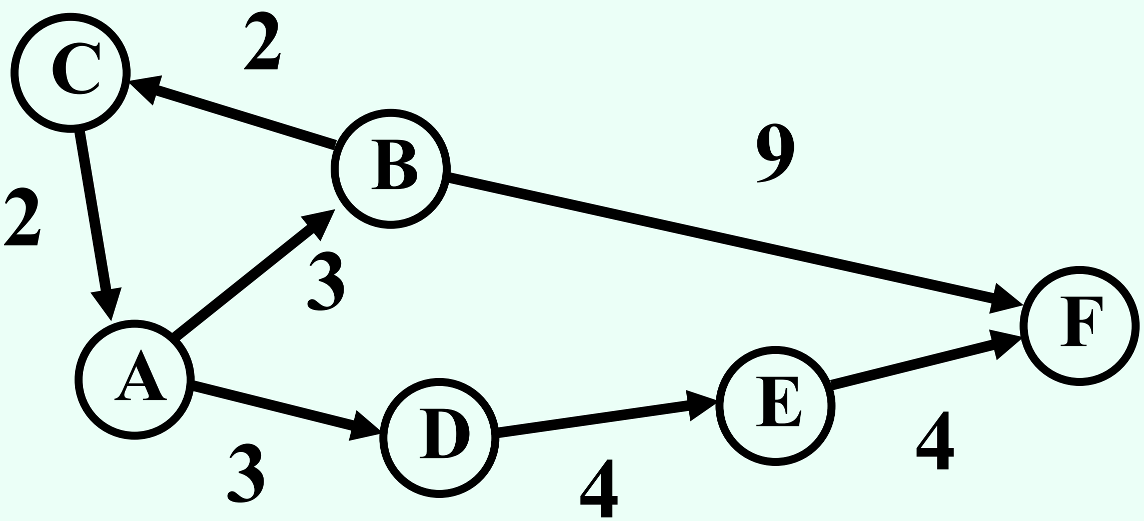


search tree nodes and states are not the same thing!

Full search tree



Changing the goal: want to visit all vertices on the graph



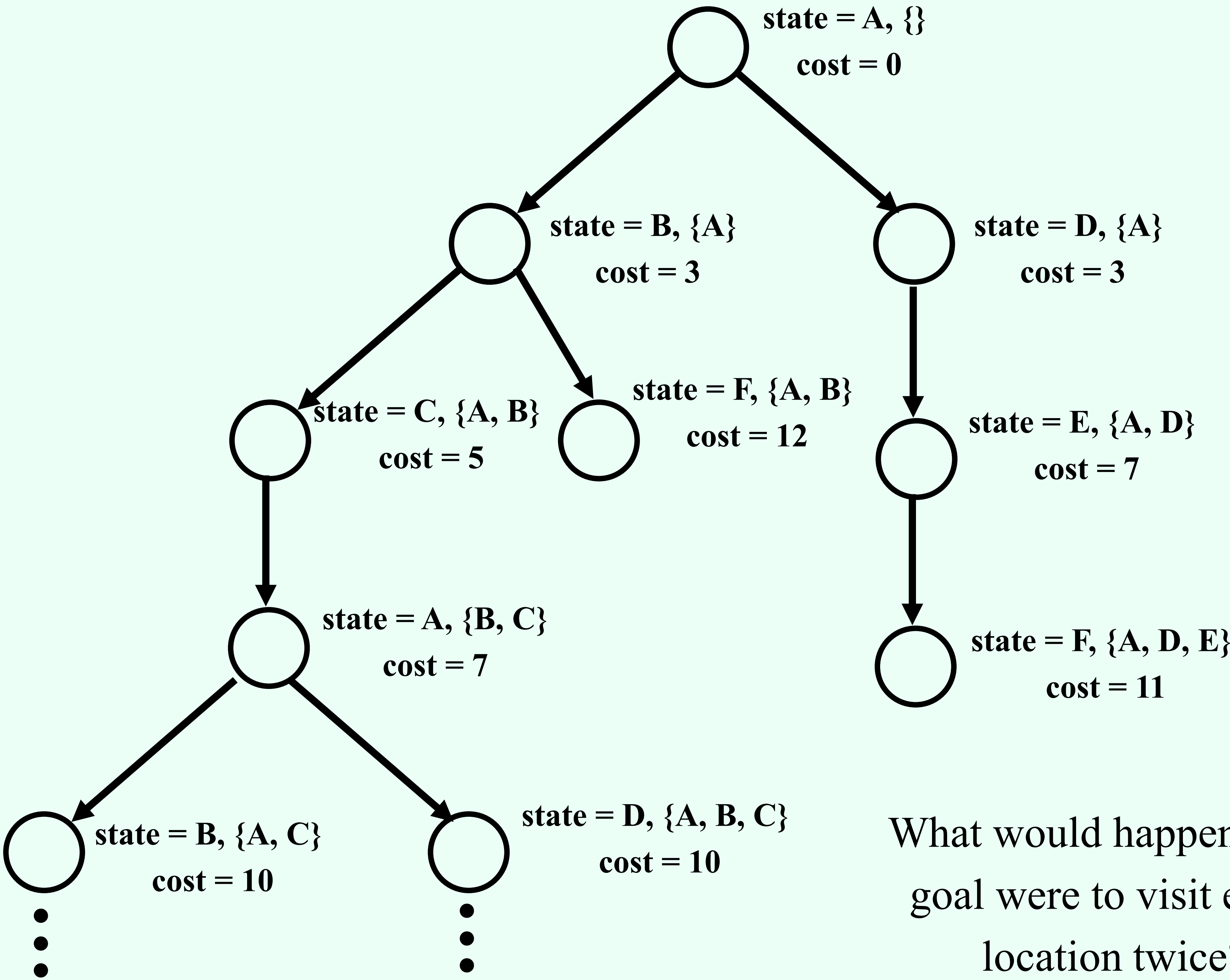
need a different definition of a state

"currently at A, also visited B, C already"

large number of states: $n*2^{n-1}$

could turn these into a graph, but…

Full search tree



What would happen if the goal were to visit every location twice?

