


# Relational Model and Algebra

Introduction to Databases  
CompSci 316 Fall 2020



1

## Announcements (Thu. Aug. 20)

- **Project details posted on Sakai**
  - Read it carefully!
  - Think about fixed vs. open project (some project videos from last semester will be available on sakai soon – **keep them private**)
  - Roster for discussion sessions available on sakai (teammates have to be from the same discussion session)
- **You do not have to form your teams or decide fixed/open projects right now.** Names of team members and project choice are due on 9/8, so you will have some time (and the class/discussion sections are still in flux)
- **Survey has been sent – Due by tomorrow 08/21 night EDT**
  - To know about your time zones, expectations, available resources, project / team-member preference etc.
  - Please respond on time – **there is a 2% weight for communication!**
- **Monday's discussion sessions: Installation and practice SQL**
  - Emails coming soon

2

## Today's plan

- Revisit relational model
- Simple SQL queries and its semantic
- Start relational algebra

3

## The famous “Beers” database

Your database for HW1!

4

## “Beers” as a Relational Database

See online database for more tuples

Bar	
name	address
The Edge	108 Morris Street
Satisfaction	995 W. Main Street

Serves		
bar	beer	price
The Edge	Budweiser	2.50
The Edge	Corona	3.00
Satisfaction	Budweiser	2.25

What is an example of a

- Relation
- Attribute
- Tuple
- Schema
- Instance

Beer	
name	brewer
Budweiser	Anheuser-Busch Inc.
Corona	Grupo Modelo
Dixie	Dixie Brewing

drinker	bar	times_a_week
Ben	Satisfaction	2
Dan	The Edge	1
Dan	Satisfaction	2

What is

- Set semantic
  - in relational model
- Bag semantic
  - In SQL (why)

Drinker	
name	address
Amy	100 W. Main Street
Ben	101 W. Main Street
Dan	300 N. Duke Street

drinker	beer
Amy	Corona
Dan	Budweiser
Dan	Corona
Ben	Budweiser

What is

- Set semantic
  - in relational model
- Bag semantic
  - In SQL (why)

5

## Basic queries: SFW statement

- **SELECT**  $A_1, A_2, \dots, A_n$   
**FROM**  $R_1, R_2, \dots, R_m$   
**WHERE** *condition*

In HW1, you can only use SFW
- SELECT, FROM, WHERE are often referred to as SELECT, FROM, WHERE “**clauses**”
- Each query must have a SELECT and a FROM
- WHERE is optional

6

### Example: reading a table

- `SELECT *`  
`FROM Serves`

Serves		
bar	beer	price
The Edge	Budweiser	2.50
The Edge	Corona	3.00
Satisfaction	Budweiser	2.25

- Single-table query
- \* is a shorthand for "all columns"

7

### Example: ORDER BY

- `SELECT *`  
`FROM Serves`  
`ORDER BY beer`

Serves		
bar	beer	price
The Edge	Budweiser	2.50
The Edge	Corona	3.00
Satisfaction	Budweiser	2.25

- Equivalent to "ORDER BY beer asc" (asc is default option)
- For descending order, use "desc"
- Can combine multiple orders
- What does this return?
  - `ORDER BY beer asc, price desc`

8

### Example: some columns and DISTINCT

- `SELECT beer`  
`FROM Serves`  
  
Returns a bag
- Only want unique values? Use `DISTINCT`
- `SELECT DISTINCT beer`  
`FROM Serves`  
  
Returns a set

Serves		
bar	beer	price
The Edge	Budweiser	2.50
The Edge	Corona	3.00
Satisfaction	Budweiser	2.25

9

### Example: selecting few rows

- `SELECT beer AS mybeer`  
`FROM Serves`  
`WHERE price < 2.75`
- `SELECT S.beer`  
`FROM Serves S`  
`WHERE bar = 'The Edge'`  
  
What does these return?
- `SELECT` list can contain expressions  
Can also use built-in functions such as `SUBSTR`, `ABS`, etc.
- `NOT EQUAL TO`: Use `<>`
- `LIKE` matches a string against a pattern  
`%` matches any sequence of zero or more characters

Serves		
bar	beer	price
The Edge	Budweiser	2.50
The Edge	Corona	3.00
Satisfaction	Budweiser	2.25

10

### Example: Join

- Find addresses of all bars that 'Dan' frequents
- Which tables do we need?

