

Utility theory

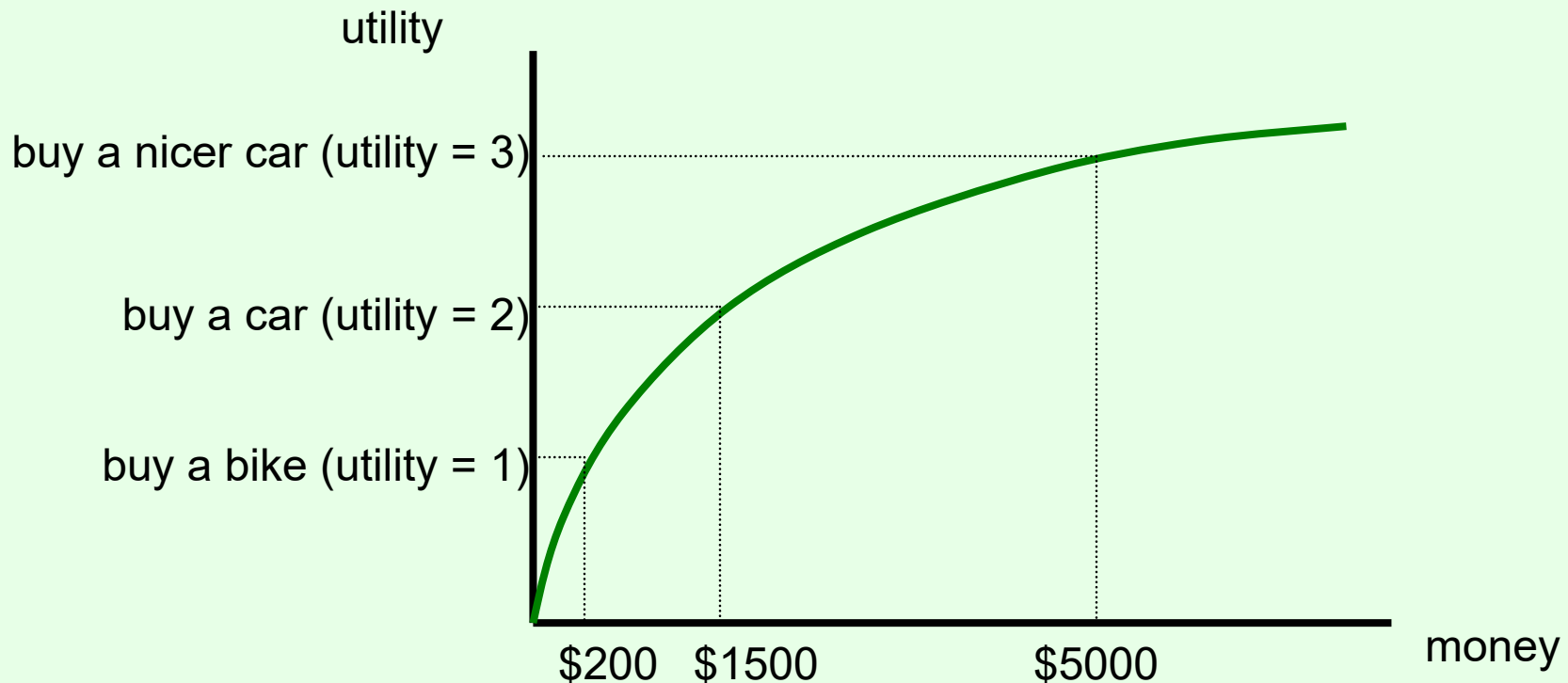
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Risk attitudes