Key concepts in search

- Set of states that we can be in
  - Including an initial state…
  - … and goal states (equivalently, a goal test)
- For every state, a set of actions that we can take
  - Each action results in a new state
  - Typically defined by successor function
    - Given a state, produces all states that can be reached from it
- Cost function that determines the cost of each action (or path = sequence of actions)
- Solution: path from initial state to a goal state
  - Optimal solution: solution with minimal cost

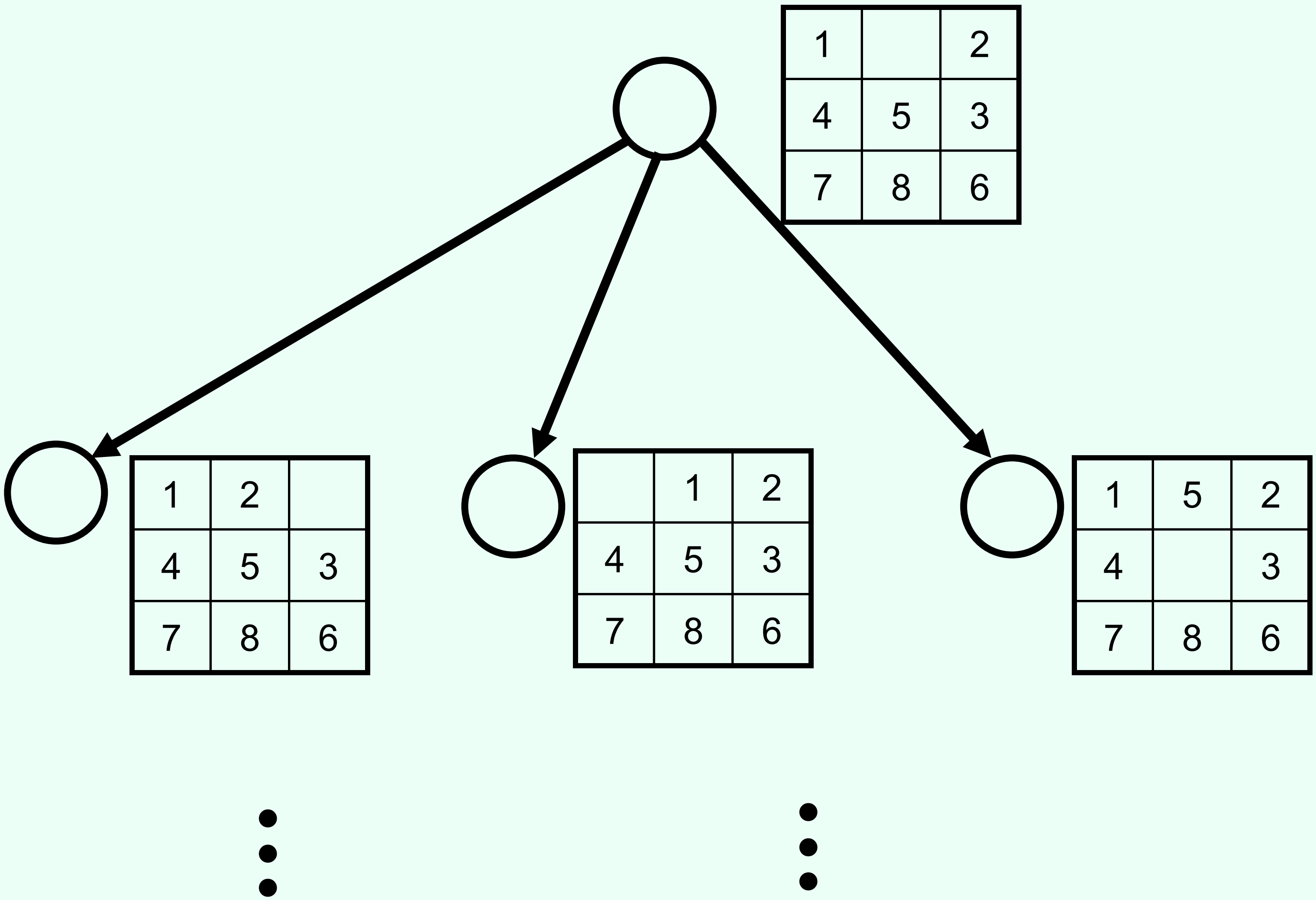
8-puzzle

1		2
4	5	3
7	8	6

1	2	3
4	5	6
7	8	

goal state

8-puzzle



Generic search algorithm

- Fringe = set of nodes generated but not expanded

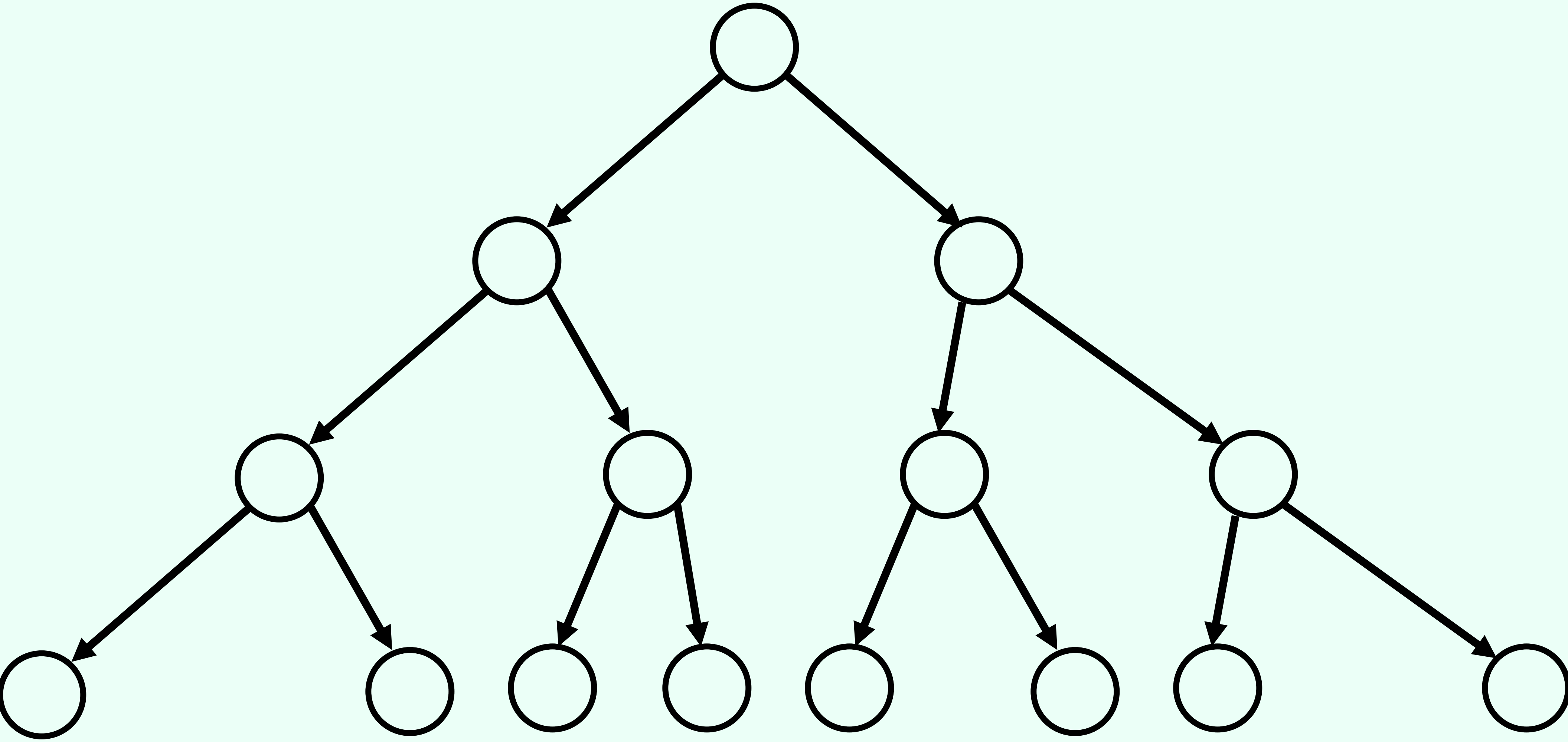
- fringe := {node with initial state}
- loop:
  - if fringe empty, declare failure
  - choose and remove a node v from fringe
  - check if v's state s is a goal state; if so, declare success
  - if not, expand v, insert resulting nodes into fringe