11

### Example: Join

- Find addresses of all bars that 'Dan' frequents
- Which tables do we need?
- How do we combine them?

Bar	
name	address
The Edge	108 Morris Street
Satisfaction	905 W. Main Street

Beer		
name	brewer	price
Budweiser	Anheuser-Busch Inc.	
Corona	Grupo Modelo	
Dixie	Dixie Brewing	

Drinker	
name	address
Amy	100 W. Main Street
Ben	101 W. Main Street
Dan	300 N. Duke Street

Serves		
bar	beer	price
The Edge	Budweiser	2.50
The Edge	Corona	3.00
Satisfaction	Budweiser	2.25

Frequents		
drinker	bar	times_a_week
Ben	Satisfaction	2
Dan	The Edge	1
Dan	Satisfaction	2

Likes	
drinker	beer
Amy	Corona
Dan	Budweiser
Dan	Corona
Ben	Budweiser

12

### Example: Join

- Find addresses of all bars that 'Dan' frequents

Bar	
name	address
The Edge	108 Morris Street
Satisfaction	905 W. Main Street

drinker	bar	times_a_week
Ben	Satisfaction	2
Dan	The Edge	1
Dan	Satisfaction	2

**Frequents**

13

### Semantics of SFW

- SELECT  $E_1, E_2, \dots, E_n$   
FROM  $R_1, R_2, \dots, R_m$   
WHERE *condition*

For each  $t_1$  in  $R_1$ :  
 For each  $t_2$  in  $R_2$ : ...  
 For each  $t_m$  in  $R_m$ :

- Apply "FROM"  
Form "cross-product" of  $R_1, \dots, R_m$

If *condition* is true over  $t_1, t_2, \dots, t_m$ :

- Apply "WHERE"  
Only consider satisfying rows

Compute and output  $E_1, E_2, \dots, E_n$  as a row

- Apply "SELECT"  
Output the desired columns

14

### Step 1: Illustration of Semantics of SFW

- NOTE: This is "NOT HOW" the DBMS outputs the result, but "WHAT" it outputs!  
Form a "Cross product" of two relations

- SELECT B.address  
FROM Bar B, Frequents F  
WHERE B.name = F.bar  
AND F.drinker = 'Dan'

Bar	
name	address
The Edge	108 Morris Street
Satisfaction	905 W. Main Street

Frequents		
drinker	bar	times_a_week
Ben	Satisfaction	2
Dan	The Edge	1
Dan	Satisfaction	2

15

### Step 2: Illustration of Semantics of SFW

- NOTE: This is "NOT HOW" the DBMS outputs the result, but "WHAT" it outputs!  
Discard rows that do not satisfy WHERE condition

- SELECT B.address  
FROM Bar B, Frequents F  
WHERE B.name = F.bar  
AND F.drinker = 'Dan'

Bar	
name	address
The Edge	108 Morris Street
Satisfaction	905 W. Main Street

Frequents		
drinker	bar	times_a_week
Ben	Satisfaction	2
Dan	The Edge	1
Dan	Satisfaction	2

16

### Step 3: Illustration of Semantics of SFW

- NOTE: This is "NOT HOW" the DBMS outputs the result, but "WHAT" it outputs!  
Output the "address" output of rows that survived

- SELECT B.address  
FROM Bar B, Frequents F  
WHERE B.name = F.bar  
AND F.drinker = 'Dan'

Bar	
name	address
The Edge	108 Morris Street
Satisfaction	905 W. Main Street

Frequents		
drinker	bar	times_a_week
Ben	Satisfaction	2
Dan	The Edge	1
Dan	Satisfaction	2

