Due Date: September 16, 11:59pm

In all problems, prove the correctness of your algorithm and analyze its running time.

**Problem 1:** [10pts] Given an undirected graph $G = (V, E)$, describe an algorithm to detect whether $G$ has a cycle of odd length.

**Problem 2:** [10pts] Given an undirected graph $G = (V, E)$, for a vertex $v \in V$, $G - v$ is obtained by removing $v$ and its incident edges from $G$. Describe an $O(|V| + |E|)$ time algorithm to find a vertex $v \in V$ such that $G - v$ consists of more than one connected components, or report that no such vertex exists.

**Problem 3:** [10pts] Let $G = (V, E)$ be a directed graph with integral and non-negative edge weights. Suppose we have $s, t \in V$ such that the distance from $s$ to $t$ is $O(|V|)$. Describe an $O(|V| + |E|)$ time algorithm to find the length of the shortest path from $s$ to $t$. 