- Key question in search: Which of the generated nodes do we expand next?

Uninformed search

- Given a state, we only know whether it is a goal state or not
- Cannot say one nongoal state looks better than another nongoal state
- Can only traverse state space blindly in hope of somehow hitting a goal state at some point
  - Also called blind search
  - Blind does **not** imply unsystematic!

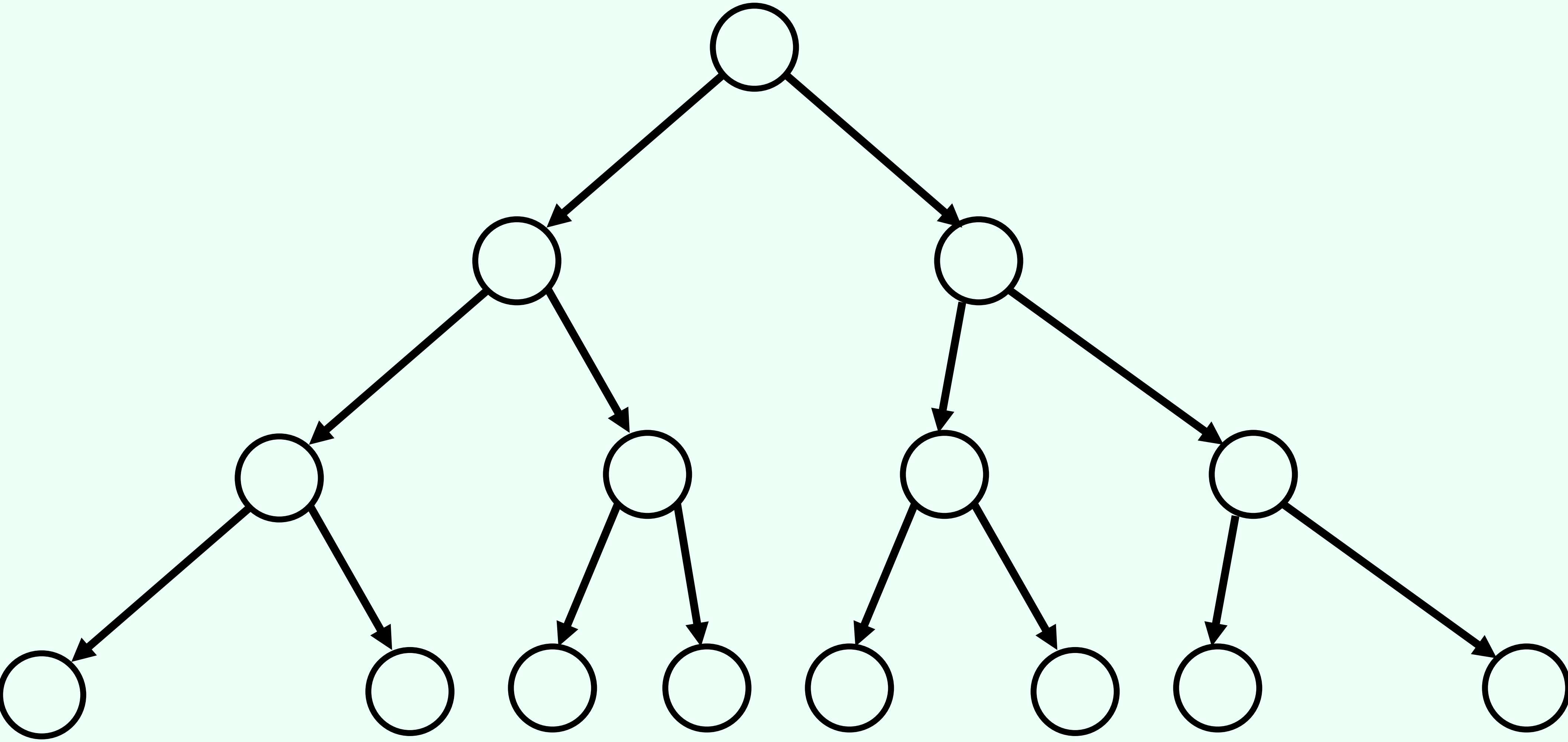
Breadth-first search



Properties of breadth-first search

- Nodes are expanded in the same order in which they are generated
  - Fringe can be maintained as a First-In-First-Out (FIFO) queue
- BFS is complete: if a solution exists, one will be found
- BFS finds a shallowest solution
  - Not necessarily an optimal solution
- If every node has b successors (the branching factor), first solution is at depth d, then fringe size will be at least b^d at some point
  - This much space (and time) required ☹

Depth-first search



Implementing depth-first search

- Fringe can be maintained as a Last-In-First-Out (LIFO) queue (aka. a stack)
- Also easy to implement recursively:
- DFS(node)
  - If goal(node) return solution(node);
  - For each successor of node
    - Return DFS(successor) unless it is *failure*;
  - Return *failure;*