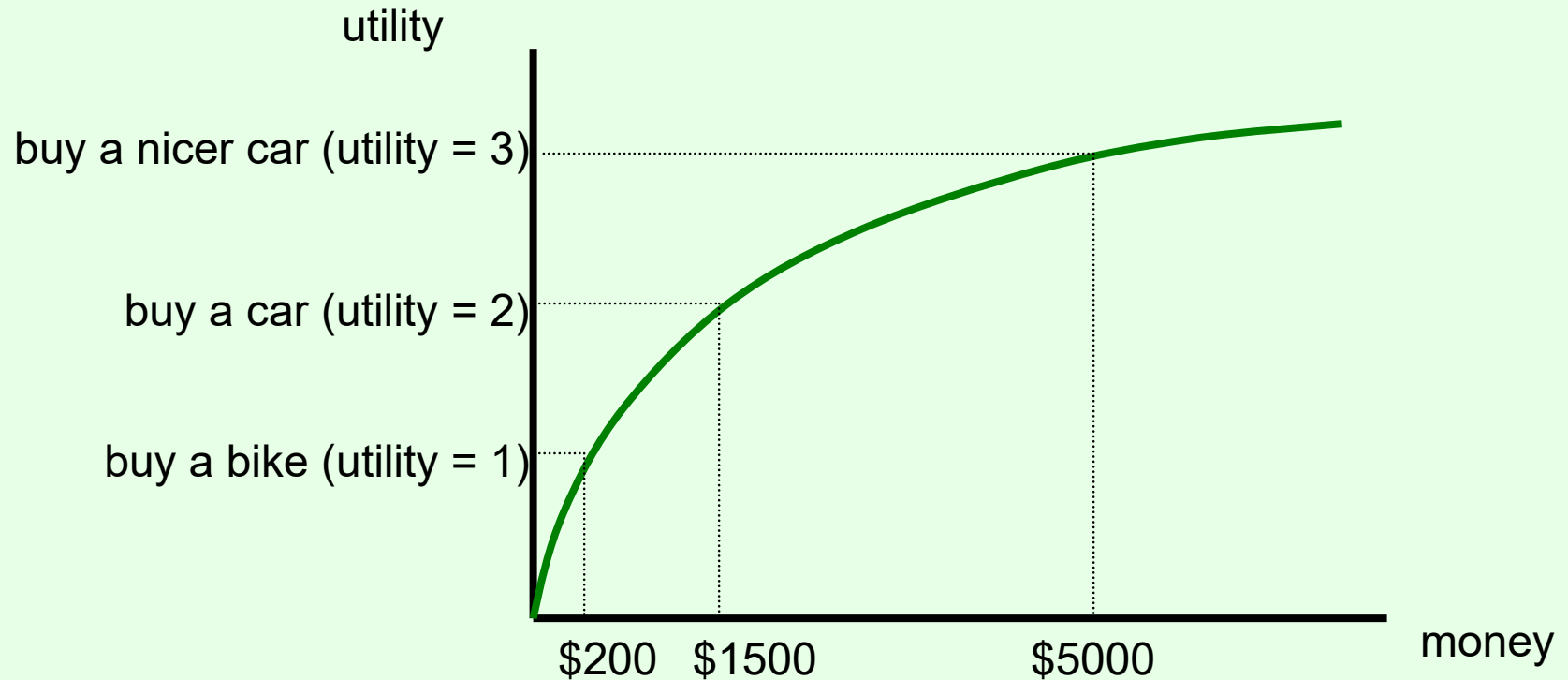
- Which would you prefer?
 - A lottery ticket that pays out \$10 with probability .5 and \$0 otherwise, or
 - A lottery ticket that pays out \$3 with probability 1
- How about:
 - A lottery ticket that pays out \$100,000,000 with probability .5 and \$0 otherwise, or
 - A lottery ticket that pays out \$30,000,000 with probability 1
- Usually, people do not simply go by expected value
- An agent is **risk-neutral** if she only cares about the expected value of the lottery ticket
- An agent is **risk-averse** if she always prefers the expected value of the lottery ticket to the lottery ticket
 - Most people are like this
- An agent is **risk-seeking** if she always prefers the lottery ticket to the expected value of the lottery ticket

Decreasing marginal utility

- Typically, at some point, having an extra dollar does not make people much happier (**decreasing marginal utility**)