17

### Final output: Illustration of Semantics of SFW

- NOTE: This is "NOT HOW" the DBMS outputs the result, but "WHAT" it outputs!  
Output the "address" output of rows that survived

- SELECT B.address  
FROM Bar B, Frequents F  
WHERE B.name = F.bar  
AND F.drinker = 'Dan'

Bar	
name	address
The Edge	108 Morris Street
Satisfaction	905 W. Main Street

Frequents		
drinker	bar	times_a_week
Ben	Satisfaction	2
Dan	The Edge	1
Dan	Satisfaction	2

18

# SQL vs. C++, Java, Python...

19

# SQL vs. C++, Java, Python...

**SQL is declarative**

- Programmer specifies **what** answers a query should return, but **not how** the query is executed
- DBMS picks the best execution strategy based on availability of indexes, data/workload characteristics, etc.
- **Not** a “Procedural” or “Operational” language like C++, Java, Python
- There are several ways to write a query, but equivalent queries always provide the same (equivalent) results
- **SQL (+ its execution and optimizations) is based on a strong foundation of “Relational Algebra”**

20

# Relational algebra

A language for querying relational data based on “operators”

- **Core operators:**
  - Selection, projection, cross product, union, difference, and renaming
- Additional, **derived** operators:
  - Join, natural join, intersection, etc.
- Compose operators to make complex queries

21

# Selection

- Input: a table  $R$
- Notation:  $\sigma_p R$ 
  - $p$  is called a **selection condition** (or **predicate**)
- Purpose: filter rows according to some criteria
- Output: same columns as  $R$ , but only rows of  $R$  that satisfy  $p$  (set!)

Example: Find beers with price < 2.75

bar	beer	price
The Edge	Budweiser	2.50
The Edge	Corona	3.00
Satisfaction	Budweiser	2.25

$\sigma_{price < 2.75}$  Serves

No actual deletion!      Equivalent SQL query?

22

# More on selection

- Selection condition can include any column of  $R$ , constants, comparison ( $=, \leq$ , etc.) and Boolean connectives ( $\wedge$ : and,  $\vee$ : or,  $\neg$ : not)
  - Example: Serves tuples for “The Edge” or price  $\geq 2.75$ 

$$\sigma_{bar='The Edge' \vee price \geq 2.75} Serves$$
- You must be able to evaluate the condition over **each single row** of the input table!
  - Example: the most expensive beer at any bar
 
$$\sigma_{price \geq \text{every price in Serves User}} \text{User}$$
**WRONG!**

bar	beer	price
The Edge	Budweiser	2.50
The Edge	Corona	3.00
Satisfaction	Budweiser	2.25

23

# Projection

- Input: a table  $R$
- Notation:  $\pi_L R$ 
  - $L$  is a list of columns in  $R$
- Purpose: output chosen columns
- Output: same rows, but only the columns in  $L$  (set!)

Example: Find all the prices for each beer

bar	beer	price
The Edge	Budweiser	2.50
The Edge	Corona	3.00
Satisfaction	Budweiser	2.25

Output of  $\pi_{beer} Serves$ ?

24

## Cross product

- Input: two tables  $R$  and  $S$
- Notation:  $R \times S$
- Purpose: pairs rows from two tables
- Output: for each row  $r$  in  $R$  and each  $s$  in  $S$ , output a row  $rs$  (concatenation of  $r$  and  $s$ )

**Bar x Frequents**

Bar	
name	address
The Edge	108 Morris Street
Satisfaction	905 W. Main Street

Frequents		
drinker	bar	times_a_week
Ben	Satisfaction	2
Dan	The Edge	1
Dan	Satisfaction	2

Note: ordering of columns does not matter, so  $R \times S = S \times R$  (commutative)

25

## Derived operator: join

(A.k.a. “theta-join”: most general joins)