Properties of depth-first search

- Not complete (might cycle through nongoal states)
- If solution found, generally not optimal/shallowest
- If every node has b successors (the branching factor), and we search to at most depth m, fringe is at most bm
  - Much better space requirement ☺
  - Actually, generally don't even need to store all of fringe
- Time: still need to look at every node
  - $b^m + b^{m-1} + \ldots + 1$ (for $b>1$, $O(b^m)$)
  - Inevitable for uninformed search methods…

Combining good properties of BFS and DFS

- Limited depth DFS: just like DFS, except never go deeper than some depth d
- Iterative deepening DFS:
  - Call limited depth DFS with depth 0;
  - If unsuccessful, call with depth 1;
  - If unsuccessful, call with depth 2;
  - Etc.
- Complete, finds shallowest solution
- Space requirements of DFS
- May seem wasteful timewise because replicating effort
  - Really not that wasteful because **almost all effort at deepest level**
  - $db + (d-1)b^2 + (d-2)b^3 + ... + 1b^d$ is $O(b^d)$ for $b > 1$

Let's start thinking about cost

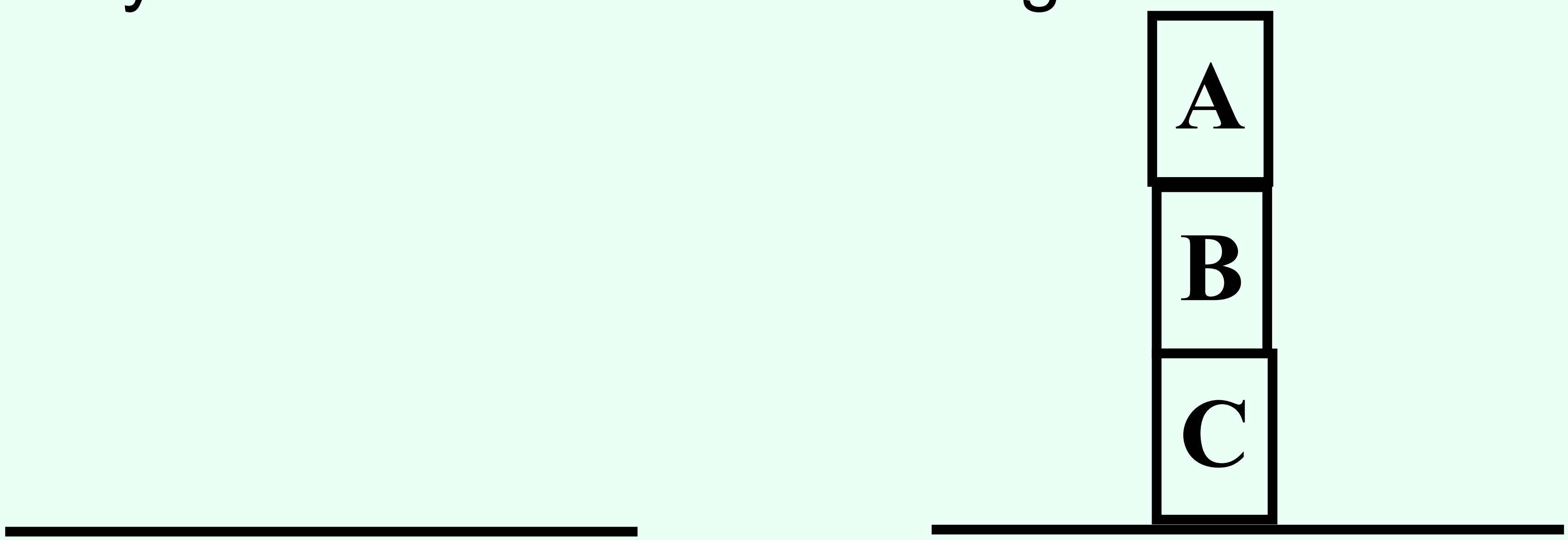
- BFS finds shallowest solution because always works on shallowest nodes first
- Similar idea: always work on the lowest-cost node first (uniform-cost search)
- Will find optimal solution (assuming costs increase by at least constant amount along path)
- Will often pursue lots of short steps first
- If optimal cost is C, and cost increases by at least L each step, we can go to depth C/L
- Similar memory problems as BFS
  - Iterative lengthening DFS does DFS up to increasing costs

Searching backwards from the goal

- Sometimes can search backwards from the goal
  - Maze puzzles
  - Eights puzzle
  - Reaching location F
  - What about the goal of “having visited all locations”?
- Need to be able to compute predecessors instead of successors
- What’s the point?

Predecessor branching factor can be smaller than successor branching factor

- Stacking blocks:
  - only action is to add something to the stack



In hand: A, B, C — *In hand: nothing*

Start state — Goal state

We'll see more of this…

Bidirectional search

- Even better: search from both the start and the goal, in parallel!