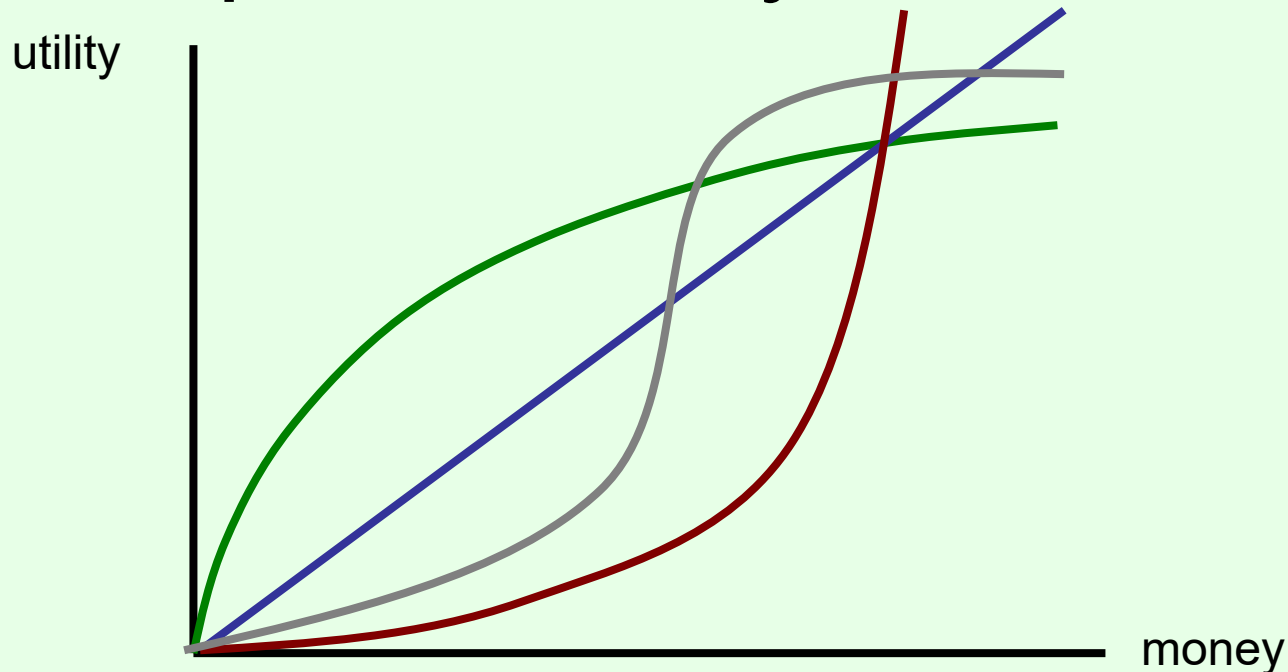


Maximizing expected utility



- Lottery 1: get \$1500 with probability 1
 - gives expected utility 2
- Lottery 2: get \$5000 with probability .4, \$200 otherwise
 - gives expected utility $.4*3 + .6*1 = 1.8$
 - (expected amount of money = $.4*\$5000 + .6*\$200 = \$2120 > \1500)
- So: maximizing expected utility is consistent with risk aversion