- Input: two tables  $R$  and  $S$
- Notation:  $R \bowtie_p S$ 
  - $p$  is called a **join condition** (or **predicate**)
- Purpose: relate rows from two tables according to some criteria
- Output: for each row  $r$  in  $R$  and each row  $s$  in  $S$ , output a row  $rs$  if  $r$  and  $s$  satisfy  $p$
- Shorthand for  $\sigma_p(R \times S)$

Predicate  $p$  only has equality ( $A = 5 \wedge B = 7$ ): **equijoin**

**One of the most important operations!**

26

## Join example

Ambiguous attribute?  
Use Bar.name

- Extend Frequents relation with addresses of the bars

$Frequents \bowtie_{bar=name} Bar$

Bar	
name	address
The Edge	108 Morris Street
Satisfaction	905 W. Main Street

Frequents		
drinker	bar	times_a_week
Ben	Satisfaction	2
Dan	The Edge	1
Dan	Satisfaction	2

27

## Join Types

- Theta Join
- Equi-Join
- Natural Join
- Later, (left/right) outer join, semi-join

28

## Derived operator: natural join

- Input: two tables  $R$  and  $S$
- Notation:  $R \bowtie S$  (i.e. no subscript)
- Purpose: relate rows from two tables, and
  - Enforce equality between identically named columns
  - Eliminate one copy of identically named columns
- Shorthand for  $\pi_L(R \bowtie_p S)$ , where
  - $p$  equates each pair of columns common to  $R$  and  $S$
  - $L$  is the union of column names from  $R$  and  $S$  (with duplicate columns removed)

29

## Natural join example

$Serves \bowtie Likes$

$= \pi_2(Serves \bowtie_? Likes)$

$= \pi_{bar,beer,price,drinker}(Serves \bowtie_{Serves.beer=Likes.beer} Likes)$

Serves		
bar	beer	price
The Edge	Budweiser	2.50
The Edge	Corona	3.00
Satisfaction	Budweiser	2.25

Likes	
drinker	beer
Amy	Corona
Dan	Budweiser
Dan	Corona
Ben	Budweiser

Serves $\bowtie$ Likes			
bar	beer	price	drinker
The Edge	Budweiser	2.50	Dan
The Edge	Budweiser	2.50	Ben
The Edge	Corona	3.00	Amy
The Edge	Corona	3.00	Dan
...	....	....	

Natural Join is on beer

Only one column for beer in the output

What happens if the tables have two or more common columns?

30

## Union

Important for set operations:  
Union Compatibility

- Input: two tables  $R$  and  $S$
- Notation:  $R \cup S$ 
  - $R$  and  $S$  must have identical schema
- Output:
  - Has the same schema as  $R$  and  $S$
  - Contains all rows in  $R$  and all rows in  $S$  (with duplicate rows removed)

Example on board

31

## Difference

Important for set operations:  
Union Compatibility

- Input: two tables  $R$  and  $S$
- Notation:  $R - S$ 
  - $R$  and  $S$  must have identical schema
- Output:
  - Has the same schema as  $R$  and  $S$
  - Contains all rows in  $R$  that are not in  $S$

Example on board

32

## Derived operator: intersection

Important for set operations:  
Union Compatibility

- Input: two tables  $R$  and  $S$
- Notation:  $R \cap S$ 
  - $R$  and  $S$  must have identical schema
- Output:
  - Has the same schema as  $R$  and  $S$
  - Contains all rows that are in both  $R$  and  $S$
- How can you write it using other operators?

33

What if you move  $\sigma$  to the top? Still correct? More or less efficient?