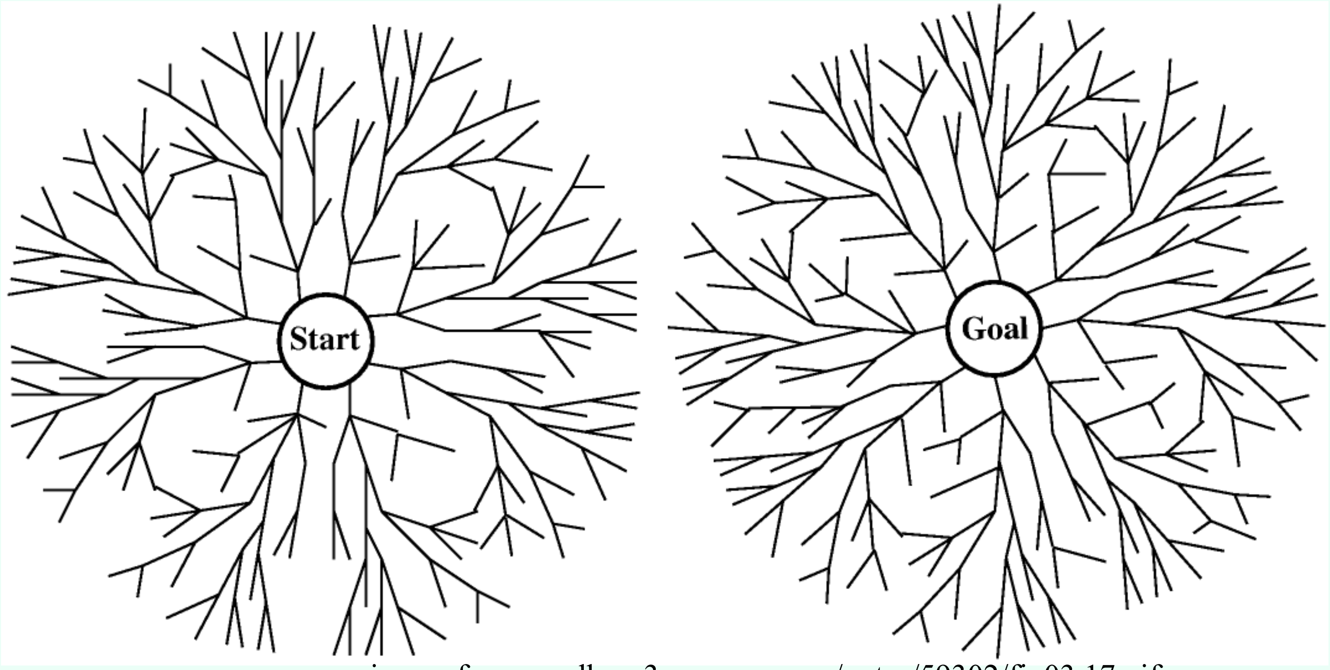


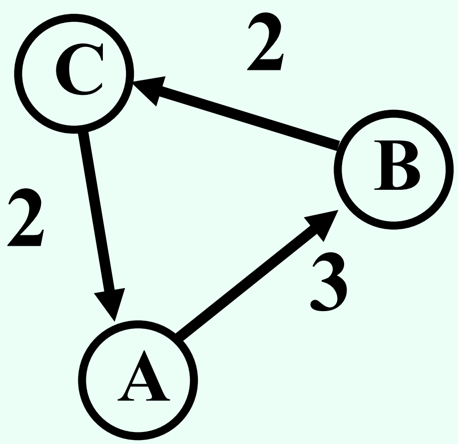
image from cs-alb-pc3.massey.ac.nz/notes/59302/fig03.17.gif

- If the shallowest solution has depth d and branching factor is b on both sides, requires only $O(b^{d/2})$ nodes to be explored!

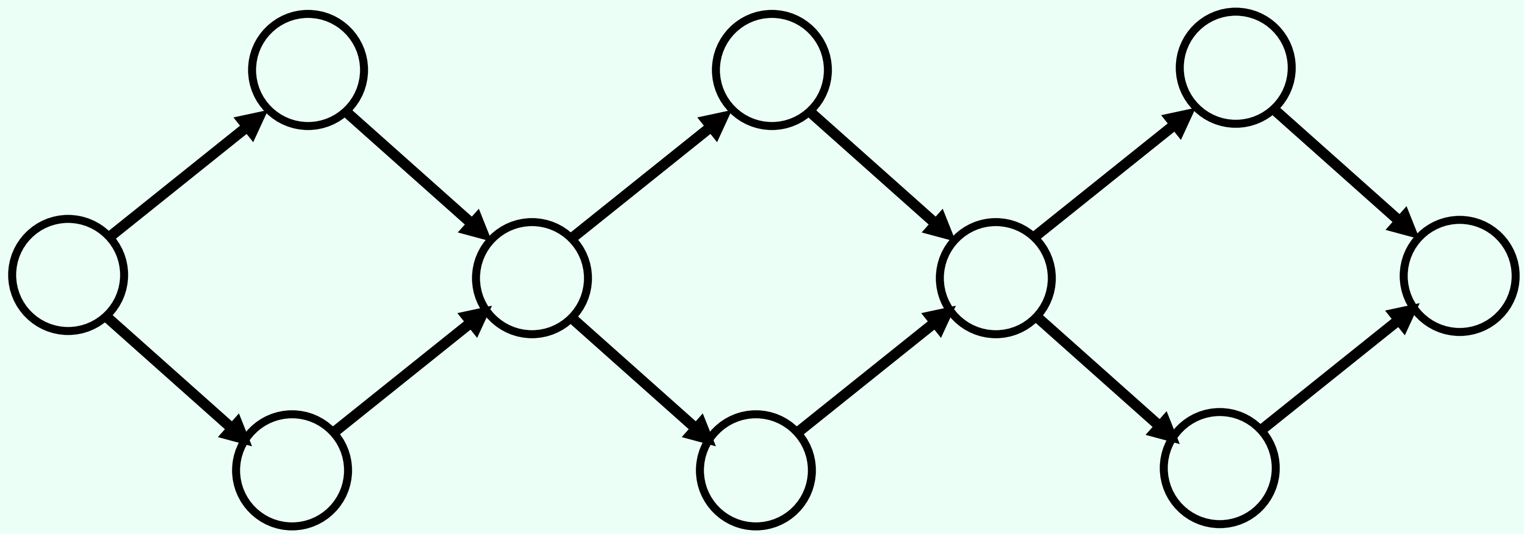
Making bidirectional search work

- Need to be able to figure out whether the fringes intersect
  - Need to keep at least one fringe in memory…
- Other than that, can do various kinds of search on either tree, and get the corresponding optimality etc. guarantees
- Not possible (feasible) if backwards search not possible (feasible)
  - Hard to compute predecessors
  - High predecessor branching factor
  - Too many goal states

Repeated states



cycles



exponentially large search trees (try it!)