Different possible risk attitudes under expected utility maximization



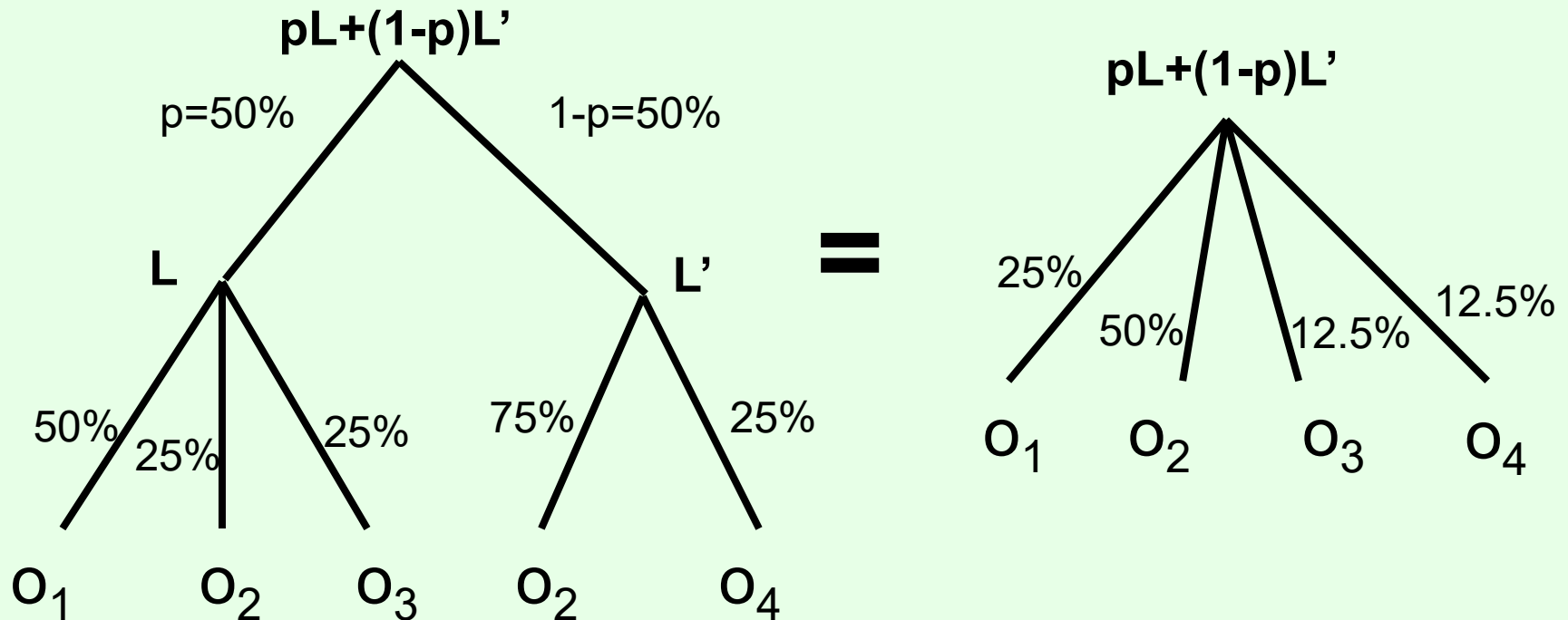
- **Green** has decreasing marginal utility → risk-averse
- **Blue** has constant marginal utility → risk-neutral
- **Red** has increasing marginal utility → risk-seeking
- **Grey**'s marginal utility is sometimes increasing, sometimes decreasing → neither risk-averse (everywhere) nor risk-seeking (everywhere)

What is utility, anyway?

- Function $u: O \rightarrow \mathfrak{R}$ (O is the set of “outcomes” that lotteries randomize over)
- What are its units?
 - It doesn’t really matter
 - If you replace your utility function by $u'(o) = a + bu(o)$, your behavior will be unchanged
- Why would you want to maximize expected utility?
 - This is a question about **preferences over lotteries**

Compound lotteries

- For two lottery tickets L and L' , let $pL + (1-p)L'$ be the “compound” lottery ticket where you get lottery ticket L with probability p , and L' with probability $1-p$



Sufficient conditions for expected utility

- $L \geq L'$ means that L is (weakly) preferred to L'
 - (\geq should be complete, transitive)
- **Expected utility theorem.** Suppose
 - (continuity axiom) for all L, L', L'' , $\{p: pL + (1-p)L' \geq L''\}$ and $\{p: pL + (1-p)L' \leq L''\}$ are closed sets, and
 - (independence axiom – more controversial) for all $L, L', L'', p > 0$, we have $L \geq L'$ if and only if $pL + (1-p)L'' \geq pL' + (1-p)L''$

Then, there exists a function $u: O \rightarrow \mathfrak{R}$ so that $L \geq L'$ if and only if L gives a higher expected value of u than L'