COMPSCI 590.7: Computational Microeconomics - Practice Midterm

Your name:

Please read instructions carefully. Do not worry if you cannot finish everything. Do not write down disorganized answers in the hope of getting partial credit; it's better to do a few questions completely right. Please write your answers down clearly (think before you write). You can use extra pages.
Good luck!
–Vince
Total available points: 100 points.

Problem 1: True or False (20 points).

Label each of the following statements as true or false. You are not required to give any explanation.

1. Suppose we offer someone to buy a lottery ticket for a price of \$50. The lottery ticket pays out \$100 with probability .6, and 0 otherwise. If the person does not accept our offer, then, this person cannot be an expected-utility maximizer.
2. In some games, there is a correlated equilibrium that both players prefer to all Nash equilibria.
3. It is easier to compute a correlated equilibrium than it is to compute a Nash equilibrium.
4. If there is a unique backward induction solution, then it will be the unique Nash equilibrium of that game.
5. The maximin rule (an alternative's score is its score in its worst pairwise election) satisfies the Condorcet criterion (if an alternative wins all its pairwise elections, it must win the election).
6. All positional scoring rules (where an alternative gets a number of points depending on the position in which it is ranked in the vote, e.g., plurality, veto/anti-plurality, Borda) are computationally easy to run.
7. All rules satisfying the Condorcet criterion are computationally easy to run.
8. The Dutch and first-price sealed-bid auctions are strategically completely equivalent.
9. The second-price sealed-bid auction for a single item is a special case of the Clarke (VCG) mechanism.
10. The reverse second-price sealed-bid auction for a single task (lowest bidder is awarded the task, is paid the second-lowest bid) is a special case of the Clarke (VCG) mechanism.

Problem 2: A multi-unit auction (40 points).

Let us consider an auction in which multiple (N) units of the same item are for sale, that is, the units are homogeneous. In this context, a single-minded bid by bidder i consists of a pair (n_i, v_i), where n_i is the number of units i wants, and v_i is i's valuation if she gets at least this many units (otherwise, it is 0).

a. (10 points). Consider the following bids (all from different bidders) in an auction with 18 units: $(9, 63), (8, 56), (7, 49), (6, 42), (5, 35)$. **Determine** which bidders win, and also **determine** what their VCG (Clarke) payments are.

b (10 points). Write a simple integer program for the winner determination problem in this context (with single-minded bidders).

c (10 points). Recall that in the (NP-hard) KNAPSACK problem, we are in a room full of treasure; each object o has a weight w_o and a value v_o. We can only carry a total weight of up to W with us, and we wish to maximize the total value that we take with us. **Discuss** the relationship between the KNAPSACK problem and the winner determination problem from **b**. **Is** our winner determination problem NP-hard?

d (10 points). In reality, it is unlikely that bidders are single-minded. Rather, bidder i will have a separate value $v_i(k)$ for obtaining k units, for each k. Suppose that each bidder i's function v_i is concave, that is, for all i, k, we have $v_i(k+2) - v_i(k+1) \leq v_i(k+1) - v_i(k)$ (in other words, the marginal value of a unit is nonincreasing in the number of units a bidder already has). **Give** a natural algorithm/procecure for solving the winner determination problem for such concave functions. Also, **describe** how the algorithm can be used as an elicitation algorithm that asks value queries (queries of the form, "What is your value for getting k units?").

Problem 3: A Bayesian version of matching pennies (40 points).

In this problem, we are going to combine the game of matching pennies with our two-card deck (King and Jack) from the poker game in class. The game will be non-zero-sum. It works as follows. Player 1 draws a card (King or Jack), which Player 2 does not see. Actually, the deck is biased: the probability of drawing the King is 0.6 (the Jack, 0.4). Then the players play matching pennies, but the card influences Player 1's utility for winning. Player 1 gets utility 2 if:

- she drew a King and both players played Heads;
- she drew a Jack and both players played Tails.

Player 1 gets utility 1 if:

- she drew a King and both players played Tails;
- she drew a Jack and both players played Heads.

Player 2 gets 0 in all of these cases.

If the players did not play the same thing, then Player 1 gets utility 0, and Player 2 gets utility 1.

a. (10 points). Draw the extensive form of the game (first, Nature draws the King or the Jack for Player 1; then, Player 1 plays (observing her card); then, Player 2 plays (not observing anything)). If you have more than two nodes in an information set, you can connect them with a single dotted line.

b. (10 points). Convert the game to normal form. Please do this carefully. (Also, please make Player 1 the row player.) An example to check that you're doing it right: If Player 1 plays the pure strategy HT (Heads on King, Tails on Jack), and Player 2 plays T, then the utilities are (.8, .6) (.4 of the time Player 1 draws the Jack, plays T, and gets 2; .6 of the time Player 1 draws the King, plays H, so Player 2 gets 1).

c. (10 points). Which pure strategies (if any) are strictly dominated?

d. (10 points). Solve for the Nash equilibrium of this game. (Hint: Player 1 will randomize over two pure strategies. One is a strategy that should look good intuitively, because it does well in all cases; the other one is mixed in to keep Player 2 indifferent between H and T.)