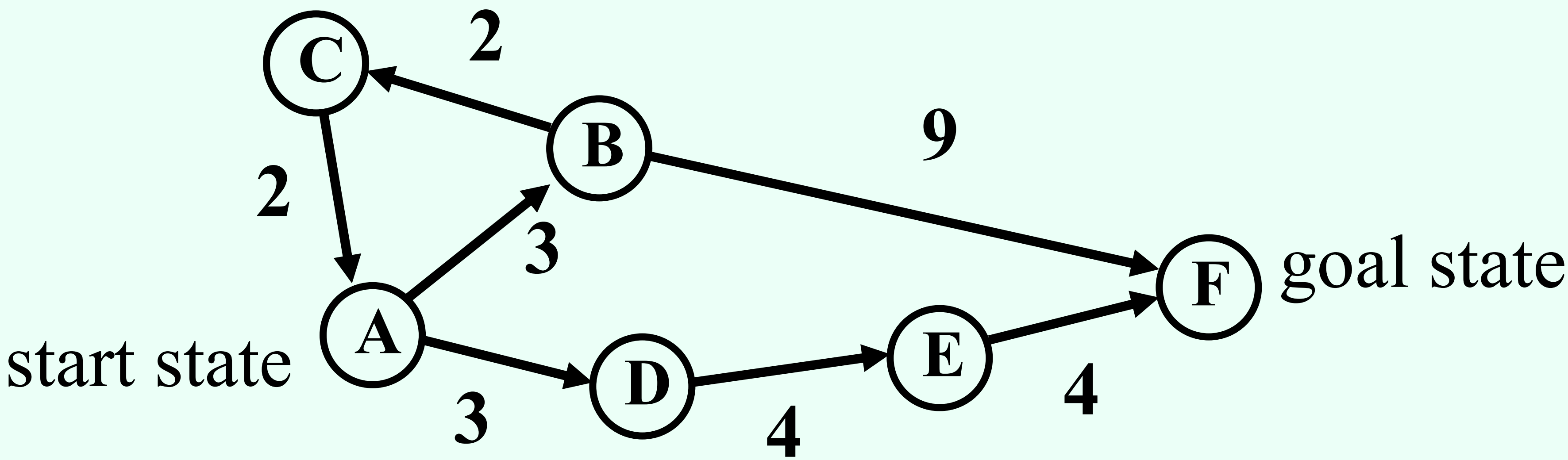
- Repeated states can cause incompleteness or enormous runtimes
- Can maintain list of previously visited states to avoid this
  - If new path to the same state has greater cost, don't pursue it further
  - Leads to time/space tradeoff
- "Algorithms that forget their history are doomed to repeat it" [Russell and Norvig]

Informed search

- So far, have assumed that no nongoal state looks better than another
- Unrealistic
  - Even without knowing the road structure, some locations seem closer to the goal than others
  - Some states of the 8s puzzle seem closer to the goal than others
- Makes sense to expand closer-seeming nodes first

Heuristics

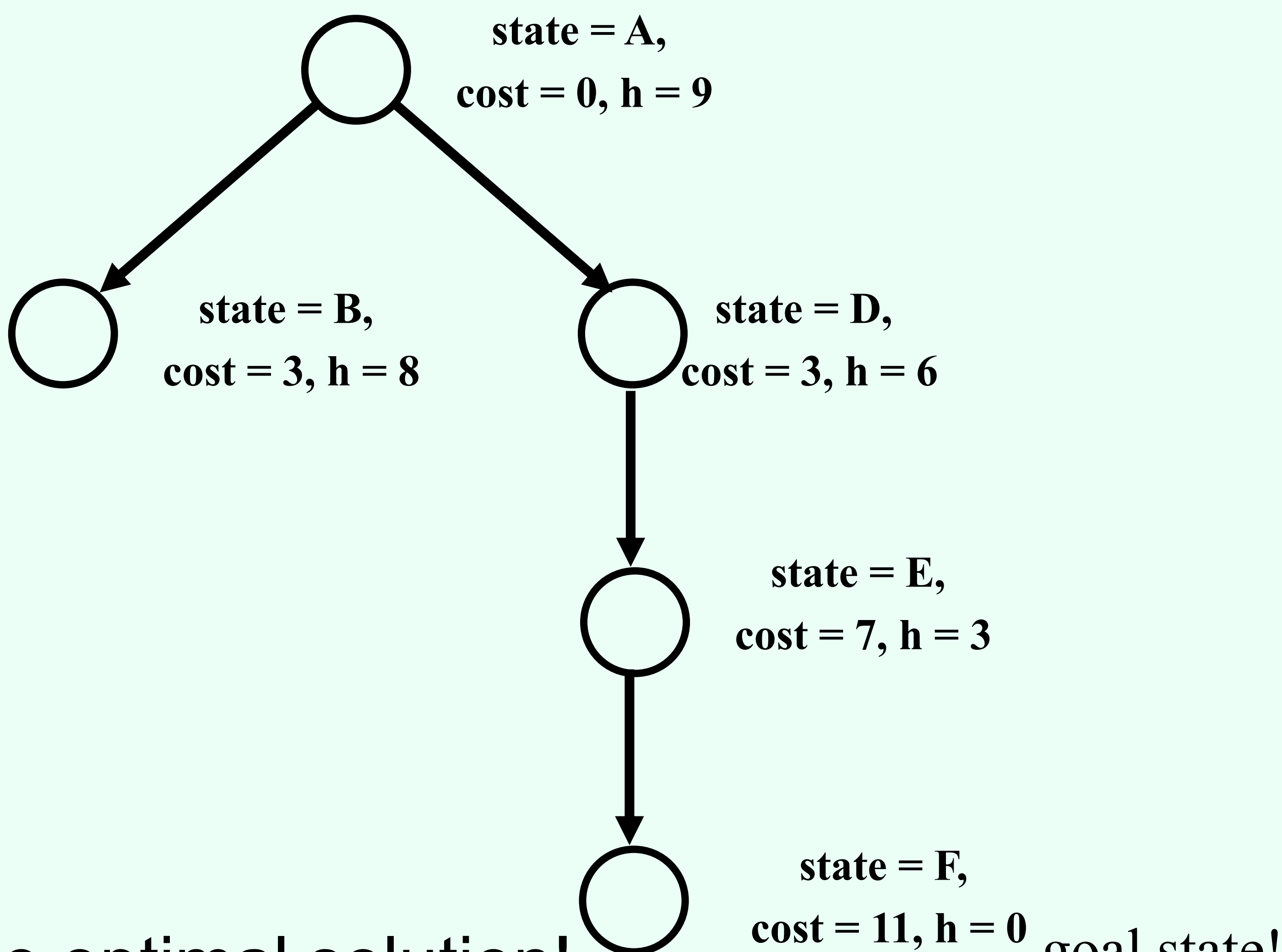
- Key notion: heuristic function h(n) gives an estimate of the distance from n to the goal
  - h(n)=0 for goal nodes
- E.g. straight-line distance for traveling problem



- Say: h(A) = 9, h(B) = 8, h(C) = 9, h(D) = 6, h(E) = 3, h(F) = 0
- We're adding something new to the problem!
- Can use heuristic to decide which nodes to expand first