## Expression tree notation

Also called logical Plan tree

- Find addresses of all bars that 'Dan' frequents

Bar	
name	address
The Edge	108 Morris Street
Satisfaction	905 W. Main Street

Frequents		
drinker	bar	times_a_week
Ben	Satisfaction	2
Dan	The Edge	1
Dan	Satisfaction	2

Equivalent to

$$\pi_{address} (Bar \bowtie_{drinker=name} (\sigma_{drinker='Dan'} Frequents))$$

34

## Using the same relation multiple times

- Find drinkers who frequent both "The Edge" and "Satisfaction"

Frequents		
drinker	bar	times_a_week
Ben	Satisfaction	2
Dan	The Edge	1
Dan	Satisfaction	2

**WRONG!**

$$\pi_{drinker} (Frequents \bowtie_{bar='The Edge' \wedge bar='Satisfaction' \wedge drinker=drinker} Frequents)$$

**Rename!**

$$\pi_{uid_1} ( \bowtie_{b1='The Edge' \wedge b2='Satisfaction' \wedge d1=d2} \rho_{(d1,b1,t1)} Frequents \bowtie \rho_{(d2,b2,t2)} Frequents )$$

35

## Renaming

- Input: a table  $R$
- Notation:  $\rho_S R$ ,  $\rho_{(A_1,A_2,\dots)} R$ , or  $\rho_{S(A_1,A_2,\dots)} R$
- Purpose: "rename" a table and/or its columns
- Output: a table with the same rows as  $R$ , but called differently
- Used to
  - Avoid confusion caused by identical column names
  - Create identical column names for natural joins
- As with all other relational operators, it doesn't modify the database
  - Think of the renamed table as a copy of the original

36

## Summary of core operators

- Selection:  $\sigma_p R$
- Projection:  $\pi_L R$
- Cross product:  $R \times S$
- Union:  $R \cup S$
- Difference:  $R - S$
- Renaming:  $\rho_{S(A_1, A_2, \dots)} R$ 
  - Does not really add “processing” power

37

## Summary of derived operators

- Join:  $R \bowtie_p S$
- Natural join:  $R \bowtie S$
- Intersection:  $R \cap S$
- Many more
  - Semijoin, anti-semijoin, quotient, ...

38

## Exercise

Frequents(drinker, bar, times\_of\_week)  
Bar(name, address)  
Drinker(name, address)

- Bars that drinkers in address “300 N. Duke Street” do not frequent

39

## Exercise

Frequents(drinker, bar, times\_of\_week)  
Bar(name, address)  
Drinker(name, address)

- Bars that drinkers in address “300 N. Duke Street” do not frequent

40

## A trickier exercise

Frequents(drinker, bar, times\_of\_week)  
Bar(name, address)  
Drinker(name, address)

- For each bar, find the drinkers who frequent it max no. times a week

41

## A trickier exercise

Frequents(drinker, bar, times\_of\_week)  
Bar(name, address)  
Drinker(name, address)

- For each bar, find the drinkers who frequent it max no. times a week
  - Who do NOT visit a bar max no. of times?
  - Whose times\_of\_weeks is lower than somebody else's for a given bar

A deeper question:  
When (and why) is “-” needed?

42

## Monotone operators

43

Add more rows to the input...

What happens to the output?

- If some old output rows may need to be removed
  - Then the operator is **non-monotone**
- Otherwise the operator is **monotone**
  - That is, old output rows always remain “correct” when more rows are added to the input
- Formally, for a monotone operator  $op$ :  
 $R \subseteq R'$  implies  $op(R) \subseteq op(R')$  for any  $R, R'$

43

## Which operators are non-monotone?

44

- Selection:  $\sigma_p R$  Monotone
- Projection:  $\pi_L R$  Monotone
- Cross product:  $R \times S$  Monotone
- Join:  $R \bowtie_p S$  Monotone
- Natural join:  $R \bowtie S$  Monotone
- Union:  $R \cup S$  Monotone
- Difference:  $R - S$  Monotone w.r.t.  $R$ ; non-monotone w.r.t.  $S$
- Intersection:  $R \cap S$  Monotone

44

## Why is “-” needed for “highest”?

45

- Composition of monotone operators produces a **monotone query**
  - Old output rows remain “correct” when more rows are added to the input
- Is the “highest” query monotone?

45

## Extensions to relational algebra

46

- Duplicate handling (“bag algebra”)
- Grouping and aggregation
- “Extension” (or “extended projection”) to allow new column values to be computed
- (Coming later)

46