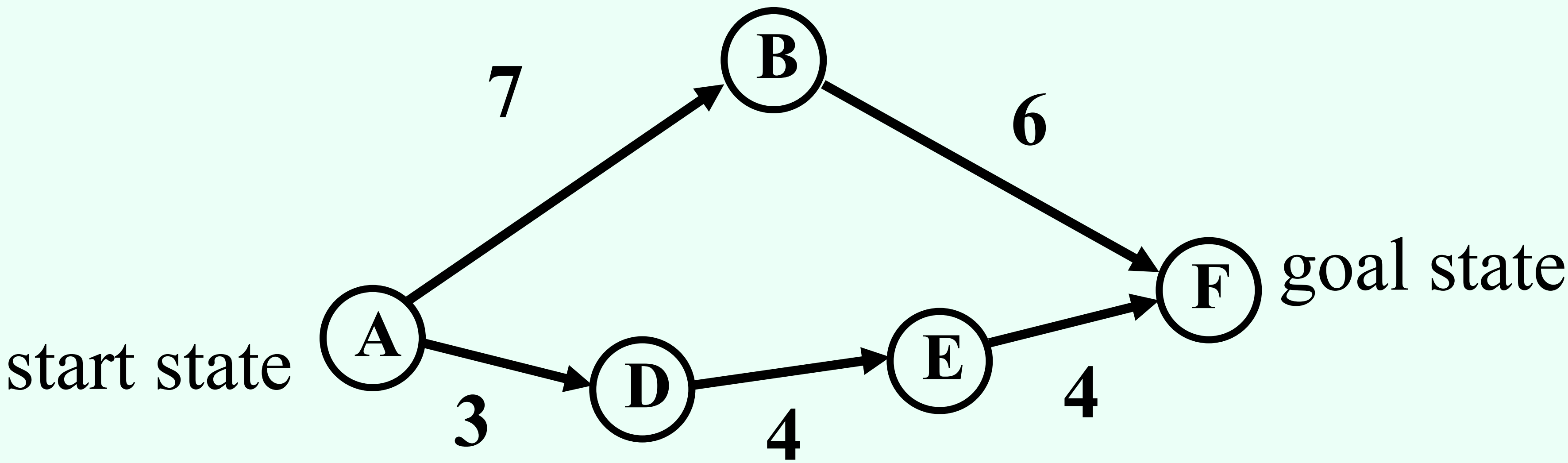
Greedy best-first search

- Greedy best-first search: expand nodes with lowest h values first



- Rapidly finds the optimal solution!
- Does it always?

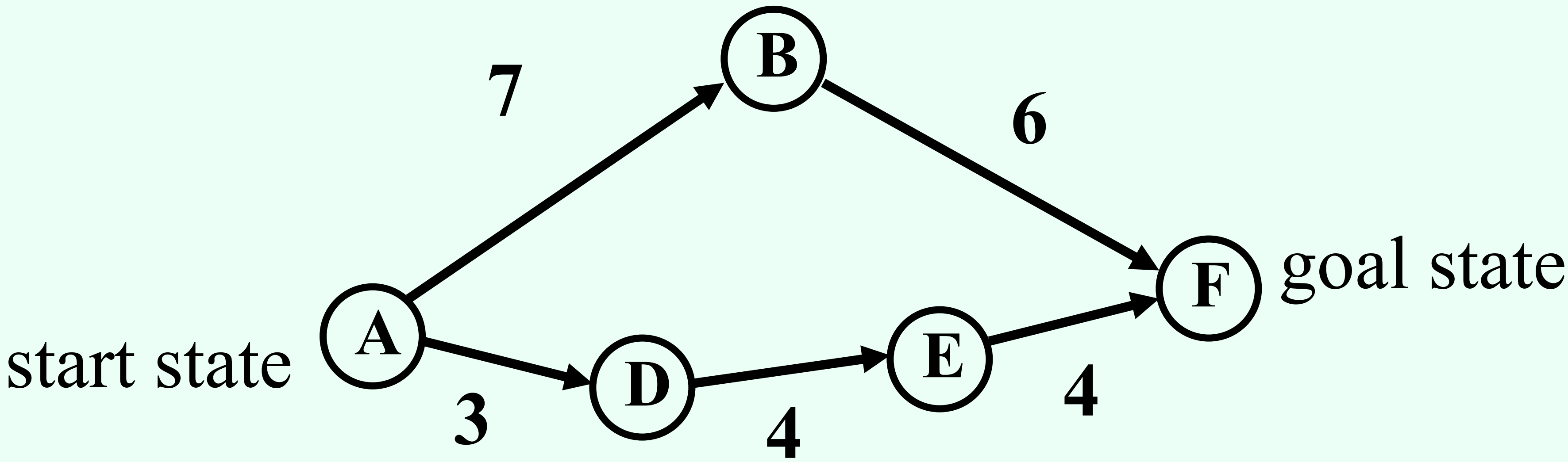
A bad example for greedy



- Say: h(A) = 9, h(B) = 5, h(D) = 6, h(E) = 3, h(F) = 0
- Problem: greedy evaluates the promise of a node only by how far is left to go, does not take cost occurred already into account

A*

- Let g(n) be cost incurred already on path to n
- Expand nodes with lowest g(n) + h(n) first



- Say: h(A) = 9, h(B) = 5, h(D) = 6, h(E) = 3, h(F) = 0

- Note: if h=0 everywhere, then just uniform cost search

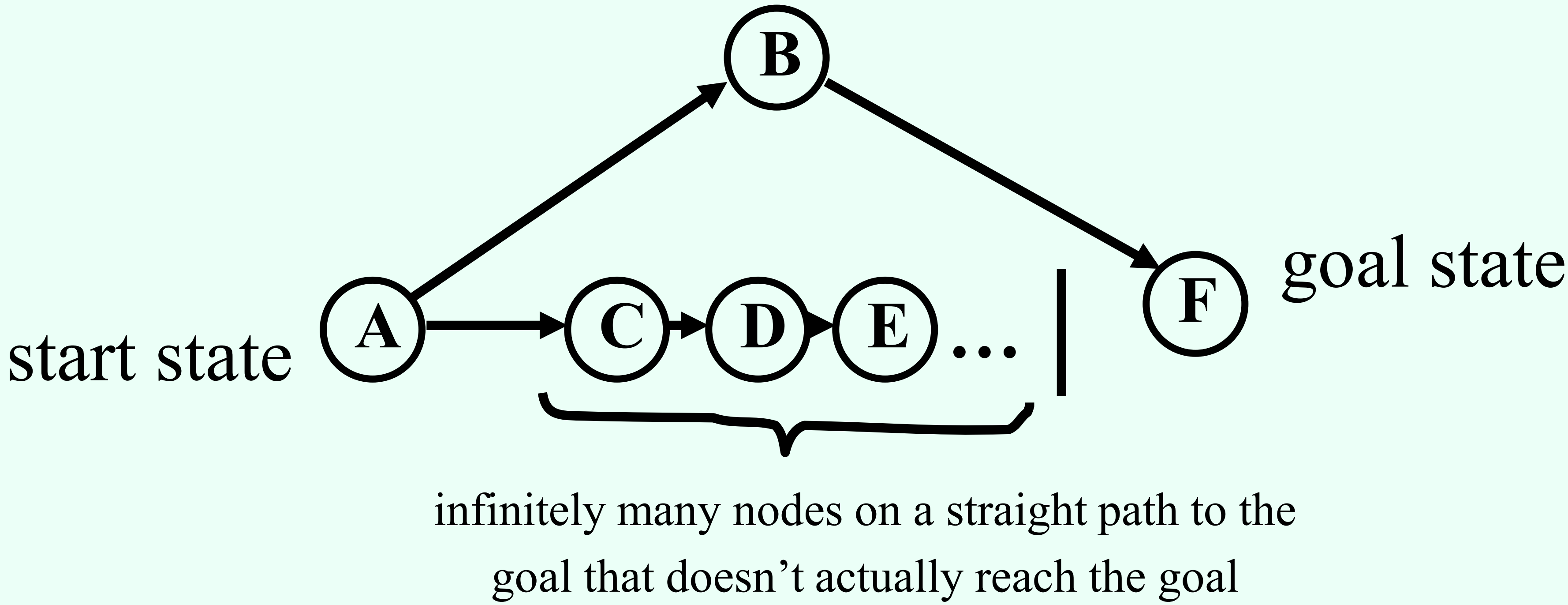
Admissibility

- A heuristic is admissible if it never overestimates the distance to the goal
  - If n is the optimal solution reachable from n', then g(n) ≥ g(n') + h(n')
- Straight-line distance is admissible: can't hope for anything better than a straight road to the goal
- Admissible heuristic means that A* is always optimistic

Optimality of A*

- If the heuristic is admissible, A* is optimal (in the sense that it will never return a suboptimal solution)
- Proof:
  - Suppose a suboptimal solution node n with solution value C > C* is about to be expanded (where C* is optimal)
  - Let n* be an optimal solution node (perhaps not yet discovered)
  - There must be some node n' that is currently in the fringe and on the path to n*
  - We have $g(n) = C > C^* = g(n^*) \geq g(n') + h(n')$
  - But then, n' should be expanded first (contradiction)

A* is not complete (in contrived examples)



- No optimal search algorithm can succeed on this example (have to keep looking down the path in hope of suddenly finding a solution)

Consistency

- A heuristic is consistent if the following holds: if one step takes us from n to n', then $h(n) \leq h(n') +$ cost of step from n to n'
  - Similar to triangle inequality
  - Equivalently, $g(n)+h(n) \leq g(n')+h(n')$
- Implies admissibility
- It's strange for an admissible heuristic not to be consistent!
  - Suppose $g(n)+h(n) > g(n')+h(n')$. Then at n', we know the remaining cost is at least $h(n)-(g(n')-g(n))$, otherwise the heuristic wouldn't have been admissible at n. But then we can safely increase $h(n')$ to this value.

A* is optimally efficient

- A* is optimally efficient in the sense that any other optimal algorithm must expand at least the nodes A* expands, if the heuristic is consistent
- Proof:
  - Besides solution, A* expands exactly the nodes with $g(n)+h(n) < C^*$ (due to consistency)
    - Assuming it does not expand non-solution nodes with $g(n)+h(n) = C^*$
  - Any other optimal algorithm must expand at least these nodes (since there may be a better solution there)
- Note: This argument assumes that the other algorithm uses the same heuristic h

A* and repeated states

- Suppose we try to avoid repeated states
- Ideally, the second (or third, …) time that we reach a state the cost is at least as high as the first time
  - Otherwise, have to update everything that came after
- This is guaranteed if the heuristic is consistent

Proof

- Suppose n and n’ correspond to same state, n’ is cheaper to reach, but n is expanded first
- n’ cannot have been in the fringe when n was expanded because $g(n') < g(n)$, so
  - $g(n') + h(n') < g(n) + h(n)$
- So n’ is generated (eventually) from some other node n’’ currently in the fringe, after n is expanded
  - $g(n) + h(n) \le g(n'') + h(n'')$
- Combining these, we get
  - $g(n') + h(n') < g(n'') + h(n'')$, or equivalently
  - $h(n'') > h(n')$ + cost of steps from n’’ to n’
    - Violates consistency

Iterative Deepening A*

- One big drawback of A* is the space requirement: similar problems as uniform cost search, BFS
- Limited-cost depth-first A*: some cost cutoff c, any node with g(n)+h(n) > c is not expanded, otherwise DFS
- IDA* gradually increases the cutoff of this
- Can require lots of iterations
  - Trading off space and time…
  - RBFS algorithm reduces wasted effort of IDA*, still linear space requirement
  - SMA* proceeds as A* until memory is full, then starts doing other things

More about heuristics

1		2
4	5	3
7	8	6

- One heuristic: number of misplaced tiles
- Another heuristic: sum of Manhattan distances of tiles to their goal location
  - Manhattan distance = number of moves required if no other tiles are in the way
- Admissible? Which is better?
- Admissible heuristic h_1 dominates admissible heuristic h_2 if $h_1(n) \geq h_2(n)$ for all n
  - Will result in fewer node expansions
- "Best" heuristic of all: solve the remainder of the problem optimally with search
  - Need to worry about computation time of heuristics…

Designing heuristics

- One strategy for designing heuristics: relax the problem (make it easier)
- *"Number of misplaced tiles"* heuristic corresponds to relaxed problem where tiles can jump to any location, even if something else is already there
- *"Sum of Manhattan distances"* corresponds to relaxed problem where multiple tiles can occupy the same spot
- Another relaxed problem: only move 1,2,3,4 into correct locations
- The ideal relaxed problem is
  - easy to solve,
  - not much cheaper to solve than original problem
- Some programs can successfully automatically create heuristics

Macro-operators

- Perhaps a more human way of thinking about search in the eights puzzle:

1	2	3
8		4
7	6	5

→ sequence of operations = macro-operation

8	2	1
7		3
6	5	4

- We swapped two adjacent tiles, and rotated everything
- Can get all tiles in the right order this way
  - Order might still be rotated in one of eight different ways; could solve these separately
- Optimality?
- Can AI think about the problem this way? Should it?